SOLUTIONS

PROGRESS TEST-6

RBPA

RB-1806-1809 & RBK-1804

JEE MAIN PATTERN

Test Date: 04-12-2017

Mentors® Eduserv™

Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd., Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

PHYSICS

1. **(B)**

Component of $\vec{v}$ along $\vec{a}$ = $(\vec{v}.\hat{a})$

= $(6\hat{i} + 2\hat{j} - 2\hat{k}).\dfrac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$

= $\dfrac{6+2-2}{\sqrt{3}} = \dfrac{6}{\sqrt{3}} = 2\sqrt{3}$

In vector form = $(2\sqrt{3})\,\hat{a}$

= $2\sqrt{3}\,\dfrac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$

= $2\,(\hat{i}+\hat{j}+\hat{k})$

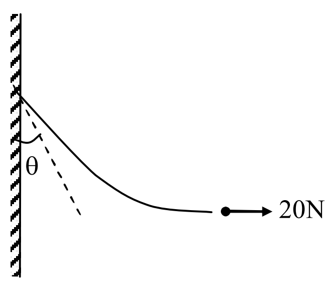
2. **(C)**

Let acceleration of mass m relative to wedge down the plane is a_r. Its absolute acceleration in horizontal direction is a_r cos 60° – a(towards right). Hence, let N be the normal reaction between the mass and the wedge. Then

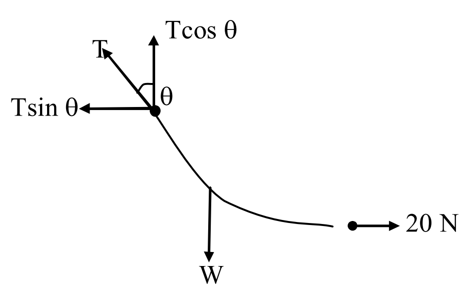
N sin θ = Ma = m(a_r cos 60° – a)

or ar = $\dfrac{(M+m)a}{m\cos 60°} = \dfrac{2(M+m)a}{m}$

3. **(C)**



Free Body Diagram



T sin θ = 20

T cos θ = W

tan θ = $\dfrac{20}{W}$

W = 20 cot θ

W = 20 cot 30º

W = $20\sqrt{3}$ = 20 × 1.732

Weight W = 34.64 N

mass 'm' = $\frac{W}{g}$ = 3.5 kg.

4. **(A)**

Since 'M' is at rest the tension in the string

= $\frac{Mg}{2}$ Let acceleration of m and m' is 'f' one will move downward and other will move upward

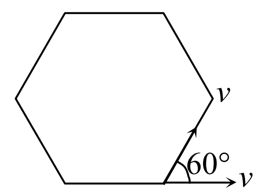
$mg - \frac{Mg}{2} = mf$ ....(i)

$\frac{Mg}{2} - m'g = m'f$ ....(ii)

Solving equation (i) and (ii)

$\frac{4}{M} = \frac{1}{m} + \frac{1}{m_1}$

5. Velocity of approach = $v - \frac{v}{2} = \frac{v}{2}$



$\therefore$ time taken = $\frac{\text{initial separation}}{\text{velocity of approach}} = \frac{2a}{v}$

$\therefore$ **(C)**

6. At maximum height $v_1 = v\cos\theta$

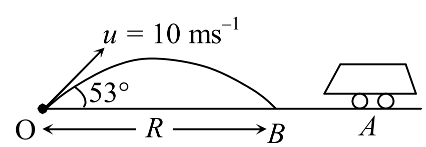
At half of maximum height $v_2 = \sqrt{v^2\cos^2\theta + (v_y)^2}$

For $v_{vertical}$ $v_y^2 = v^2\sin^2\theta - 2g\frac{H}{2} = v^2\sin^2\theta - g\frac{v^2\sin^2\theta}{2g} = \frac{v^2\sin^2\theta}{2}$

$\frac{v_1}{v_2} = \sqrt{\frac{2}{5}} \Rightarrow \tan\theta = \sqrt{3}$, $\theta = 60^0$

$\therefore$ **(D)**

7. $T = \frac{2u\sin\theta}{g} = \frac{2\times10\times\frac{4}{5}}{10} = \frac{8}{5}s$



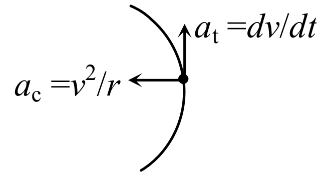
$OB = R = \frac{u^2\sin2\theta}{g} = \frac{100\times2\times\frac{4}{5}\times\frac{3}{5}}{10} = \frac{48}{5}$ m

