## SOLUTIONS

# PROGRESS TEST-8 

 GZRA
## JEE MAIN PATTERN <br> Test Date: 10-12-2017



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## PHYSICS

1. (C)
$\Delta p=2 m v \sin 45^{\circ}=2 m v \frac{1}{\sqrt{2}}=\sqrt{2} m v$

2. (A)
$u^{2}=5 g R$
$\therefore \mathrm{v}^{2}=\mathrm{u}^{2}-2 \mathrm{gR}$
$=5 g R-2 g R=3 g R$


Tangential acceleration at B is

$$
a_{\mathrm{t}}=\mathrm{g} \text { (downwards) }
$$

Centripetal acceleration at $B$ is
$\mathrm{a}_{\mathrm{C}}=\frac{\mathrm{v}^{2}}{\mathrm{R}}=3 \mathrm{~g}$
$\therefore$ Total acceleration will be
$a=\sqrt{a_{C}^{2}+a_{t}^{2}}=g \sqrt{10}$
3. (D)
4. (B)


At: A

$\mathrm{T}^{\prime} \cos \beta=\mathrm{mg}$

$$
\mathrm{T}^{\prime}=\frac{\mathrm{mg}}{\cos \beta}
$$

At: B


$$
\mathrm{T}^{\prime \prime}-\mathrm{mg} \cos \beta=\frac{\mathrm{mv}^{2}}{\ell}
$$

$$
\text { at } B ; \quad v=0
$$

$$
\mathrm{T}^{\prime \prime}=\mathrm{mg} \cos \beta
$$

$$
\frac{\mathrm{T}^{\prime \prime}}{\mathrm{T}^{\prime}}=\frac{\mathrm{mg} \cos \beta}{\mathrm{mg} / \cos \beta}
$$

$$
\frac{\text { Tensionat } \mathrm{B}}{\text { Tension at } \mathrm{A}}=\frac{\mathrm{T}^{\prime \prime}}{\mathrm{T}^{\prime}}=\cos ^{2} \beta
$$

5. Let $x$ be the distance between the particles after $t$ seconds.

Then

$$
\begin{equation*}
x=v t-\frac{1}{2} a t^{2} \tag{i}
\end{equation*}
$$

For $x$ to be maximum ,

$$
\frac{\mathrm{dx}}{\mathrm{dt}}=0 \quad \text { or } \quad \mathrm{t}=\frac{\mathrm{v}}{\mathrm{a}}
$$

From (i), we get

$$
x=\frac{v^{2}}{2 a}
$$

$$
\therefore \quad \text { (b) }
$$

6. Maximum acceleration of slab can be

$$
\mathrm{a}=0.6 \times 10 \times 9.8=\frac{6 \times 9.8}{40}=\frac{58.8}{40}=1.47 \mathrm{~ms}^{-2}
$$

Hence block over slab will slip and $a=\frac{0.4 \times 10 \times 9.8}{40}=0.98 \mathrm{~ms}^{-2}$
$\therefore$ (A)
7. (D)

Let N be the normal reaction between m and M ,
Equilibrium of M
$N \sin 45^{\circ}=k x$
Equilibrium of $m$ in vertical direction gives

$$
\begin{equation*}
\mathrm{N} \cos 45^{\circ}=\mathrm{mg} \tag{ii}
\end{equation*}
$$

From Eqs. (i) and (ii), we get

$$
x=\frac{m g}{\mathrm{k}}
$$

8. (C)

The work done by man is negative of magnitude of decrease in potential energy of chain.

