SOLUTIONS

WEEKLY TEST-A

RBA & RBPA

(JEE ADVANCED PATTERN)

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PHYSICS

1. **(A, B)**

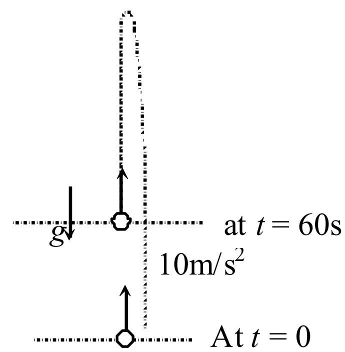
(C) Charges are unequal, but repulsion force will be $\frac{K(q)(3q)}{r^2}$ for both the spheres. As weight of both balls are same, so both will deflect equally.

(A,D) $\rightarrow$ If the ball is heavier, it will deflect less.

2. **(A,B,C,D)**

Since net force on negative charge is always directed towards fixed positive charge, the torque on negative charge about positive charge is zero. Therefore angular momentum of negative charge about fixed positive charge is conserved.

3. Total height $= \frac{1}{2}\times 10\times(60)^2 + \frac{(10\times 60)^2}{2\times 10}$ = 36000 m = 36 km



Total time $= 60 + 60 + \sqrt{\frac{2\times 36000}{10}}$

$= 60s + 60s + 60\sqrt{2}s$

$= \left(120 + 60\sqrt{2}\right)$s

$\therefore$ **(B) (D)**

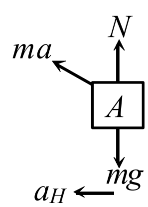
4. From FBD of *A* with respect to *B*

$a_v = 0$

$mg = N - ma\sin\theta$

$\Rightarrow \quad N = mg - ma\sin\theta$

$ma\cos\theta = ma_H \quad \Rightarrow \quad a_H = a\cos\theta$



If block *B* is having friction then, for a_H =0

$ma\cos\theta \le \mu N = \mu(mg - ma\sin\theta)$

$\mu \ge \frac{a\cos\theta}{g - a\sin\theta}$

$\therefore$ **(B) (D)**

5. $F_x = -\frac{dU}{dx} = 7 \Rightarrow a_x = \frac{F_x}{m} = \frac{7}{5}$

$F_y = -\frac{dU}{dx} = -24 \Rightarrow a_y = \frac{F_y}{m} = \frac{-24}{5}$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{24}{5}\right)^2}$ **=** 5 m/s²

$\vec{a} = \left(\frac{7}{5}\right)\hat{i} - \left(\frac{24}{5}\right)\hat{j}$ and $\vec{v} = 14.4\hat{i} + 4.2\hat{j}$

$\vec{a}.\vec{v} = \frac{7}{5} \times 14.4 - \frac{24}{5} \times 4.2 = 0$

Hence $\vec{a}$ is perpendicular to $\vec{v}$.

$V_x = u_x + a_x t$ $V_y = v_y + a_y t$

$V_x = 14.4 + \left(\frac{7}{5}\right)^4$ $V_y = (4.2) - \left(\frac{24}{5}\right)^4$

$v = \sqrt{V_x^2 + V_y^2} =$ 25 m/s

$\therefore$ **(A) (B) (C) & (D)**

6. $F = \frac{-K}{r}$

where K is proportionally constant.

$\frac{mV^2}{r} = \frac{-K}{r}$ $i.e., \ V \propto r^0$

$T = \frac{2\pi r}{V} = \frac{2\pi r}{\sqrt{\frac{K}{m}}}$ $i.e., \ T \propto r$.

So choices (*a*) and (*d*) are wrong and choices (*b*) and (*c*) are correct.

