SOLUTIONS

PROGRESS TEST-8

GZR-1901(A), GZRS-1901, GZR-1901-1907

GZRK-1901-1902

JEE MAIN PATTERN

Test Date: 10-12-2017

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PHYSICS

1. **(B)**

As tortoise moves near the axis, ω increases because of conservation of angular momentum.

2. **(A)**

$$X_{cm} = \int x\,dm / \int dm$$

Where $dm = \lambda dx$

3. **(C)**

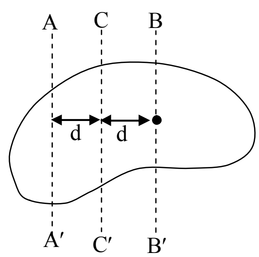
$$r_{cm} = \frac{Mr_0 - mr}{M - m}$$

4. **(C)**

$$a_{cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2} = \frac{m(0) + m(a)}{m + m} = a/2$$

5. **(C)**

Let BB′ be an axis passing through centre of mass and parallel to CC′.



$\therefore\ I_{AA'} = I_{BB'} + 4\ Md^2$

$= I_{CC'} - Md^2 + 4\ Md^2$

$I_{AA'} = I_{CC'} + 3\ Md^2$

6. **(B)**

The rod will rotate about point A with angular acceleration:

$$\alpha = \frac{\tau}{I} = \frac{Fx}{\frac{ml^2}{3}} = \frac{3Fx}{ml^2}$$

$$\therefore\ a = \frac{l}{2}\ \alpha = \frac{3}{2}\ \frac{Fx}{ml}$$

or $a \propto x$

i.e., a-x graph is a straight line passing through origin.

7. **(B)**

$\tau_0 = 0$

8. **(A)**

$h_n = e^{2n}h_0$ or $e^n = \sqrt{\frac{h_n}{h_0}}$

9. **(B)**

$P_3 = \sqrt{P_1^2 + P_2^2}$

$m_3v_3 = \sqrt{5^2 + 12^2}$

or $m_3v_3 = 13$

or $m_3 = \frac{13}{6.5} = 2\text{kg}$

$\therefore$ Total mass = 1 + 2 + 2 = 5 kg

10. **(A)**

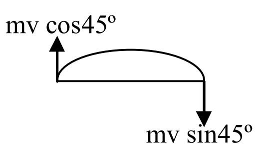
From momentum conservation

(1000) (50) = 1250 v

v = 40 m/s

11. **(C)**

$\Delta p = 2mv\sin 45^\circ = 2\,mv\frac{1}{\sqrt{2}} = \sqrt{2}\,mv$



12. **(B)**

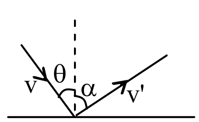
$e = (v_2 - v_1)/v$, $mv = mv_1 + mv_2$

$1/2mv_2^2 = mg \times J$

$\Rightarrow v_2 = 10$ m/s , $16 = v_1 + 10$, $v_1 = 6$m/s

$e = (10-6)/16 = 1/4$

13. **(B)**



$v'\cos\alpha = ev\cos\theta$

$v'\sin\alpha = v\sin\theta$

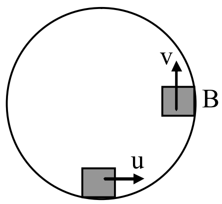
$\Rightarrow \alpha = \tan^{-1}\left(\frac{1}{e}\tan\theta\right)$

14. (A)

$u^2 = 5gR$

$\therefore v^2 = u^2 - 2gR$

$= 5gR - 2gR = 3gR$



Tangential acceleration at B is

$a_t = g$ (downwards)

Centripetal acceleration at B is

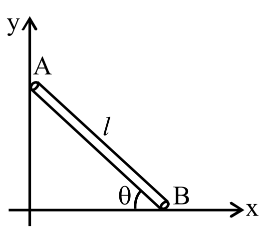
$a_C = \frac{v^2}{R} = 3g$

$\therefore$ Total acceleration will be

$a = \sqrt{a_C^2 + a_t^2} = g\sqrt{10}$

15. (C)