9. (B)

$$
\mathrm{W}=\int_{(0,0)}^{(1,1)} \overrightarrow{\mathrm{F}} \cdot \overrightarrow{\mathrm{~d}} \mathrm{x}
$$

Here $\overrightarrow{\mathrm{d}} \mathrm{s}=\mathrm{dx} \hat{\mathrm{i}}+\mathrm{dy} \hat{\mathrm{j}}+\mathrm{dz} \hat{\mathrm{k}}$

$$
\therefore \quad W=\int_{(0,0)}^{(1,1)}\left(x^{2} d y+y d x\right)
$$

$$
\begin{aligned}
& =\int_{(0,0)}^{(1,1)}\left(y^{2} d y+x \cdot d x\right)(\text { as } x=y) \\
\therefore & W=\left[\frac{y^{3}}{3}+\frac{x^{2}}{2}\right]_{(0,0)}^{(1,1)}=\frac{5}{6} J
\end{aligned}
$$

10. (C)

$$
\text { K.E. }=F x ; \quad P=\left(\frac{F^{2}}{m}\right) t ; \quad K=\left(\frac{F^{2}}{2 m}\right) t^{2}
$$

11. (C)

$$
P=F v
$$

$$
\text { or } \mathrm{P}=\left(\mathrm{mv} \frac{\mathrm{dv}}{\mathrm{dx}}\right) \mathrm{v}
$$

$$
\text { or } \int_{0}^{\mathrm{v}} \mathrm{v}^{2} \mathrm{dv}=\int_{0}^{\mathrm{x}} \frac{\mathrm{P}}{\mathrm{~m}} \mathrm{dx}
$$

$$
\frac{v^{3}}{3}=\frac{P x}{m} \text { or } v=\left(\frac{3 P x}{m}\right)^{1 / 3}
$$

12. (D)


After some time friction becomes more than mgsin$\theta$, then body will retard. Thus speed is maximum when, total force or acc. is zero.
$m g \sin \theta-\mu m g \cos \theta=0$
$\Rightarrow \mu=\tan \theta \Rightarrow 0.3 x=3 / 4$
$\Rightarrow \mathrm{x}=2.5 \mathrm{~m}$
13. (D)
$\mathrm{H}=\frac{\mathrm{u}_{1}^{2} \sin ^{2} \theta_{1}}{2 \mathrm{~g}}=\frac{\mathrm{u}_{2}^{2} \sin ^{2} \theta_{2}}{2 \mathrm{~g}}$
14. (A)
15. (A)
16. (D)

For a mole of an ideal gas, the equation of state is $P V=R T$
or $T=\frac{P V}{R}$
which is proportional to the product PV
At $x, P V=\left(4 \times 10^{5}\right)\left(1 \times 10^{-4}\right)=40 \mathrm{Nm}$
At $y, P V=\left(1 \times 10^{5}\right)\left(5 \times 10^{-4}\right)=50 \mathrm{Nm}$
At $z, P V=\left(1 \times 10^{5}\right)\left(1 \times 10^{-4}\right)=10 \mathrm{Nm}$
Thus, $T$ is maximum at $y$ since $P V$ is the highest and $T$ is minimum at $z$ since $P V$ is the smallest
17. (B)

$\mathrm{P}=\frac{2 \mathrm{Kx}}{\mathrm{A}}$
$=\frac{2 \times 100}{1} \times \frac{1}{2}$
$=100 \mathrm{~N} / \mathrm{m}^{2}$
18. (B)
$U=U_{0} \mathrm{~V} \Rightarrow \mathrm{nC}_{\mathrm{V}} \mathrm{T}=\mathrm{U}_{0} \mathrm{~V} \Rightarrow \mathrm{~T} \propto \mathrm{~V}$ isobaric process
$\mathrm{C}=\mathrm{C}_{\mathrm{V}}+\frac{\mathrm{P}}{\mathrm{n}} \frac{\mathrm{dV}}{\mathrm{dT}}$
$\frac{\mathrm{dV}}{\mathrm{dT}}=$ constant
$\frac{\mathrm{P}}{\mathrm{n}}=\frac{\mathrm{RT}}{\mathrm{V}}=\frac{\mathrm{RT}}{\operatorname{constant} \mathrm{T}}$
$\mathrm{C}=\mathrm{C}_{\mathrm{V}}+\frac{\mathrm{R}}{\text { constant }}$
$C=C_{V}+R=\frac{5}{2} R+R=\frac{7}{2} R$
19. (C)
$\Delta \mathrm{Q}=\mathrm{nC}_{\mathrm{P}} \Delta \mathrm{T}=\frac{7}{2} \mathrm{nR} \Delta \mathrm{T} \quad\left(\mathrm{C}_{\mathrm{P}}=\frac{7}{2} \mathrm{R}\right)$