$\therefore$ **(B) and (C)**

7. (B, C, D)

8. (A)

9. (D)

$$E = \frac{\sigma}{\varepsilon_0 k} = \frac{5 \times 10^{-4}}{8.85 \times 10^{-12} \times 5.4} = 10^7 \text{ Vm}^{-1}$$

$\therefore$ **(D)**

10. (C)

$V = Ed = 10^7 \times 5 \times 10^{9} = 0.05$ V

$\therefore$ **(C)**

11. (B)

12. (C)

13. (B)

From above

$$2t = \frac{t^3}{6}$$

$\Rightarrow$ $t^2 = 12$

$\Rightarrow$ $t = 2\sqrt{3}$ sec.

14. (C)

Sol. a = t = 4

$\therefore$ after 4 seconds $V_B = 2$ m/s

$$V_p = \frac{4^2}{2} = 8 \text{ m/s}$$

$\therefore$ $V_{rel} = 8 - 2 = 6$ m/s.

15. (A)

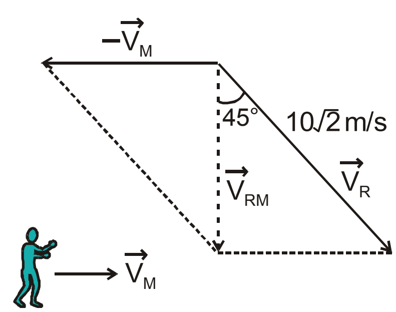
16. (B)

In the first case :

From the figure it is clear that

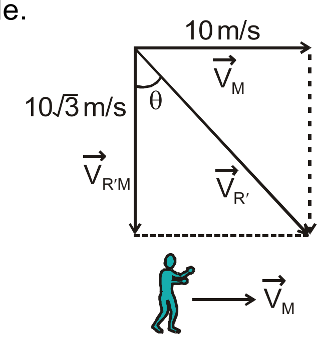
$\vec{V}_{RM}$ is 10 m/s downwards and

$\vec{V}_M$ is 10 m/s towards right.



In the second case :

Velocity of rain as observed by man becomes $\sqrt{3}$ times in magnitude.



∴ New velocity of rain

$$\vec{V}_{R'} = \vec{V}_{R'M} + \vec{V}_M$$

∴ The angle rain makes with vertical is

$$\tan\theta = \frac{10}{10\sqrt{3}} \text{ or } \theta = 30°$$

∴ Change in angle of rain = 45 – 30 = 15°.

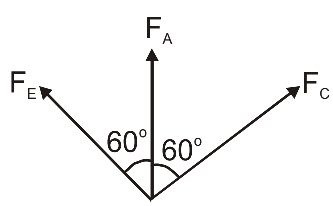
17. (A)

(A- r) ; (B -q) ; (C-s) ; (D - p)

(A) $F = K(3q)q\dfrac{\sqrt{\frac{2}{3}}(2a)}{8a^3} = \dfrac{1}{2}\sqrt{\dfrac{3}{2}}\dfrac{Kq^2}{a^2}$

(B) $F = 2\dfrac{Kq^2}{(2a)^2}\times\cos 45° + \dfrac{Kq^2}{(2\sqrt{2}\,a)^2} = \dfrac{Kq^2}{8a^2}(2\sqrt{2}+1)$

(C) Putting addinional charge 'q' on D



$\therefore F_{net} = F_A + 2F_C\cos 60°$

$= \dfrac{2Kq^2}{4a^2} + \dfrac{2Kq^2}{a^2}$

$= \dfrac{5}{2}\dfrac{Kq^2}{a^2}$

(D) $F = q.E = q\,.5Kq\,\dfrac{a}{(\sqrt{2}a)^3}$

$= \dfrac{5}{2\sqrt{2}}\left(\dfrac{Kq^2}{a^2}\right)$.