Let l be the length of the rod and θ the angle of rod with x-axis (horizontal) at some instant of time. Co-ordinates of the centre of rod at this instant of time are



$x = \frac{l}{2}\cos\theta$

and $y = \frac{l}{2}\sin\theta$

Squaring and adding Eqs. (1) and (2), we get:

$x^2 + y^2 = \frac{l^2}{4}$

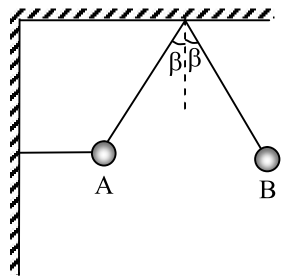
Which is an equation of a circle of radius $\frac{l}{2}$ and centre at origin.

16. **(D)**

$$\omega = \frac{8\sin 30^\circ + 6\sin 30^\circ}{10} = 0.7 \text{ rad / sec}$$

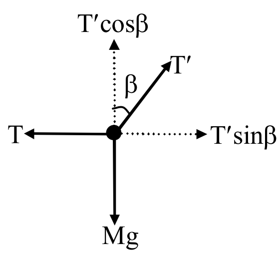
17. **(A)**

18. **(B)**



At : A

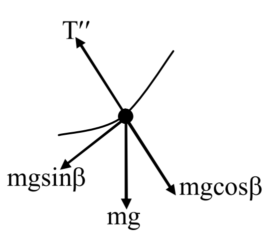
T′cosβ



$T'\cos\beta = mg$

$T' = \frac{mg}{\cos\beta}$

At : B



$T'' - mg\cos\beta = \frac{mv^2}{\ell}$

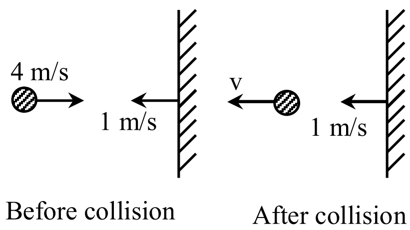
at B ; $v = 0$

$T'' = mg\cos\beta$

$\frac{T''}{T'} = \frac{mg\cos\beta}{mg/\cos\beta}$

$\frac{\text{Tension at B}}{\text{Tension at A}} = \frac{T''}{T'} = \cos^2\beta$

19. (D)



Let v be the velocity of ball after collision, collision is elastic

$\therefore$ e =1

or

relative velocity of separation = relative velocity of approach

$\therefore$ v – 1 = 4 + 1

or v = 6 m/s (away from the wall)

20. (A)

From conservation of mechanical energy

$\frac{1}{2}kx^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ ... (1)

from COLM

$O = m_1v_1 - m_2v_2$ .....(2)

and v_r = relative velocity of the two

then from (1) & (2) we get

$kx^2 = mv_r^2$

$\therefore v_r = \left(\sqrt{\frac{3k}{2m}}\right)x$

21. (A)

COLM

$mv_0 = mv + 2mv$

$\Rightarrow v = \frac{v_0}{3}$

$\therefore e = \frac{\frac{2v_0}{3} - \frac{v_0}{3}}{v_0} = \frac{1}{3}$

22. (C)

Maximum expansion in spring is given by

$$\frac{1}{2}kx_{max}^2 = \frac{1}{2}mv^2 + \frac{1}{2}2mv^2 \quad (\because v_1 = v_2 = v) \qquad \ldots(i)$$

Also $mv_0 = mv + 2mv$

$$\therefore\ x_{max} = \sqrt{\frac{2m}{3k}}\,v_0$$

23. (D)

Let N be the normal reaction between m and M,

Equilibrium of M

$N \sin 45^o = kx \qquad \ldots$ (i)

Equilibrium of m in vertical direction gives

$N \cos 45^o = mg \qquad \ldots$ (ii)

From Eqs. (i) and (ii), we get

$$x = \frac{mg}{k}$$

24. (A)