$$
\Delta U=n C_{V} \Delta T=\frac{5}{2} n R \Delta T
$$

$$
\left(\mathrm{C}_{\mathrm{V}}=\frac{5}{2} \mathrm{R}\right)
$$

and $\Delta \mathrm{W}=\Delta \mathrm{Q}-\Delta \mathrm{U}=\mathrm{nR} \Delta \mathrm{T}$
$\therefore \Delta \mathrm{Q}: \Delta \mathrm{U}: \Delta \mathrm{W}=7: 5: 2$
20. (D)

Heat released by water
$\Delta \mathrm{Q}=80 \times 1 \times 30=2400 \mathrm{cal}$
Mass of Ice melt

$$
\begin{gathered}
2400=\mathrm{m} \times 80 \quad[\Delta \mathrm{Q}=\mathrm{mL}] \\
\therefore \mathrm{m}=\frac{2400}{80}=30 \mathrm{gm}
\end{gathered}
$$

21. (B)
22. (A) $x \times 540=y \times 80+y \times 1 \times 100$

$$
\Rightarrow 540 x=180 y \quad \text { or } \quad \frac{x}{y}=\frac{1}{3}
$$

23. (C)
$(3 \mathrm{~L}) \alpha_{\text {eff }} \Delta \theta=\mathrm{L} \alpha \Delta \theta+2 \mathrm{~L}(2 \alpha)(\Delta \theta)$
$\therefore \alpha_{\text {eff }}=\frac{5}{3} \alpha$
24. (B)

$$
\begin{aligned}
\frac{\mathrm{mL}}{\mathrm{t}_{1}} & =\frac{\mathrm{K}_{1} \mathrm{~A}\left(\mathrm{~T}_{1}-\mathrm{T}_{2}\right)}{\mathrm{L}} \\
\frac{\mathrm{~mL}}{\mathrm{t}_{2}} & =\frac{\mathrm{K}_{2} \mathrm{~A}\left(\mathrm{~T}_{1}-\mathrm{T}_{2}\right)}{\mathrm{L}} \\
\frac{\mathrm{~K}_{1}}{\mathrm{~K}_{2}} & =\frac{\mathrm{t}_{2}}{\mathrm{t}_{1}}
\end{aligned}
$$

25. (D)
$\tau \propto \frac{\mathrm{Kr}^{2}}{\ell}$
$\therefore \quad \tau_{1}=\tau_{2}$
$\frac{\mathrm{K}_{1} \mathrm{r}_{1}^{2}}{\ell_{1}}=\frac{\mathrm{K}_{2} \mathrm{r}_{2}^{2}}{\ell_{2}}$
26. (D)
$N=\frac{P V}{K T}$
$\frac{\mathrm{N}_{\mathrm{A}}}{\mathrm{N}_{\mathrm{B}}}=\frac{\mathrm{PV}}{\mathrm{KT}} \times \frac{\mathrm{K} 2 \mathrm{~T}}{2 \mathrm{P}(\mathrm{V} / 4)}=\frac{4}{1}$
27. (C)

$$
\begin{aligned}
& g_{\text {eff }}=g+a=\frac{2 u}{t} \\
& \Rightarrow a=\frac{2 u}{t}-g=\frac{2 u-g t}{t}
\end{aligned}
$$

28. (A)
$\frac{3}{4}$ th energy is lost i.e., $\frac{1}{4}$ th kinetic energy is left. Hence, its velocity becomes $\frac{\mathrm{v}_{0}}{2}$ under a retardation of $\mu \mathrm{g}$ in time $\mathrm{t}_{0}$.
$\therefore \frac{\mathrm{v}_{0}}{2}=\mathrm{v}_{0}-\mu \mathrm{g}_{0}$
or $\mu \mathrm{g} \mathrm{t}_{0}=\frac{\mathrm{v}_{0}}{2}$ or $\mu=\frac{\mathrm{v}_{0}}{2 \mathrm{~g} \mathrm{t}_{0}}$
29. 