18. (C)

(A - R) ; (B - P), (C – S) ; (D – Q)

If S_1 is closed, then $\frac{kQ_A}{a}+\frac{kQ}{2a}=0$ $\quad Q_A=-\frac{Q}{2}$

If S_2 is closed, then $\frac{kQ_B}{2a}=0$ $\quad Q_B = 0$

If S_3 is closed, then $\frac{kQ}{3a}+\frac{kQ_C}{3a}=0$ $\quad Q_C=-Q$

If S_4 is closed, charge on shell *B* is *Q*

19. (B)

(A-s), (B-p), (C-r), (D-q)

The initial velocity of A relative to B is $\vec{u}_{AB}=\vec{u}_A-\vec{u}_B$ = $(8\hat{i}-8\hat{j})$ m/s $\therefore$ $u_{AB} = 8\sqrt{2}$ m/s

Acceleration of A relative to B is -

$\vec{a}_{AB}=\vec{a}_A-\vec{a}_B=(-2\hat{i}+2\hat{j})$ m/s^2 $\therefore$ $a_{AB}=2\sqrt{2}$ m/s^2

since B observes initial velocity and constant acceleration of A in opposite directions, Hence B observes A moving along a straight line.

From frame of B

Hence time when v_{AB} = 0 is $t=\frac{u_{AB}}{a_{AB}}$ = 4 sec.

The distance between A & B when v_{AB} = 0 is $S=\frac{u_{AB}^2}{2a_{AB}}=16\sqrt{2}$ m

The time when both are at same position is -

$T=\frac{2u_{AB}}{a_{AB}}$ = 8 sec.

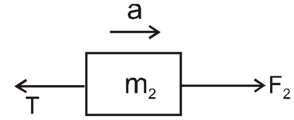
Magnitude of relative velocity when they are at same position in $u_{AB}=8\sqrt{2}$ m/s.

20. (D)

(A-q), (B-r), (C-q), (D-r)

Let a be acceleration of two block system towards right

$\therefore\ a=\frac{F_2-F_1}{m_1+m_2}$



The F.B.D. of m_2 is

$\therefore\ F_2 - T = m_2 a$

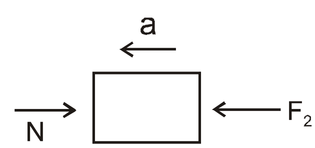
Solving T = $\frac{m_1 m_2}{m_1+m_2}\left(\frac{F_2}{m_2}+\frac{F_1}{m_1}\right)$

(B) Replace F_1 by – F_1 is result of A

$\therefore$ T = $\frac{m_1 m_2}{m_1+m_2}\left(\frac{F_2}{m_2}-\frac{F_1}{m_1}\right)$

(C) Let a be acceleration of two block system towards left

$\therefore$ a = $\frac{F_2-F_1}{m_1+m_2}$



The FBD of m_2 is

$\therefore$ $F_2 - N_2 = m_2 a$

Solving N = $\frac{m_1 m_2}{m_1+m_2}\left(\frac{F_1}{m_1}+\frac{F_2}{m_2}\right)$

(D) Replace F_1 by $-F_1$ in result of C

N = $\frac{m_1 m_2}{m_1+m_2}\left(\frac{F_2}{m_2}-\frac{F_1}{m_1}\right)$

CHEMISTRY

21. (A), (C)

22. (B), (C), (D)

23. (B)

The velocity corresponding to the maxima is the most probable velocity which is given by the expression.

$$C_{mp}=\sqrt{\frac{2RT}{M}}$$

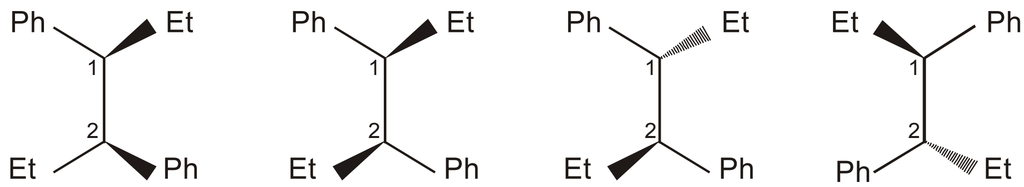
$$RT=\frac{C_{mp}^2 M}{2}=\frac{(200)^2\times 100\times 10^{-3}}{2}=40000\times 50\times 10^{-3}\ \text{J mol}^{-1}=40\times 50\ \text{J mol}^{-1}$$