$AB = \frac{8}{5}\times5 = 8$ m

$OA = OB + AB = \frac{48}{5} + 8 = 17.6$ m

$\therefore$ **(D)**

8. $v = 2t^2$, $r = 100$ m



$a_t = \frac{dv}{dt} = 4t$

$a_t\,(t = 5s) = 20\ ms^{-2}$

$v\,(t = 5s) = 50\ ms^{-1}$

$a_c(t = 5\,s) = \frac{v^2}{r} = \frac{50 \times 50}{100} = 25\ ms^{-2}$

$a = \sqrt{a_c^2 + a_t^2} = \sqrt{1025} \approx 32\ ms^{-1}$

$\therefore$ **(D)**

9. $t_1 = \frac{2u\sin\theta}{g}, t_2 = \frac{2u\cos\theta}{g}$

$\therefore\ t_1t_2 = \frac{2u^2 \sin 2\theta}{g \times g} = \frac{2}{g}R$

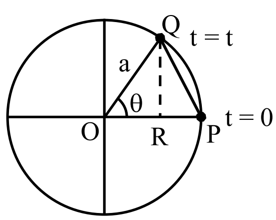
$\therefore\ t_1t_2 \propto R$

$\therefore$ **(C)**

10. **(B)** $T = m_2\sqrt{g^2 + \left(\frac{v_2^2}{r}\right)^2} = \frac{m_1v_1^2}{r}$

11. **(B)**

In time t particle has rotated an angle $\theta = \omega t$. Displacement



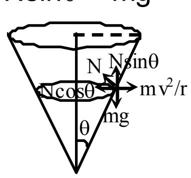
s = PQ = $\sqrt{QR^2 + PR^2}$

$= \sqrt{(a\sin\omega t)^2 + (a - a\cos\omega t)^2}$

s = 2a sin $\frac{\omega t}{2}$

12 **(D)**

$N\sin\theta = mg$



$N\cos\theta = mv^2/r$

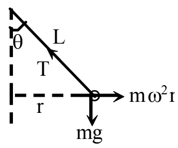
$\Rightarrow \tan\theta = rg/v^2$

$\Rightarrow \frac{r}{h} = \frac{rg}{v^2}$

$\Rightarrow h = \frac{v^2}{g} = 2.5$ cm

13. (D)

$\omega = 2/\pi$ rev/s = 4 rad/s



$T\sin\theta = m\omega^2 r$

$\Rightarrow T\sin\theta = m\omega^2(L\sin\theta)$

$\Rightarrow T = m\omega^2 L = m(4)^2 L = 16mL$

14. (D)

$\because$ Cyclist returns at initial point

$\therefore$ Displacement = 0

$v_{av} = \frac{s}{t} = \frac{R + \frac{\pi R}{2} + R}{(1/6)} = 6R\,(2 + \pi/2)$

= 21.4 km

15. (B)

$W_g + W_{Fr} = \Delta K$

$W_g = - W_{Fr}$

$\boxed{W_{Fr} = -mgh}$

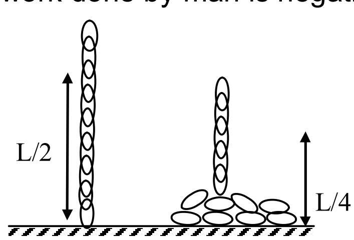
$W_f + W_g + W_f = \Delta k$

$W_f = - W_g - W_{Fr}$

$(W_F = 2mgh)$

16. (C)

The work done by man is negative of magnitude of decrease in potential energy of chain.



$\Delta U = mg\,\frac{L}{2} - \frac{m}{2}g\,\frac{L}{4} = 3\,mg\,\frac{L}{8}$

$\therefore\ W = -\,\frac{3mg\ell}{8}$

17. **(D)** $\because\ \vec{F} = -\left[\frac{\Delta U}{\Delta x}\hat{i} + \frac{\Delta U}{\Delta y}\hat{j}\right] = -[-7\hat{i} + 24\hat{j}]$

$\therefore\quad F = \sqrt{7^2 + 24^2} = 25\ N \Rightarrow a = F/m = 25/5 = 5\ m/s^2$

$\because\quad v = u + at$

$= 0 + 5 \times 2 = 10$ m/s

18. **(A)**

Power = Av^3d

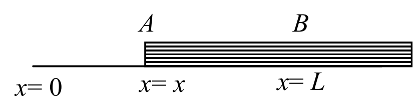
= $(Av)(v^2d)$

= (volume per sec) × (pressure)

$= \frac{4000 \times 10^{-6}}{60} \times (130 \times 10^{-3} \times 13.6 \times 10^3 \times 10)$

= 1.15 watt

19. Total frictional force on $AB = \int_x^L \frac{Mg}{L}Kx\,dx$



$\therefore$ Heat generated

$$= \int_0^L \frac{MgK}{L}\left[\frac{L^2 - x^2}{2}\right]dx = \frac{KmgL^2}{3}$$

$\therefore$ **(B)**

20. **[C]**

The capacitor is charged by a battery of 25 V. Let the magnitude of surface charge density on each plate be σ. Before inserting the dielectric slab, electric field strength between the plates,