(B) $\frac{\frac{1}{2} \mathrm{mu}_{1}^{2} \cos ^{2} \theta_{1}}{\frac{1}{2} m u_{2}^{2} \cos ^{2} \theta_{2}}=\frac{4}{1}$
$\Rightarrow \frac{\mathrm{u}_{1} \cos \theta_{1}}{\mathrm{u}_{2} \cos \theta_{2}}=2$
and $\frac{\mathrm{u}_{1}^{2} \sin ^{2} \theta_{1}}{\mathrm{u}_{2}^{2} \sin ^{2} \theta_{2}}=\frac{4}{1}$
or $\frac{\mathrm{u}_{1} \sin \theta_{1}}{\mathrm{u}_{2} \sin \theta_{2}}=\frac{2}{1}$
from equation no. (1) and (2)

$$
\frac{\mathrm{u}_{1} \sin \theta_{1} \cdot \mathbf{u}_{1} \cos \theta_{1}}{\mathrm{u}_{2} \sin \theta_{2} \cdot \mathbf{u}_{2} \cos \theta_{2}}=\frac{4}{1}
$$

30. (C) $\mathrm{T}_{\text {mix }}=\frac{\mathrm{m}_{1} \mathrm{~s}_{1} \mathrm{~T}_{1}+\mathrm{m}_{2} \mathrm{~s}_{2} \mathrm{~T}_{2}}{\mathrm{~m}_{1} \mathrm{~s}_{1}+\mathrm{m}_{2} \mathrm{~s}_{2}}$

## CHEMISTRY

31. (D)
32. (C)
33. (A)
34. (D)
35. (B)

The velocity corresponding to the maxima is the most probable velocity which is given by the expression.
$C_{m p}=\sqrt{\frac{2 R T}{M}}$
$R T=\frac{\mathrm{C}_{\mathrm{mp}}^{2} \mathrm{M}}{2}=\frac{(200)^{2} \times 100 \times 10^{-3}}{2}=40000 \times 50 \times 10^{-3} \mathrm{~J} \mathrm{~mol}^{-1}=40 \times 50 \mathrm{~J} \mathrm{~mol}^{-1}$
No. of moles $=\frac{300}{100}=3$
$E=\frac{3}{2} n R T=\frac{3}{2} \times 3 \times 40 \times 50=9 \times 10^{3} \mathrm{~J}=9 \mathrm{~kJ}$
36. (D)
37. (B)

$$
\mathrm{V}_{\text {real }}=\frac{\text { Molar mass }}{\text { density }}=\frac{18}{0.36} .
$$

$$
V_{\text {ideal }}=\frac{\mathrm{nRT}}{\mathrm{P}}=\frac{1 \times 0.082 \times 500}{1} .
$$

$$
\text { So, } Z=\frac{V_{\text {real }}}{V_{\text {ideal }}}=\frac{50}{0.082 \times 500}=\frac{50}{41} .
$$

38. (B)


During deexcitation emission of light results
39. (A)
40. (C)
$\lambda=\frac{\mathrm{h}}{\mathrm{mv}} \lambda=\frac{\mathrm{h}}{\sqrt{2 \mathrm{mKE}}}$
$\lambda_{\text {req }}=\frac{\mathrm{h}}{\sqrt{\mathrm{Zm} 9 \mathrm{KE}}}=\frac{1}{3} \frac{\lambda}{\sqrt{2 \mathrm{mKG}}}=\frac{\lambda}{3}$
41. (C)


1,4,4-Trimethylcyclobutene
42. (D)
43. (B)

44. (A)

45. (C)

46. (A)

47. (B)

48. (C)
49. (C)

50. (C)
51. (C)
52. (B)
53. (C)

54. (A)
(IV) $\mathrm{LiClO}_{4}>\mathrm{NaClO}_{4}>\mathrm{KClO}_{4}>\mathrm{RbClO}_{4}>\mathrm{CsClO}_{4}$
55. (A)
(iii) $\mathrm{C}_{12} \mathrm{O}_{9}-\mathrm{sp}^{2}$;