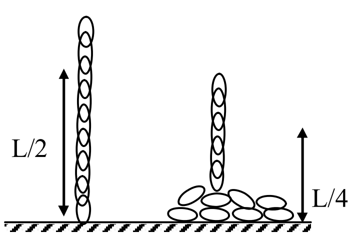
COLM gives

$$M\sqrt{2gL} = (M+m)v$$

$$\therefore\ v = \frac{M\sqrt{2gL}}{M+m}\ \therefore\ h = \frac{v^2}{2g}$$

25. (C)

The work done by man is negative of magnitude of decrease in potential energy of chain.



$$\Delta U = mg\,\frac{L}{2} - \frac{m}{2}g\,\frac{L}{4} = 3\,mg\,\frac{L}{8}$$

$$\therefore\ W = -\frac{3mg\ell}{8}$$

26. (B)

$$W = \int_{(0,0)}^{(1,1)} \vec{F}.\vec{d}x$$

Here $\vec{d}s = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$\therefore \quad W = \int_{(0,0)}^{(1,1)} (x^2dy + ydx) = \int_{(0,0)}^{(1,1)} (x^2dy + x.dx) \qquad \text{(as } x = y)$$

$$\therefore \quad W = \left[\frac{y^3}{3} + \frac{x^2}{2}\right]_{(0,0)}^{(1,1)} = \frac{5}{6}\text{ J}$$

27. (C)

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{f}{m}t\right)^2$$

Also, $K = f \times s$

28. (C)

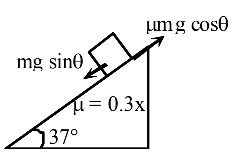
P = Fv

$$\text{or } P = \left(mv\frac{dv}{dx}\right)v$$

$$\text{or } \int_0^v v^2dv = \int_0^x \frac{P}{m}dx$$

$$\frac{v^3}{3} = \frac{Px}{m} \text{ or } v = \left(\frac{3Px}{m}\right)^{1/3}$$

29. (D)



After some time friction becomes more than mgsinθ, then body will retard. Thus speed is maximum when, total force or acc. is zero.

$mg\sin\theta - \mu mg\cos\theta = 0$

$\Rightarrow \mu = \tan\theta \Rightarrow 0.3x = 3/4$

$\Rightarrow x = 2.5\text{m}$

30. (D)

$$H = \frac{u_1^2\sin^2\theta_1}{2g} = \frac{u_2^2\sin^2\theta_2}{2g}$$

CHEMISTRY

31. (D) **32. (C)** **33. (A)** **34. (D)**

35. (B)

The velocity corresponding to the maxima is the most probable velocity which is given by the expression.

$$C_{mp} = \sqrt{\frac{2RT}{M}}$$

$$RT = \frac{C_{mp}^2 M}{2} = \frac{(200)^2 \times 100 \times 10^{-3}}{2} = 40000 \times 50 \times 10^{-3}\ J\,mol^{-1} = 40 \times 50\ J\,mol^{-1}$$

No. of moles = $\frac{300}{100} = 3$

$$E = \frac{3}{2}nRT = \frac{3}{2} \times 3 \times 40 \times 50 = 9 \times 10^3 J = 9kJ$$

36. (D)

37. (B)

$$V_{real} = \frac{\text{Molar mass}}{\text{density}} = \frac{18}{0.36}.$$

$$V_{ideal} = \frac{nRT}{P} = \frac{1 \times 0.082 \times 500}{1}.$$

$$\text{So, } Z = \frac{V_{real}}{V_{ideal}} = \frac{50}{0.082 \times 500} = \frac{50}{41}.$$

38. (B)

$$2S \xrightarrow[\text{excitation}]{\text{Energy 4p}} 2p \xrightarrow[\text{Excitation}]{\text{Energy up}} 3d \xrightarrow[\text{deexcitations}]{\text{Energy down}} 3p$$

$$3p \xrightarrow[\text{excitation}]{\text{Energy up}} 4S \xrightarrow[\text{deexcitation}]{\text{Energy down}} 3p$$

During deexcitation emission of light results

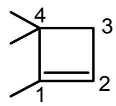
39. (A)

40. (C)

$$\lambda = \frac{h}{mv} \quad \lambda = \frac{h}{\sqrt{2mKE}}$$

$$\lambda_{req} = \frac{h}{\sqrt{zm9KE}} = \frac{1}{3}\frac{\lambda}{\sqrt{2mKG}} = \frac{\lambda}{3}$$