No. of moles = $\frac{300}{100}=3$

$$E=\frac{3}{2}nRT=\frac{3}{2}\times 3\times 40\times 50=9\times 10^3 J=9kJ$$

24. (A), (B), (C)

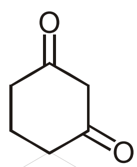
25. (A), (B), (D)



Compound (I) and (II) are optically active, so can not have plane of symmetry as well as centre of symmetry, (III) and (IV) are optically inactive and possess centre of symmetry not plane of symmetry (II) has two fold axis of symmetry but (III) has no axis of symmetry.

26. (A), (B)

(A) Ester is poor –M group

(B) 

has higher enol content due to active methylene group

27. (A), (B), (C), (D)

(I) and (II) do not have chiral centres, but these are geometrical isomers. (I) is cis while (II) is trans isomers. (III) and (IV) are geometrical isomer and each have two chiral centres, so these are also optical isomers.

Since, Geometrical isomers are diastereomers. So, (I) and (II) as well as (III) and (IV) are diastereomers.

28. (A), (B), (C)

29. (D)

30. (B)

31. (A)

32. (A)

33. (B)

In B most stable cation is formed hence maximum rate of reaction.

34. (C)

Step-1 is r.d.s and Step-2 is fast reaction.

35. (C)

36. (D)

37. (B)

(A — Q); (B — S); (C — P); (D — R)

The given equilibrium is

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

$$K_C = \frac{[SO_3]^2}{[SO_2]^2[O_2]} \quad \text{… (i)}$$

(A) If $[SO_3] = [SO_2]$

Then $K_C = \frac{1}{[O_2]}$

$[O_2] = \frac{1}{K_C} = \frac{1}{100} = 0.01$ mole/litre

Total moles of O_2 present = 0.01 × 10 = 0.1 mole

∵ Volume of vessel is 10 litre

(B) When $[SO_3] = 2[SO_2]$

Then $K_C = \frac{[SO_2]^2}{[SO_2]^2[O_2]}$ from Eq. (i)

$100 = \frac{4}{[O_2]}$

$[O_2]$ = 4/100 = 0.04 mole/litre

Total moles of O_2 in vessel at equilibrium = 0.04 × 10 = 0.4 mole

(C) When $[SO_3] = 3[SO_2]$

Then $K_C = \frac{[SO_2]^2}{[SO_2]^2[O_2]}$ from Eq. (i)

$100 = \frac{4}{[O_2]}$

$[O_2]$ = 9/100 = 0.09 mole/litre

Total moles of O_2 in vessel at equilibrium = 0.09 × 10 = 0.9 mole

(D) When $[SO_3] = 0.5[SO_2]$

Then $K_C = \frac{[SO_2]^2}{[SO_2]^2[O_2]}$ from Eq. (i)

$100 = \frac{4}{[O_2]}$

$[O_2]$ = 0.25/100 = 0.0025 mole/litre

Total moles of O_2 in vessel at equilibrium = 0.0025 $\times$ 10 = 0.025 mole

38. (C)

(A — R); (B — Q); (C — S); (D — P)

39. (B)

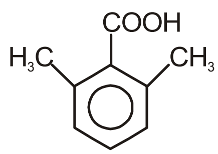
$A \rightarrow Q$; $B \rightarrow R$; $C \rightarrow S$; $D \rightarrow P$

40. (A)

($a \rightarrow p$; $b \rightarrow q,s$; $c \rightarrow r$; $d \rightarrow q,s$)