(iv) $\mathrm{N}_{3} \mathrm{P}_{3} \mathrm{Cl}_{6}-\mathrm{sp}^{2} \& \mathrm{sp}^{3}$;

56. (A)
(i) $\mathrm{LiF}>\mathrm{NaF}>\mathrm{KF}>\mathrm{RbF}$ : Lattice energy
(iii) $\mathrm{Li}^{+}<\mathrm{Mg}^{2+}<\mathrm{Al}^{3+}$ : Hydration energy
57. (A)
58. (C)

Due to size of Nitrogen is smaller than another.
59. (D)
$1 \& 3$ have $x-x$ bond absent.
(1) $\mathrm{B}_{2} \mathrm{H}_{6}$

(2) $\mathrm{C}_{2} \mathrm{H}_{6}$

(3) $\mathrm{Al}_{2} \mathrm{H}_{6}$

(4)

60. (C)

## MATHEMATICS

61. (B)

Let the equation of chord be $y=m x+c$; Joint equation of $O A \& O B$ is
$4 x^{2}+y^{2}-x\left(\frac{y-m x}{c}\right)+4 y\left(\frac{y-m x}{c}\right)=0$
$\because \mathrm{OA} \perp \mathrm{OB} \Rightarrow\left(4+\frac{\mathrm{m}}{\mathrm{c}}\right)+\left(1+\frac{4}{\mathrm{c}}\right)=0$
$\Rightarrow 5 \mathrm{c}+\mathrm{m}+4=0$
$\therefore y=m x+c \Rightarrow y+4 x+c(5 x-1)=0$
$\Rightarrow$ passing through the intersection of
$y+4 x=0$ and $5 x-1=0$
62. (D)

Only $(3,-4)$ satisfies equation of the circle.
63. (C)
64. (C)

$$
P \equiv \frac{x}{\cos \frac{\pi}{4}}=\frac{y}{\sin \frac{\pi}{4}}=6 \sqrt{2} \Rightarrow x=6, y=6
$$

Since $P(6,6)$ lie on circle

$$
\begin{equation*}
72+12(g+f)+c=0 \tag{i}
\end{equation*}
$$

Since $y=x$ touches the circle, then

$$
\begin{align*}
& 2 x^{2}+2 x(g+f)+c=0 \text { has equal roots } D=0 \\
& 4(g+f)^{2}=8 c \Rightarrow(g+f)^{2}=2 c \tag{ii}
\end{align*}
$$

From, we get

$$
(12(g+f))^{2}=[-(c+72)]^{2} \Rightarrow 144(2 c)=(c+72)^{2} \Rightarrow(c-72)^{2}=0 \Rightarrow c=72
$$

65. (C)
66. (A)

$$
\alpha-\beta=\sum_{r=1}^{100}\left(a_{2 r}-a_{2 r-1}\right)=100 d
$$

67. (D)
68. (C)
$(x-1)(x-0)+(y-0)(y-1)=0$
69. (C)

Let the equation of one of the circles be
$x^{2}+y^{2}+2 g x+2 f y+c=0$
Since it passes through origin,
$\therefore \quad c=0$.
So, the equation becomes

$$
x^{2}+y^{2}+2 g x+2 f y=0
$$

Since it cuts the circle $x^{2}+y^{2}+6 x-4 y+2=0$
orthogonally,
$\therefore \quad 2 \mathrm{~g}(3)+2 \mathrm{f}(-2)=0+2$
$\Rightarrow \quad-6(-\mathrm{g})+4(-\mathrm{f})=2$
Thus, the locus of the centre $(-g, f)$ is
$-6(-g)+4(-f)=2$ or $3 x-2 y+1=0$
70. (B)
$\sin x+\cos x=\sqrt{2}$
71. (B)

The equation of the straight line passing through the points of intersection of given circle is
$\left(x^{2}+y^{2}+5 x+1\right)-\left(x^{2}+y^{2}-3 x+7 y-25\right)=0$
i.e., $8 x-15 y+26=0$