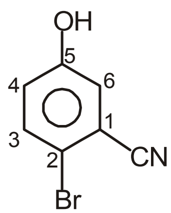
41. (C)



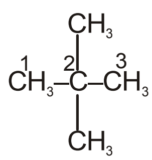
1,4,4-Trimethylcyclobutene

42. (D)

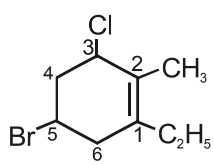
43. (B)



44. (A)

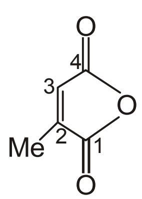


45. (C)

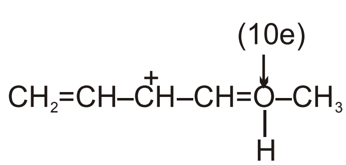


: 5-Bromo-3-chloro-1-ethyl-2-methylcyclohex-1-ene

46. (A)

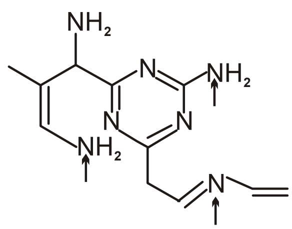


47. (B)



48. (C)

49. (C)

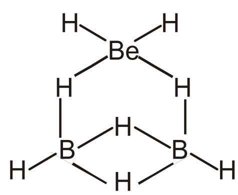


50. (C)

51. (C)

52. (B)

53. (C)

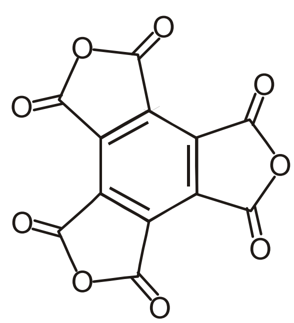


54. (A)

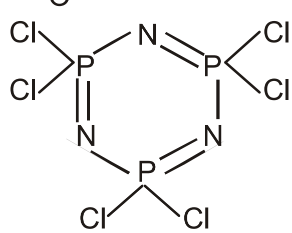
(IV) $LiClO_4 > NaClO_4 > KClO_4 > RbClO_4 > CsClO_4$

55. (A)

(iii) $C_{12}O_9$ – sp^2 ;



(iv) $N_3P_3Cl_6$ – sp^2 & sp^3 ;



56. (A)

(i) LiF > NaF > KF > RbF : Lattice energy

(iii) $Li^+ < Mg^{2+} < Al^{3+}$:Hydration energy

57. (A)

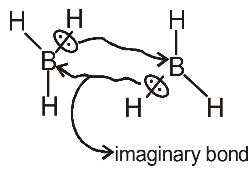
58. (C)

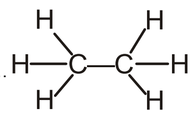
Due to size of Nitrogen is smaller than another.

59. (D)

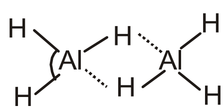
1 & 3 have x – x bond absent.

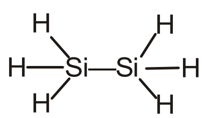
(1) B_2H_6



(2) C_2H_6 .... 

(3) Al_2H_6



(4)  x-x bond are present

60. (C)

MATHEMATICS

61. (B)

Let the equation of chord be y = mx + c ; Joint equation of OA & OB is

$$4x^2 + y^2 - x\left(\frac{y-mx}{c}\right) + 4y\left(\frac{y-mx}{c}\right) = 0$$

$$\because OA \perp OB \Rightarrow \left(4+\frac{m}{c}\right) + \left(1+\frac{4}{c}\right) = 0$$

$$\Rightarrow 5c + m + 4 = 0$$

$$\therefore y = mx + c \Rightarrow y + 4x + c(5x-1) = 0$$

$\Rightarrow$ passing through the intersection of

$y + 4x = 0$ and $5x - 1 = 0$

62. (D)

Only (3, –4) satisfies equation of the circle.