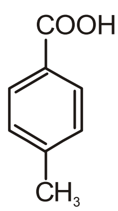
$\because pK_a$ of benzoic acid = 4.22 $\therefore pK_b = 9.78$

and pK_a of phenol = 9.7 $\therefore pK_b = 4.3$



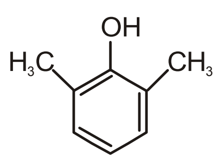
is more acidic than benzoic acid

$\therefore$ its $P^{K}b > 9.78$



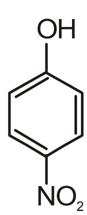
is less acidic than benzoic acid but more acidic than phenol

$\therefore$ its $pK_b < 9.78$ or $4.3 < pK_b < 9.78$



is less acidic than phenol

$\therefore$ its $pK_b > 4.3$



is more acidic than phenol but less acidic than benzoic acid

$\therefore$ it $pK_b < 9.78$ or $4.3 < pK_b < 9.78$

MATHEMATICS

41. (A, C)

$$f(x)=\frac{x^3}{3}+\frac{x^2}{2}+x+2 \Rightarrow \quad f'(x)=x^2+x+1$$

so f(x) is always increasing so, minimum of f(g(x)) will concide with minima of g(x).

42. (A, B, C, D)

Here $f'(x)>0, \forall x \in R$

$\Rightarrow f(x)$ is increasing function

Also f(–x)= –f(x) $\Rightarrow f(x)$ is odd function

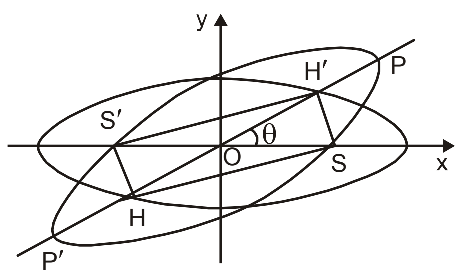
Now $\alpha+\beta>0 \Rightarrow \alpha>-\beta$

$\Rightarrow f(\alpha)>f(-\beta) \Rightarrow f(\alpha)>-f(\beta) \Rightarrow f(\alpha)+f(\beta)>0$

Also, $\alpha+\beta<0 \Rightarrow f(\alpha)<-f(\beta)$

$\Rightarrow f(\alpha)+f(\beta)<0$

43. (A, B, C)



(A) Clearly 'O' is the mid point of SS' and HH'

(B) $AA'=SH'+S'H'=SH'+SH=PP'=2a$ (say)

$\therefore\ OH'=ae', OS=ae$

$H'(ae'\cos\theta, ae'\sin\theta)$ lies on

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow \theta=\cos^{-1}\sqrt{\frac{e^2+e'^2-1}{e^2e'^2}} \quad \Rightarrow \theta=\frac{\pi}{2} \text{ if } e^2+e'^2=1$$

(C) $HH'=SS'$, if $e'=e$

44. (A, B, C)

Curve through the intersection of S_1 and S_2 is given by $S_1 + \lambda S_2 = 0$

$$\Rightarrow x^2(\sin^2\theta + \lambda\cos^2\theta) + 2(h\tan\theta - \lambda h'\cot\theta)xy + (\cos^2\theta + \lambda\sin^2\theta)y^2$$

$$+(32 + 16\lambda)x + (16 + 32\lambda)y + 19(1 + \lambda) = 0$$

45. (A, B, C, D)

Given that either $px^2 + qy^2 + r = 0$ or $(x - 1)^2 + y^2 = 1$

(i) two straight lines if $r = 0$ and $p\,q < 0$

(ii) circle, if $p = 1$ and r is of opposite sign to that of p

(iii) a hyperbola if $pq < 0$ and $r \neq 0$

(iv) a circle and an ellipse if $pq > 0$ and $p \neq q$ and $pr < 0$

46. (B, D)

f(x) is undifined at $x = -2$

$\Rightarrow$ g(f(x)) is also undefined at $x = -2$

Also g(x) is undefined at $x = \pm 1,\ n\pi \quad (n \in I)$

Now $f(x) = 1 \Rightarrow \dfrac{2x+1}{x+2} = 1 \Rightarrow x = 1$

$$f(x) = -1 \Rightarrow \frac{2x+1}{x+2} = -1 \Rightarrow x = -1$$

$$f(x) = n\pi \Rightarrow \frac{2x+1}{x+2} = n\pi \Rightarrow x = \frac{2n\pi - 1}{2 - n\pi} \qquad \{n \in I\}$$