Also, centre of the circle $x^{2}+y^{2}-2 x=0$ is $(1,0)$.
$\therefore$ Distance of the point $(1,0)$ from the straight line
$=\frac{|8(1)-15(0)+26|}{\sqrt{64+225}}=\frac{34}{17}=2$
72. (B)

The equation of the line $L$ be $y-2=m(x-8), m<0$
coordinates of $P$ and $Q$ are $P\left(8-\frac{2}{m}, 0\right)$ and $Q(0,2-8 m)$.
So, $\mathrm{OP}+\mathrm{OQ}=8-\frac{2}{\mathrm{~m}}+2-8 \mathrm{~m}=10+\frac{2}{(-\mathrm{m})}+8(-\mathrm{m}) \geq 10+2 \sqrt{\frac{2}{(-\mathrm{m})} \times 8(-\mathrm{m})} \geq 18$
So, absolute minium value of $O P+O Q=18$
73. (B)

The parabola $\mathrm{y}=\mathrm{x}^{2}+1$ and $\mathrm{x}=\mathrm{y}^{2}+1$ are symmetrical about $\mathrm{y}=\mathrm{x}$.
Therefore, the tangent at point $A$ is parallel to $y=x$. Therefore, $\frac{d y}{d x}=2 x$ or $2 x=1$
or $x=\frac{1}{2}$ and $y=\frac{5}{4}$
$\therefore \mathrm{A} \equiv\left(\frac{1}{2}, \frac{5}{4}\right)$ and $\mathrm{B} \equiv\left(\frac{5}{4}, \frac{1}{2}\right)$
Hence, Radius $=\frac{1}{2} \sqrt{\left(\frac{1}{2}-\frac{5}{4}\right)^{2}+\left(\frac{5}{4}-\frac{1}{2}\right)^{2}}=\frac{3}{8} \sqrt{2}$
$\therefore$ Area $=\frac{9 \pi}{32}$
74. (A)

The family of parabola is
$y=\frac{a^{3} x^{2}}{3}+\frac{a^{2} x}{2}-2 a$
and the vertex is $\mathrm{A}(-\mathrm{B} / 2 \mathrm{~A},-\mathrm{D} / 4 \mathrm{~A}) \equiv(\mathrm{h}, \mathrm{k})$. Therefore,
$h=-\frac{a^{2} / 2}{2\left(a^{3} / 3\right)}=-\frac{3}{4 a}$
and $\mathrm{k}=\frac{\left(\mathrm{a}^{2} / 2\right)^{2}-\left\{4 \mathrm{a}^{3}(-2 \mathrm{a}) / 3\right\}}{4\left(\mathrm{a}^{3} / 3\right)}=\frac{-35 \mathrm{a}}{16}$
Eliminating a, required locus is $x y=105 / 64$.
75. (C)

$$
\begin{aligned}
& \mathrm{y}^{2}=8 \mathrm{x} \\
\Rightarrow & \mathrm{a}=2 \\
\Rightarrow & \frac{2 \cdot \mathrm{PS} \cdot \mathrm{SQ}}{\mathrm{PS}+\mathrm{SQ}}=4 \\
\Rightarrow & \mathrm{SQ}=3
\end{aligned}
$$


76. (D)
required equation is $x^{2}-(\alpha+\beta) x+\alpha \beta=0$

$$
\begin{aligned}
& \alpha+\beta=\frac{1}{10-\sqrt{72}}+\frac{1}{10+6 \sqrt{2}}=\frac{5}{7} \\
& \alpha \cdot \beta=\frac{1}{28}
\end{aligned}
$$

77. (A)