$E = \frac{\sigma}{\varepsilon_0} = \frac{V}{d}$

or $E = \frac{\sigma}{\varepsilon_0} = \frac{25}{5 \times 10^{-3}} = 5000$ N/C

The capacitor is disconnected from the battery but charge on it will not change so that σ has the same value. When a dielectric slab of thickness 3mm is placed between the plates, the thickness of air between the plates will be 5 – 3 = 2 mm. Electric field strength in air will have the same value (5000 N/C) but inside the dielectric, it will be $\frac{5000}{K} = \frac{5000}{10}$

= 500 N/C

so potential difference = $E_{air}\,d_{air} + E_{med}\,d_{med}$

$= 5000 \times (2 \times 10^{-3}) + 500 \times (3 \times 10^{-3})$

= 11.5 V

21. Assuming sphere is complete then charge on it = $2Q$

So potential at point P due to this spherical charge = $\frac{1}{4\pi\varepsilon_0}\frac{2Q}{d}$

Hence potential due to hemisphere = $\frac{1}{4\pi\varepsilon_0}\frac{Q}{d}$

$\therefore$ **(A)**

22. (A)

$E_0 = \frac{kq}{1^2} + \frac{kq}{2^2} + \frac{kq}{4^2} + \frac{kq}{8^2} \ldots\ldots\ldots\ldots\infty$

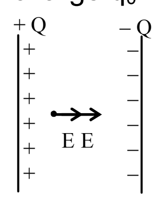
$E = kq\left(1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\ldots\infty\right)$

$E = kq\left(\frac{q}{1-r}\right) = kq \cdot \frac{1}{1-\left(\frac{1}{4}\right)} = \frac{kq\,4}{3}$

$12 \times 10^3 = \frac{9\times10^9\times4}{3} \times q \quad \therefore \quad q = 1\mu C$

23. (A)

Initially force on charge q_0 is



$F = q_0(2E) = 2q_0E$

If separation between the plates is doubled E= same

$\therefore$ F = same

24. $\frac{1}{C_\infty} = \frac{1}{C_\infty + C} + \frac{2}{C} = \frac{3C + 2C_\infty}{C(C_\infty + C)}$

$2C_\infty^2 + 2CC_\infty - C^2 = 0 \Rightarrow C_\infty = C\left(\frac{-1+\sqrt{3}}{2}\right)$

$\therefore$ **(C)**

25. (B)

Potential at centre = $V_q + V_{induced}$

$= V_q + 0$

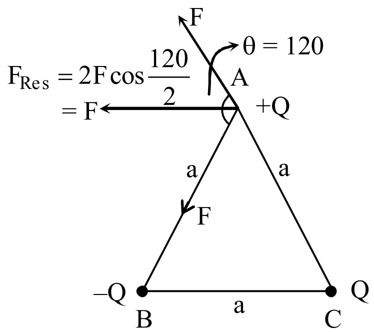
$= V_q = \frac{q}{4\pi\epsilon_0 r}$

Potential at P = potential at C

$V_q + V_{induced} = \frac{q}{4\pi \in_0 r}$

$V_{induced} = \frac{q}{4\pi \in_0 r} - \frac{q}{4\pi \in_0 r_1} = \frac{kq}{r} - \frac{kq}{r_1}$

26. (D)



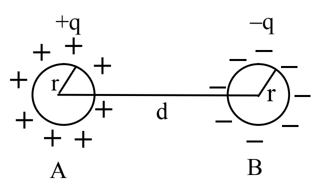
$\therefore$ Force perpendicular to BC is zero.

27. [C]

$C_{eff} = \frac{\varepsilon_0 A}{d}$ since effective capacitance between plates A and E is zero

$\therefore U = \frac{1}{2} CV^2 = \frac{\varepsilon_0 A}{2d} V^2$

28. (B)



Step I :

Step II : $V_A - V_B = \frac{Kq}{r} - \left(-\frac{Kq}{r}\right) = \frac{2Kq}{r}$

Step III : $C = \frac{q}{V_A - V_B} - \frac{qr}{2Kq} = 2\pi\varepsilon_0 r$

29. (C)

$U = \frac{1}{2} CV^2$

$U = \frac{1}{2}\left[\frac{\varepsilon_0 b(\ell - x)}{d} + \frac{\varepsilon_0 bx}{d} K\right] v^2 = \frac{\varepsilon_0 bv^2}{2d} (\ell - x + xK)$

$U = \frac{\varepsilon_0 bv^2}{2d} [\ell + k (K - 1)]$

30 (D)

$C_{eq} = C_1 + C_2 = 2 + 2 = 4\mu F$

$q = C_{eq} V = 4\mu F \times 8 = 32\mu q$

CHEMISTRY

31. (D)

Equivalent mass of $KMnO_4$ in acidic, basic and neutral medium are 31.6 158 and 52.6. The ratio will be 31.6 : 158 : 52.6.