63. (C)

64. (C)

$$P \equiv \frac{x}{\cos\frac{\pi}{4}} = \frac{y}{\sin\frac{\pi}{4}} = 6\sqrt{2} \Rightarrow x = 6,\ y = 6$$

Since P(6,6) lie on circle

$72 + 12(g + f) + c = 0$ ......(i)

Since y = x touches the circle, then

$2x^2 + 2x(g + f) + c = 0$ has equal roots D = 0

$4(g + f)^2 = 8c \Rightarrow (g + f)^2 = 2c$ ......(ii)

From, we get

$(12(g + f))^2 = [-(c + 72)]^2 \Rightarrow 144(2c) = (c + 72)^2 \Rightarrow (c - 72)^2 = 0 \Rightarrow c = 72$

65. (C)

66. (A)

$$\alpha - \beta = \sum_{r=1}^{100} (a_{2r} - a_{2r-1}) = 100\,d$$

67. (D)

68. (C)

$(x - 1)(x - 0) + (y - 0)(y - 1) = 0$

69. (C)

Let the equation of one of the circles be

$x^2 + y^2 + 2gx + 2fy + c = 0$

Since it passes through origin,

$\therefore$ c = 0.

So, the equation becomes

$x^2 + y^2 + 2gx + 2fy = 0$

Since it cuts the circle $x^2 + y^2 + 6x - 4y + 2 = 0$

orthogonally,

$\therefore\ 2g(3) + 2f(-2) = 0 + 2$

$\Rightarrow\ -6(-g) + 4(-f) = 2$

Thus, the locus of the centre (–g, f) is

$-6(-g) + 4(-f) = 2$ or $3x - 2y + 1 = 0$

70. (B)

$\sin x + \cos x = \sqrt{2}$

71. (B)

The equation of the straight line passing through the points of intersection of given circle is

$(x^2 + y^2 + 5x + 1) - (x^2 + y^2 - 3x + 7y - 25) = 0$

i.e., $8x - 15y + 26 = 0$ ....(i)

Also, centre of the circle $x^2 + y^2 - 2x = 0$ is (1, 0).

$\therefore$ Distance of the point (1, 0) from the straight line ......(1)

$$= \frac{|8(1) - 15(0) + 26|}{\sqrt{64 + 225}} = \frac{34}{17} = 2$$

72. (B)

The equation of the line L be $y - 2 = m(x - 8)$, $m < 0$

coordinates of P and Q are P $\left(8 - \frac{2}{m}, 0\right)$ and Q $(0, 2 - 8m)$.

So, $OP + OQ = 8 - \frac{2}{m} + 2 - 8m = 10 + \frac{2}{(-m)} + 8(-m) \geq 10 + 2\sqrt{\frac{2}{(-m)} \times 8(-m)} \geq 18$

So, absolute minium value of OP + OQ = 18

73. (B)

The parabola $y = x^2 + 1$ and $x = y^2 + 1$ are symmetrical about $y = x$.

Therefore, the tangent at point A is parallel to $y = x$. Therefore, $\frac{dy}{dx} = 2x$ or $2x = 1$

or $x = \frac{1}{2}$ and $y = \frac{5}{4}$

$$\therefore A \equiv \left(\frac{1}{2}, \frac{5}{4}\right) \text{ and } B \equiv \left(\frac{5}{4}, \frac{1}{2}\right)$$

Hence, Radius $= \frac{1}{2}\sqrt{\left(\frac{1}{2} - \frac{5}{4}\right)^2 + \left(\frac{5}{4} - \frac{1}{2}\right)^2} = \frac{3}{8}\sqrt{2}$

$$\therefore \text{Area} = \frac{9\pi}{32}$$

74. (A)

The family of parabola is

$$y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$$

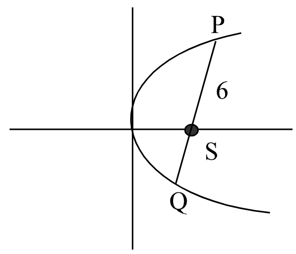
and the vertex is A(–B/2A, –D/4A) ≡ (h, k). Therefore,

$$h=-\frac{a^2/2}{2(a^3/3)}=-\frac{3}{4a}$$

and $k=\frac{(a^2/2)^2-\{4a^3(-2a)/3\}}{4(a^3/3)}=\frac{-35a}{16}$

Eliminating a, required locus is xy = 105/64.