Let $S=\sum_{n=1}^{\infty} \frac{a_{n}}{2^{n}}$, then

$$
\begin{aligned}
S= & \frac{a_{1}}{2}+\frac{a_{2}}{2^{2}}+\frac{a_{3}}{2^{3}}+\sum_{n=3}^{\infty} \frac{a_{n+1}}{2^{n+1}}=0+\frac{1}{4}+\frac{2}{8}+\sum_{n=3}^{\infty} \frac{a_{n}+a_{n-1}+a_{n-2}}{2^{n+1}} \\
& =\frac{1}{2}+\frac{1}{2} \sum_{n=3}^{\infty} \frac{a_{n}}{2^{n}}+\frac{1}{4} \sum_{n=3}^{\infty} \frac{a_{n-1}}{2^{n-1}}+\frac{1}{8} \sum_{n=3}^{\infty} \frac{a_{n-2}}{2^{n-2}}=\frac{1}{2}+\frac{1}{2}\left(S-\frac{0}{2}-\frac{1}{4}\right)+\frac{1}{4}\left(S-\frac{0}{2}\right)+\frac{1}{8} S \\
\Rightarrow S & =\frac{3}{8}+\frac{7}{8} S \Rightarrow \frac{1}{8} S=\frac{3}{8} \Rightarrow S=3
\end{aligned}
$$

78. (D)
b $>0$
Also, $(\alpha-1)^{2}+\beta^{2}=(\alpha-3)^{2}+\beta^{2} \Rightarrow \alpha=2$

$$
\cos 60^{\circ}=\frac{A A_{1}^{2}+A A_{2}^{2}-A_{1} A_{2}^{2}}{2 A A_{1} \times A A_{2}} \Rightarrow\left(1+\beta^{2}\right)=2 \beta^{2}-2 \Rightarrow \beta=\sqrt{3}
$$

$\therefore \quad$ Equation of circle having centre $(2, \sqrt{3})$ and radius 2 is

$$
x^{2}+y^{2}-4 x-2 \sqrt{3} y+3=0
$$


79. (C)

$$
\begin{aligned}
& a_{n}=\frac{n(n+1)}{\left(\frac{n(n+1)}{2}\right)^{2}}=4\left(\frac{1}{n}-\frac{1}{n+1}\right) \\
& S_{n}=4\left(1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\ldots+\frac{1}{n}-\frac{1}{(n+1)}\right) \\
& S_{n}=4\left(1-\frac{1}{n+1}\right) \\
& S_{\infty}=4 .
\end{aligned}
$$

80. (B)

$$
\begin{aligned}
& \alpha+\beta=a \text { and } \alpha \beta=-(a+b) \\
& \frac{\alpha^{2}+2 \alpha+1}{\alpha^{2}+2 \alpha+b}+\frac{\beta^{2}+2 \beta+1}{\beta^{2}+2 \beta+b}=\frac{(\alpha+1)^{2}}{(\alpha+1)^{2}+(b-1)}+\frac{(\beta+1)^{2}}{(\beta+1)^{2}+(b-1)}
\end{aligned}
$$

81. (B)

$$
\alpha+\beta=\frac{3}{2}
$$

$$
\alpha \beta=-3
$$

$$
\therefore(\alpha+\beta)^{2}-2 \alpha \beta+4
$$

$$
=\frac{9}{4}+6+4=\frac{49}{4}
$$

and $\left(\alpha^{2}+2\right)\left(\beta^{2}+2\right)=\alpha^{2} \beta^{2}+2\left(\alpha^{2}+\beta^{2}\right)+4$
$=9+2\left(\frac{33}{4}\right)+4=\frac{59}{2}$
Hence, the required equation is $x^{2}-\frac{49}{4} x+\frac{59}{2}=0$
82. (B)

Equation of normal in terms of $m$ is $y=m x-4 m-2 m^{3}$. If it passes through $(a, 0)$ then $a m-4 m-2 m^{3}=0$
$\Rightarrow \mathrm{m}\left(\mathrm{a}-4-2 \mathrm{~m}^{2}\right)=0 \Rightarrow \mathrm{~m}=0, \mathrm{~m}^{2}=\frac{\mathrm{a}-4}{2}$.
For three distinct normal, $\mathrm{a}-4>0 \Rightarrow \mathrm{a}>4$
83. (A)