Acid medium	:	Basic medium	:	Neutral medium
$E_w = \frac{M_w}{5} = \frac{158}{5}$	:	$E_w = \frac{M_w}{1} = \frac{158}{1} = 158$	:	$E_w = \frac{M_w}{3} = \frac{158}{3}$
3	:	15	:	5

32. (C)

$W = \Delta E$ (For q = 0)

$$-P_{ext}\Delta V = nC_V\Delta T$$

$$-1(2-1) = 1 \times \frac{3}{2}R\,(T_f - T)$$

$$\frac{-2}{3R} = T_f - T$$

$$T_f = T - \frac{2}{3 \times 0.0821}$$

33. (C)

The van der Waals' equation for 1 mol gas will be

$$\left[P + \frac{a}{V^2}\right][V - b] = RT$$

Since, b = 0

$$\therefore \quad PV = RT - \frac{a}{V} \qquad (y = c + MX \text{ form})$$

when PV is plotted against (1/V), we get straight line with negative slope, i.e. (–a).

$$\therefore \quad \text{Slope} = \frac{21.1 - 24.6}{3} = -1.5 = -a$$

a = 1.5

34. (C)

Work done = Area under BC line

$$\frac{1}{2}(6P_1 + P_1) \times 2V_1 = 7P_1V_1$$

35. (B)

$G = H - TS$

$G = U + PV - TS$

$\Delta G = \Delta U + P\Delta V + V\Delta P - T\Delta S - S\Delta T$

From the first and second laws,

$T\Delta S = \Delta U + P\Delta V$

$\therefore \quad \Delta G = V\Delta P - S\Delta T$

At constant pressure, $\Delta P = 0$

$$\frac{\Delta G}{\Delta T} = -S$$

From eqns. (i) and (ii)

$$G = H + T\frac{\Delta G}{\Delta T} \text{ or } G = H + T\left(\frac{\partial G}{\partial T}\right)_P$$

$$-\frac{H}{T^2} = -\frac{G}{T^2} + \frac{1}{T}\left(\frac{\partial G}{\partial T}\right)_P = \left[\frac{\partial (G/T)}{\partial T}\right]_P$$

$$H = -T^2\left[\frac{\partial (G/T)}{\partial T}\right]_P$$

36. (B)

$$\Delta S = 2.303\, nR \log_{10}\left(\frac{V_2}{V_1}\right)$$

$$= 2.303 \times 2 \times 8.314 \log_{10}\left(\frac{100}{10}\right)$$

$$= 38.3 \text{ J mol}^{-1} \text{K}^{-1}$$

37. (A)

m–moles of HCl = 20 × 0.1 = 2

m–moles of MgO reacted with HCl = $\frac{2}{2} \Rightarrow 1$

mass of MgO present = 1 × 40 mg

$\therefore$ % of MgO = $\frac{40}{320} \times 100 = 12.5\%$

38. (C)

$$\frac{r_{CH_3OCH_3}}{r_{CH_4}} = \sqrt{\frac{M_{CH_4}}{M_{CH_3OCH_3}}} \times \frac{P_{CH_3OCH_3}}{P_{CH_4}}$$

$$= \sqrt{\frac{16}{46}} \times \frac{0.8}{0.2} = 2.36 : 1$$

39. (A)

$$\frac{\partial}{\partial T} \ln K_P = \frac{\Delta H}{RT^2}$$

$$\frac{\partial}{\partial T} \ln K = \frac{E_a}{RT^2}$$

40. (C)

$$0.40 = aT_1^3 + bT_1$$

$$0.40 = a \times (1000) + b \times 10$$

0.4 = 1000a + 10b

$$0.92 - aT_2^3 + bT_2$$

$$\Rightarrow 0.92 = a \times 8000 + 20b$$

from Eqs. (1) and (2)

$$a = 2 \times 10^{-3},\ b = 0.038$$

$$S_m = \int \frac{aT^3 + bT}{T} \cdot dT$$

$$= \frac{a\left[T_2^3 - T_1^3\right]}{3} + b[T_2 - T_1]$$

$$= 0.813\ J/K - mol$$

41. (C)

42. (C)

Compound 'A' can show only optical isomerism due to presence of a chiral carbon, 'B' can show only geometrical isomerism, 'C' can show both geometrical as well as optical isomerism, D can show only optical isomerism.

43. (C)

(A) & (B) Cis-1,3-dimethyl cyclohexane & (D) is trans-1,4-dimethyl cyclohexane

44. (D)

The compound P is identical with (D) because both are meso. (A, B & C) are optically active.

45. (C)

46. (D)

47. (A)

melting point $\propto$ closed packing .

p- substituted benzene has linear geometry and such molecules are closely packed in their solid state.