75. (C)



$y^2 = 8x$

$\Rightarrow$ a = 2

$\Rightarrow \frac{2.PS.SQ}{PS+SQ}=4$

$\Rightarrow$ SQ = 3

76. (D)

required equation is $x^2-(\alpha+\beta)x+\alpha\beta=0$

$$\alpha+\beta=\frac{1}{10-\sqrt{72}}+\frac{1}{10+6\sqrt{2}}=\frac{5}{7}$$

$$\alpha.\beta=\frac{1}{28}$$

77. (A)

Let $S=\sum_{n=1}^{\infty}\frac{a_n}{2^n}$, then

$$S=\frac{a_1}{2}+\frac{a_2}{2^2}+\frac{a_3}{2^3}+\sum_{n=3}^{\infty}\frac{a_{n+1}}{2^{n+1}}=0+\frac{1}{4}+\frac{2}{8}+\sum_{n=3}^{\infty}\frac{a_n+a_{n-1}+a_{n-2}}{2^{n+1}}$$

$$=\frac{1}{2}+\frac{1}{2}\sum_{n=3}^{\infty}\frac{a_n}{2^n}+\frac{1}{4}\sum_{n=3}^{\infty}\frac{a_{n-1}}{2^{n-1}}+\frac{1}{8}\sum_{n=3}^{\infty}\frac{a_{n-2}}{2^{n-2}}=\frac{1}{2}+\frac{1}{2}\left(S-\frac{0}{2}-\frac{1}{4}\right)+\frac{1}{4}\left(S-\frac{0}{2}\right)+\frac{1}{8}S$$

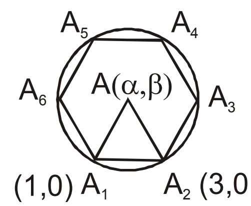
$$\Rightarrow\quad S=\frac{3}{8}+\frac{7}{8}S\Rightarrow\frac{1}{8}S=\frac{3}{8}\Rightarrow S=3$$

78. (D)

b > 0

Also, $(\alpha-1)^2+\beta^2=(\alpha-3)^2+\beta^2 \Rightarrow \alpha=2$

$$\cos 60^\circ=\frac{AA_1^2+AA_2^2-A_1A_2^2}{2AA_1\times AA_2} \Rightarrow (1+\beta^2)=2\beta^2-2 \Rightarrow \beta=\sqrt{3}$$



$\therefore$ Equation of circle having centre $(2,\sqrt{3})$ and radius 2 is

$$x^2+y^2-4x-2\sqrt{3}\,y+3=0$$

79. (C)

$$a_n=\frac{n(n+1)}{\left(\frac{n(n+1)}{2}\right)^2}=4\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$S_n=4\left(1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\ldots+\frac{1}{n}-\frac{1}{(n+1)}\right)$$

$$S_n=4\left(1-\frac{1}{n+1}\right)$$

$S_\infty=4$.

80. (B)

$\alpha+\beta=a$ and $\alpha\beta=-(a+b)$

$$\frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+b}+\frac{\beta^2+2\beta+1}{\beta^2+2\beta+b}=\frac{(\alpha+1)^2}{(\alpha+1)^2+(b-1)}+\frac{(\beta+1)^2}{(\beta+1)^2+(b-1)}$$

81. (B)

$$\alpha+\beta=\frac{3}{2}$$

$$\alpha\beta=-3$$

$$\therefore (\alpha+\beta)^2-2\alpha\beta+4$$

$$=\frac{9}{4}+6+4=\frac{49}{4}$$

and $(\alpha^2+2)(\beta^2+2)=\alpha^2\beta^2+2(\alpha^2+\beta^2)+4$

$$=9+2\left(\frac{33}{4}\right)+4=\frac{59}{2}$$

Hence, the required equation is $x^2-\frac{49}{4}x+\frac{59}{2}=0$

82. (B)

Equation of normal in terms of m is $y = mx - 4m - 2m^3$. If it passes through (a, 0) then $am - 4m - 2m^3 = 0$

$\Rightarrow m(a - 4 - 2m^2) = 0 \quad \Rightarrow \quad m = 0, m^2 = \frac{a-4}{2}$.