Let $16^{\sin ^{2} x}=y$, then $16^{\cos ^{2} x}=16^{1-\sin ^{2} x}=\frac{16}{y}$
Hence $y+\frac{16}{y}=10 \Rightarrow y^{2}-10 y+16=0 \Rightarrow y=2$ or 8
Now $16^{\sin ^{2} x}=2 \Rightarrow 2^{4 \sin ^{2} x}=(2)^{1} \Rightarrow 4 \sin ^{2} x=1$
$\therefore \quad \sin x= \pm \frac{1}{2} \Rightarrow x=\frac{\pi}{6}$
and $16^{\sin ^{2} x}=8 \Rightarrow 2^{4 \sin ^{2} x}=2^{3} \Rightarrow \sin x= \pm \frac{\sqrt{3}}{2} \Rightarrow x=\frac{\pi}{3}$
84. (B)
85. (C)

Let $\alpha, 2 \alpha$ are the roots of equation
so $\alpha+2 \alpha=3 \alpha=3 \mathrm{a} \Rightarrow \alpha=\mathrm{a}$
and $\alpha(2 \alpha)=2 \alpha^{2}=f(a)$
$\Rightarrow \quad f(a)=2 a^{2}$
Hence $f(x)=2 x^{2}$
86. (A)

Since $x_{1} x_{2}=4$
$x_{2}=\frac{4}{x_{1}}$
$\therefore \quad \frac{x_{1}}{x_{1}-1}+\frac{\frac{4}{x_{1}}}{\frac{4}{x_{1}}-1}=2 \Rightarrow \frac{x_{1}}{x_{1}-1}+\frac{4}{4-x_{1}}=2$
$4 x_{1}-x_{1}^{2}+4 x_{1}-4=2\left(x_{1}-1\right)\left(4-x_{1}\right)$
$\Rightarrow \quad x_{1}^{2}-2 x_{1}+4=0 \quad \Rightarrow \quad x^{2}-2 x+4=0$
87.
(C)

Let $f(x)=x^{2}-\frac{3 a x}{a-2}+\frac{1}{a-2}=0 \quad \therefore \quad a-2>2$
$D=\frac{9 a^{2}}{(a-2)^{2}}-4\left(\frac{1}{a-2}\right)=\frac{1}{(a-2)^{2}}\left(9 a^{2}-4 a+8\right)=\left\{8 a^{2}+(a-2)^{2}+4\right\}>9$
$f(0)=\frac{1}{a-2}>0 \quad$ and $-\frac{b}{2 a}=\frac{3 a}{2(a-2)}>0$
Since $D>0, f(0)>0-\frac{b}{2 a}>0$
Hence both roots of given equation are positive.
88. (B)

Let $\alpha_{1}$ and $\alpha_{2}$ are roots
$\alpha_{1}{ }^{2}+\alpha_{1}{ }^{2}=16$
$\alpha_{1} \alpha_{2}=p$
$\alpha_{1}+\alpha_{2}=4$
Solving we get $\alpha_{1} \alpha_{2}=0$
$\Rightarrow \mathrm{p}=0$
89. (C)
90. (A)

Let the equatoin of the circle be

$$
\begin{equation*}
x^{2}+y^{2}+2 g x+2 f y+c=0 \tag{i}
\end{equation*}
$$

Since the circle touches $y$-axis, we have $c=f^{2}$.
Clearly, the point of contact is $(0,2)$ and it lies on the circle.
So, $(0,2)$ must satisfy (i).
$\therefore \quad 4+4 \mathrm{f}+\mathrm{c}=0 \quad$ or $4+4 \mathrm{f}+\mathrm{f}^{2}=0 \quad\left[\because \mathrm{c}=\mathrm{f}^{2}\right]$
This gives $(2+\mathrm{f})^{2}=0$ or $\mathrm{f}=-2$. And, therefore, $\mathrm{c}=4$.
Also, intercept on $x$-axis is given by $2 \sqrt{g^{2}-c}$.
Now, $2 \sqrt{g^{2}-c}=3 \Rightarrow g^{2}-c=\frac{9}{4}$ or $g^{2}=\frac{9}{4}+c=\frac{9}{4}+4$, i.e., $g= \pm \frac{5}{2}$.
Hence, the required equation of the circle is

$$
x^{2}+y^{2} \pm 5 x-4 y+4=0
$$