48. (C)

49. (B)

50. (B)

51. (C)

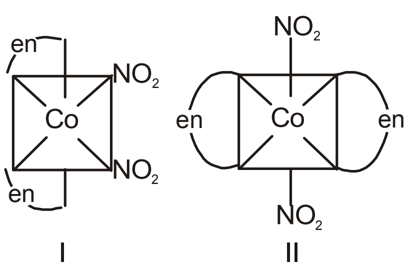
$[Fe\ (O_2)(CN)_4Cl]^{4-}$

Chlorotetracyano superoxoferrate (II) anion.

52. (C)

$$\left.\begin{array}{l} [Cu(NH_3)_4][PtCl_4] \\ [Cu(NH_3)_3Cl][PtCl_3NH_3] \\ [Cu(NH_3)Cl_3][PtCl(NH_3)_3] \\ [CuCl_4][Pt(NH_3)_4] \end{array}\right\}\text{Four isomers}$$

53. (A)



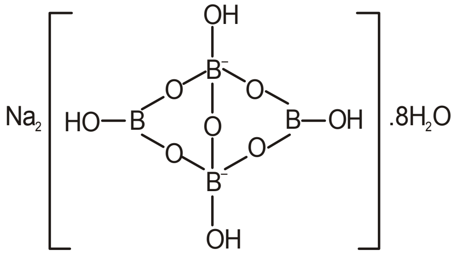
Both are geometrical isomerism

54. (D)

Due to decerasing trend of I.E. reactivity increase along the group but due to standard electrode potential reactivity of halogen in decreasing order along the group..

55. (B)

$Na_2B_4O_7.10H_2O$ exist as $Na_2 [B_4O_5(OH)_4].8H_2O$



sp^2 hybridised boron atom = 2 ; sp^3 hybridised boron atom = 2.

56. (A)

$\Delta H_{ion.} = -\Delta Heg$

57. (B)

Conceptual.

58. (C)

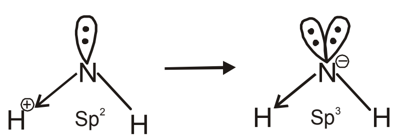
$[Zn (Br)_4]^{2-}$ = Sp^3 tetrahedral. Diamagnetic.

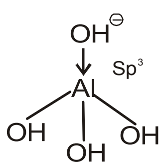
59. (B)

Strength of ligands α stability of complex.

$H_2O < NH_3 < NO_2^-$ is correct order of stability

60. (C)

(A) 

(B) 

Tetrahedral

(C) Sp^2 'S' Chr = 33 %

'P' Chr = 66%

(D) Hybridized orbitals always from 6 (sigma) bond due to π bond form by pure orbital only.

MATHEMATICS

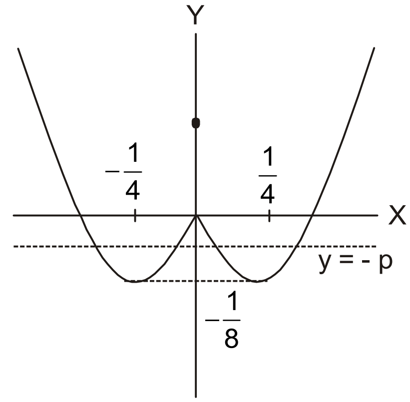
61. (A)

Let $\log_3 x = t$, then

$2t^2 - |t| = -p$

if $t \geq 0,\ 2t^2 - t = -p$

if t < 0, $2t^2 + t = -p$



$\therefore$ for 4 distinct roots $-\frac{1}{8} < -p < 0 \;\Rightarrow\; 0 < p < \frac{1}{8}$

62. (C)

$m_{SP} = \frac{2\tan\alpha}{\tan^2\alpha - 1} = -\tan 2\alpha = \tan(\pi - 2\alpha)$

since $0 < \pi - 2\alpha < \pi$

63. (A) $\sin x\sqrt{8}\,|\cos x| = 1$

$2\sin x\,|\cos x| = \frac{1}{\sqrt{2}}$

If $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\sin 2x = \sin\frac{\pi}{4} \Rightarrow 2x = n\pi + (-1)^n\frac{\pi}{4} \Rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8} \Rightarrow$ common difference $= \frac{\pi}{4}$

64. (B)

$f(x) = \sqrt{3}\sin x - \cos x + 2 = 2\sin\left(x - \frac{\pi}{6}\right) + 2$

Since f(x) is one–one and onto, f is invertible.