For three distinct normal, $a - 4 > 0 \Rightarrow a > 4$

83. (A)

Let $16^{\sin^2 x} = y$, then $16^{\cos^2 x} = 16^{1-\sin^2 x} = \frac{16}{y}$

Hence $y + \frac{16}{y} = 10 \Rightarrow y^2 - 10y + 16 = 0 \Rightarrow y = 2$ or 8

Now $16^{\sin^2 x} = 2 \Rightarrow 2^{4\sin^2 x} = (2)^1 \Rightarrow 4\sin^2 x = 1$

$\therefore \quad \sin x = \pm\frac{1}{2} \Rightarrow x = \frac{\pi}{6}$

and $16^{\sin^2 x} = 8 \Rightarrow 2^{4\sin^2 x} = 2^3 \Rightarrow \sin x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}$

84. (B)

85. (C)

Let α, 2α are the roots of equation

so $\alpha + 2\alpha = 3\alpha = 3a \Rightarrow \alpha = a$

and $\alpha(2\alpha) = 2\alpha^2 = f(a)$

$\Rightarrow \quad f(a) = 2a^2$

Hence $f(x) = 2x^2$

86. (A)

Since $x_1 x_2 = 4$

$x_2 = \frac{4}{x_1}$

$$\therefore \quad \frac{x_1}{x_1 - 1} + \frac{\frac{4}{x_1}}{\frac{4}{x_1} - 1} = 2 \Rightarrow \frac{x_1}{x_1 - 1} + \frac{4}{4 - x_1} = 2$$

$4x_1 - x_1^2 + 4x_1 - 4 = 2(x_1 - 1)(4 - x_1)$

$\Rightarrow \quad x_1^2 - 2x_1 + 4 = 0 \quad \Rightarrow \quad x^2 - 2x + 4 = 0$

87. (C)

Let $f(x) = x^2 - \frac{3ax}{a-2} + \frac{1}{a-2} = 0$ $\therefore$ $a - 2 > 2$

$D = \frac{9a^2}{(a-2)^2} - 4\left(\frac{1}{a-2}\right) = \frac{1}{(a-2)^2}(9a^2 - 4a + 8) = \{8a^2 + (a-2)^2 + 4\} > 9$

$f(0) = \frac{1}{a-2} > 0$ and $-\frac{b}{2a} = \frac{3a}{2(a-2)} > 0$

Since $D > 0$, $f(0) > 0 - \frac{b}{2a} > 0$

Hence both roots of given equation are positive.

88. (B)

Let α_1 and α_2 are roots

$\alpha_1{}^2 + \alpha_1{}^2 = 16$ ...(1)

$\alpha_1\alpha_2 = p$

$\alpha_1 + \alpha_2 = 4$ ...(2)

Solving we get $\alpha_1\alpha_2 = 0$

$\Rightarrow p = 0$

89. (C)

90. (A)

Let the equatoin of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

Since the circle touches y-axis, we have $c = f^2$.

Clearly, the point of contact is (0, 2) and it lies on the circle.

So, (0, 2) must satisfy (i).

$\therefore$ $4 + 4f + c = 0$ or $4 + 4f + f^2 = 0$ $[\because c = f^2]$

This gives $(2 + f)^2 = 0$ or $f = -2$. And, therefore, $c = 4$.

Also, intercept on x-axis is given by $2\sqrt{g^2 - c}$.

Now, $2\sqrt{g^2 - c} = 3 \Rightarrow g^2 - c = \frac{9}{4}$ or $g^2 = \frac{9}{4} + c = \frac{9}{4} + 4$, i.e., $g = \pm\frac{5}{2}$.

Hence, the required equation of the circle is

$x^2 + y^2 \pm 5x - 4y + 4 = 0$