Now $fof^{-1}(x) = x \Rightarrow 2\sin\left(f^{-1}(x) - \frac{\pi}{6}\right) + 2 = x$

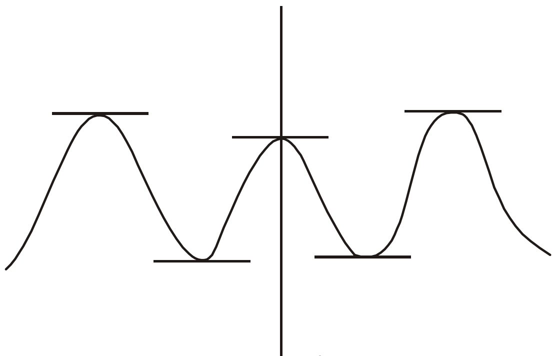
$\Rightarrow \sin\left(f^{-1}(x) - \frac{\pi}{6}\right) = \frac{x}{2} - 1 \Rightarrow f^{-1}(x) = \sin^{-1}\left(\frac{x}{2} - 1\right) + \frac{\pi}{6}$

Because $\left|\frac{x}{2} - 1\right| \leq 1$ for all $x \in [0, 4]$

Hence (B) is the correct answer.

65. (C)

Clearly g(x) is symmetric about x = 1/2



66. (B)

x = 0 $\Rightarrow$ f(2) = 2f(0) – f(1) = 2 × 2 – 3 = 1

x = 1 $\Rightarrow$ f(3) = 6 – 1 = 5

x = 2 $\Rightarrow$ f(4) = 2f(2) – f(3) = 2 × 1 – 5 = – 3

x = 3 $\Rightarrow$ f(5) = 2f(3) – f(4) = 2(5) – (– 3) = 13

Hence (B) is correct answer.

67. (B)

$3 = \lim_{x\to 0}(1+a\sin x)^{\text{cosecx}}$ $[1^{\infty}\text{form}] \Rightarrow \lim_{x\to 0} e^{\text{cosecx.asinx}} = e^{a}$

$\therefore$ $e^a = 3 \Rightarrow a = \log_e 3 = \ln 3$.

68. (B)

Using LMVT, $\frac{\tan^{-1}\beta-\tan^{-1}\alpha}{\beta-\alpha}=\frac{1}{1+c^2}$

where, $0<\alpha<c<\beta<\sqrt{3}$

So, $\frac{1}{4}<\frac{1}{1+c^2}<1$

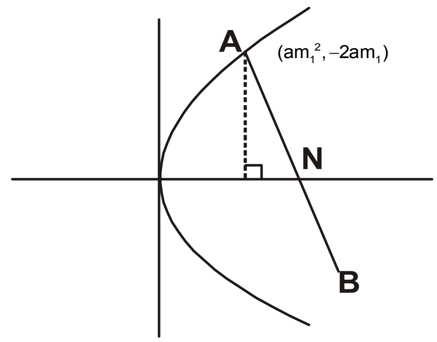
$$\Rightarrow \frac{1}{4}<\frac{\tan^{-1}\beta-\tan^{-1}\alpha}{\beta-\alpha}<1 \Rightarrow \frac{1}{4}<\frac{\tan^{-1}\left(\frac{\beta-\alpha}{1+\alpha\beta}\right)}{\beta-\alpha}<1$$

$$\Rightarrow 1<\frac{\beta-\alpha}{\cot^{-1}\left(\frac{1+\alpha\beta}{\beta-\alpha}\right)}<4$$

69. (C)

Equation of Normal through A is :

$y = m_1x - 2am_1 - am_1^3$



for the parabola $y^2 = 4ax$ where 4a = 1071

Here $N \equiv \left(\frac{1071}{2} + \frac{1071}{4}m_1^2, 0\right)$

If $B = (h,k)$, then $k = \frac{1071}{4}.m_1$

But since $k = m_1h - 2am_1 - am_1^3$

$$\Rightarrow m_1m_2m_3 = \frac{-k}{a} = \frac{-\frac{1071}{4}m_1}{\frac{1071}{4}} \Rightarrow m_2m_3 = -1$$

Hence option (C)

70. (D)

Intersection of the ellipse is possible if a > 1

$\Rightarrow b^2 - 5b + 7 > 1 \Rightarrow b^2 - 5b + 6 > 0 \Rightarrow b \in (-\infty, 2) \text{ U } (3, \infty)$

71. (C)

[sinx] + [2cosx] = –3

Possible if [sinx] = –1, [2cosx] = –2

$\Rightarrow \quad -1 \le \sin x < 0, \ -2 \le 2\cos x < -1$

$\Rightarrow \quad \pi < x < 2\pi$ and $\frac{2\pi}{3} < x < \frac{4\pi}{3} \Rightarrow \pi < x < \frac{4\pi}{3}$

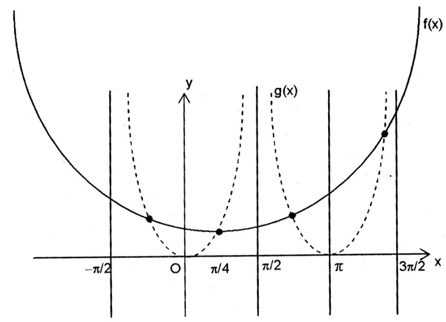
Now, f(x) = sinx + cosx = $\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

Clearly the range of f(x) is $[-\sqrt{2}, -1)$

72. (D)

Given equation is

$$16x^2 - 8\pi x + \pi^2 + 16 = |\tan x|$$



Let $f(x) = 16x^2 - 8\pi x + \pi^2 + 16$

and g(x) = | tan x |

$$\Rightarrow \quad f(x) = 16\left[\left(x - \frac{\pi}{4}\right)^2 + 1\right]$$

or y = f(x) is an upward parabola with vertex $\left(\frac{\pi}{4}, 16\right)$

Now, from graph it is clear that f(x) and g(x) intersect at infinitely many points.

73. (B)

$$m_2(x) = \min_{0 \le t \le x}(t^2 + (x-t)^2) = \min_{0 \le t \le x} 2\left(\left(t - \frac{x}{2}\right)^2 + \frac{x^2}{4}\right) = \frac{x^2}{2}$$

$$m_3(x) = \min_{0 \le t \le x}\left(\frac{t^2}{2} + (x-t)^2\right) = \min_{0 \le t \le x} \frac{3}{2}\left(\left(t - \frac{2}{3}x\right)^2 + \frac{2}{9}x^2\right) = \frac{x^2}{3}$$

. . .

$$m_n(x) = \frac{x^2}{n} \quad \therefore \quad m_n\left(\frac{1}{\sqrt{n+1}}\right) = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$\therefore \quad \sum_{n=1}^{k} m_n\left(\frac{1}{\sqrt{n+1}}\right) = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \ldots + \left(\frac{1}{k} - \frac{1}{k+1}\right) = 1 - \frac{1}{k+1} = \frac{k}{k+1}$$

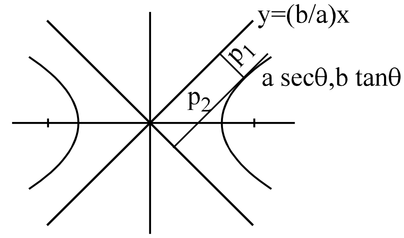
74. (D)

$$ae = \sqrt{\sec^2\theta - \tan^2\theta} = 1$$

Distance between foci = 2

75. (B)

$$p_1p_2 = \frac{a^2b^2}{a^2+b^2} = \frac{a^2.a^2(e^2-1)}{a^2e^2} = 6$$



$$\frac{2a^2}{3} = 6 \Rightarrow a^2 = 9 \Rightarrow a = 3$$

hence $2a = 6$

76. (D)

$f(1) = -6$

for maximum at $x = 1$

$$\lim_{x\to 1^-} f(x) = \tan^{-1}\alpha - 5 < -6$$

$$\tan^{-1}\alpha < -1 \Rightarrow \alpha < -\tan 1$$

77. (B)

$$x = \frac{b.(a\cos\beta) - a(b\cos\alpha)}{b-a} \qquad y = \frac{b.(a\sin\beta) - a(b\sin\alpha)}{b-a}$$

$$\Rightarrow \frac{x}{y} = \frac{\cos\beta - \cos\alpha}{\sin\beta - \sin\alpha}$$

$$\Rightarrow \frac{x}{y} = \frac{2\sin\left(\frac{\beta+\alpha}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\beta+\alpha}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)}$$

$$\Rightarrow x\cos\left(\frac{\alpha+\beta}{2}\right) = -y\sin\left(\frac{\alpha+\beta}{2}\right)$$

$$\therefore x\cos\left(\frac{\alpha+\beta}{2}\right) + y\sin\left(\frac{\alpha+\beta}{2}\right) = 0$$

78. (A)

Let $x = r\cos\theta$ and $y = r\sin\theta$, then $x^2 + y^2 = r^2$

Now $3x^2 - 4xy + 2y^2 = 12$

$$\Rightarrow r^2(3\cos^2\theta - 4\cos\theta\sin\theta + 2\sin^2\theta) = 12$$

$$\Rightarrow r^2 = \frac{24}{5 + \cos 2\theta - 4\sin 2\theta}$$

$\therefore\ m = \dfrac{24}{5+\sqrt{17}} = 3(5-\sqrt{17})$ and $n = \dfrac{24}{5-\sqrt{17}} = 3(5+\sqrt{17})$.

79. (C)

A rectangular hyperbola circumscribing a Δ also passes through its orthocentre

if $\left(ct_i, \dfrac{c}{t_i}\right)$ where i = 1, 2, 3 are the vertices of the Δ therefore orthocentre is

$\left(\dfrac{-c}{t_1t_2t_3}, -ct_1t_2t_3\right)$, where $t_1\,t_2\,t_3\,t_4 = 1$. Hence orthocentre is $\left(-ct_4, \dfrac{-c}{t_4}\right) = (-x_4, -y_4)$

80. (C)

$f(x) = \cos^2x + \cos^2 2x + \cos^2 3x = 1 + \cos^2x + \cos^2 2x - \sin^2 3x$

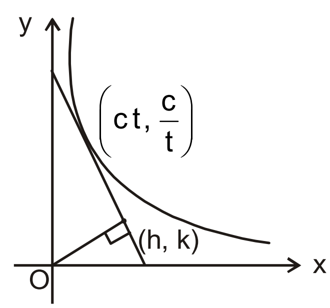
$= 1 + \cos^2x + \cos 5x.\cos x = 1 + \cos x(\cos x + \cos 5x)$

$\Rightarrow \cos x.\cos 2x.\cos 3x = 0$

$\Rightarrow x = (2n-1)\dfrac{\pi}{2}$ or $x = (2n-1)\dfrac{\pi}{4}$ or $x = (2n-1)\dfrac{\pi}{6}$

$\Rightarrow x = \dfrac{\pi}{2}, \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ $\Rightarrow$ no.of values of x = 5

81. (D)



Tangent at 't' passes through (h, k)

$\therefore h + kt^2 = 2ct$ ....(i)

& $\left(-\dfrac{1}{t^2}\right)\cdot\dfrac{k}{x} = -1$ ....(ii)

from (i) and (ii)

Locus is $\left(x^2+y^2\right)^2 = 4c^2xy$

82. (D)

Let $\dfrac{y}{x} = \alpha$, then

α will be minimum or maximum when $y = \alpha x$ ..... (1)

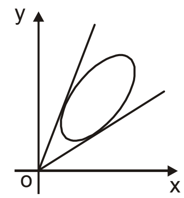
is a tangent to the ellipse

$x^2 + xy + 2y^2 - 6x - 10y + 14 = 0$ ..... (2)

Solving (1) and (2),

$(1+\alpha+2\alpha^2)\,x^2-2(3+5\alpha)\,x+14=0$

D = 0 $\Rightarrow$ $4\,(3+5\alpha)^2-56\,(1+\alpha+2\alpha^2)=0$



$\Rightarrow$ $3\alpha^2-16\alpha+5=0$ $\Rightarrow$ $\alpha=\frac{1}{3}$ or 5

$\therefore$ $m=\frac{1}{3}, M=5$

83. (A)

We have, $\int\frac{1}{4e^{-x}-9e^{x}}\,dx=\int\frac{e^x}{4-9e^{2x}}\,dx$

$=\int\frac{e^x}{2^2-(3e^x)^2}\,dx=\frac{1}{3}\int\frac{3e^x}{2^2-(3e^x)^2}\,dx=\frac{1}{3}\int\frac{1}{2^2-(3e^x)^2}\,d(3e^x)\left[\because d(3e^x)=3e^x\,dx\right]$

$=\frac{1}{3}\int\frac{dt}{2^2-t^2}$, where t = $3e^x$

$=\frac{1}{3}\times\frac{1}{2\times 2}\log\left|\frac{2+t}{2-t}\right|+c=\frac{1}{12}\log\left|\frac{2+3e^x}{2-3e^x}\right|+c$

84. (C)

radius of the first circle is half of the second circle.

$\Rightarrow$ Triangle is equilateral. $\Rightarrow$ Incentre and circumcentre coincides.

$\Rightarrow\alpha-2\beta=1$ and $\alpha+\beta=2$ $\Rightarrow(\alpha,\beta)\equiv\left(\frac{5}{3},\frac{1}{3}\right)$

85. (B)

The locus is $\frac{x^2}{16}-\frac{y^2}{48}=1$ $\Rightarrow$ $e=\sqrt{\frac{16+48}{16}}=2$

86. (B)

$f'(x)=5^x+7^x$

$f''(x)=5^x\ell n5+7^x\ell n7>0$

i.e $f'(x)$ is $\uparrow$

Maximum slope = $5^2+7^2=74$

87. (D)

P : (a secθ, a tanθ) ; N : [(a/2) (secθ + tanθ) , (a/2) (secθ + tanθ)]

$\Rightarrow$ 4h/a = 2 secθ + tanθ & 4k/a = secθ + 2 tanθ $\Rightarrow$ $x^2-y^2=3a^2/16$

88. (C)

$$y^2 - 2y + 1 = 4x^2$$

$$\Rightarrow (y-1)^2 = 4x^2 \quad \Rightarrow y - 1 = 2x, -2x$$

89. (B)

$$y - y_i = \left(\frac{dy}{dx}\right)_{(x_i, y_i)} (x - x_1)$$

$$-y_i = \frac{-y_i}{2x_i}(x_{i+1} - x_i)$$

$$x_{i+1} - x_i = 2x_i$$

$$x_{i+1} = 3x_i$$

90. (C)

If 'O' be the circumcentre then

$$OP + OR > PR \geq AD = 10$$

Also a circle through ABCD has radius $5\sqrt{2}$.