47. (A, B, D)

Since, $\sin^2\left(\dfrac{A}{2}\right) + \sin^2\left(\dfrac{B}{2}\right) + \sin^2\left(\dfrac{C}{2}\right) =$

$$\frac{1}{2}\left[3 - (\cos A + \cos B + \cos C\right] = \frac{3}{2} - \frac{1}{2}(\cos A + \cos B + \cos C)$$

but the maximum value of $\cos A + \cos B + \cos C = \dfrac{3}{2}$

$\therefore$ minimum value of $\sum \sin^2\left(\dfrac{A}{2}\right) = \dfrac{3}{2} - \dfrac{3}{4} = \dfrac{3}{4}$

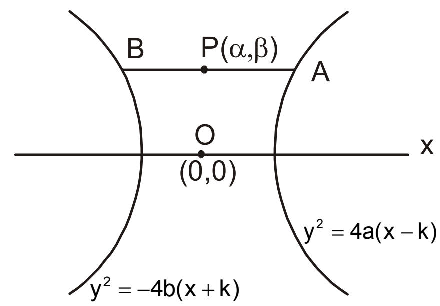
$\therefore \sum \sin^2\left(\dfrac{A}{2}\right) \geq \dfrac{3}{4}$ hence (c) is incorrect.

and (A) , (B) and (D) are correct and hold good in any triangle

48. (A, D)

Let the parabolas be $y^2 = 4a(x-k)$ and $y^2 = -4b(x+k)$.

A line parallel to common axis is y = h



Let $A = \left(\frac{h^2}{4a} + k, h\right)$, $B = \left(-k - \frac{h^2}{4b}, h\right)$.

If $P(\alpha, \beta)$ is the mid point of AB, then

$\alpha = \frac{1}{2}\left(\frac{h^2}{4a} + k - k - \frac{h^2}{4b}\right)$ and $\beta = h$.

$\therefore\ 2\alpha = \frac{h^2}{4}\left(\frac{1}{a} - \frac{1}{b}\right) \Rightarrow 2\alpha = \frac{\beta^2}{4}\left(\frac{b-a}{ba}\right)$

$\therefore$ Locus of P is $2x = \frac{y^2}{4}\left(\frac{b-a}{ba}\right)$

49. (B)

a = 1

$f(x) = 8x^3 + 4x^2 + 2bx + 1$

$f'(x) = 24x^2 + 8x + 2b = 2(12x^2 + 4x + b)$

for increasing function, $f'(x) \geq 0 \quad \forall\ x \in R$

$\therefore\ D \leq 0 \quad \Rightarrow \quad 16 - 48b \leq 0 \quad \Rightarrow \quad b \geq \frac{1}{3}$

$\therefore \lambda = \frac{1}{3}$

50. (A)

if b = 1

$f(x) = 8x^3 + 4ax^2 + 2x + a$

$f'(x) = 24x^2 + 8ax + 2$ or $2(12x^2 + 4ax + 1)$

for non monotonic, f ' (x) = 0 must have distinct roots

hence D > 0 i.e. $16a^2 - 48 > 0 \Rightarrow a^2 > 3$; $\therefore\ a > \sqrt{3}$ or $a < -\sqrt{3}$

$\therefore\ a \in \{2, 3, 4, ...\}$

sum = 5050 – 1 = 5049

sum of squares = 338350 – 1= 338349

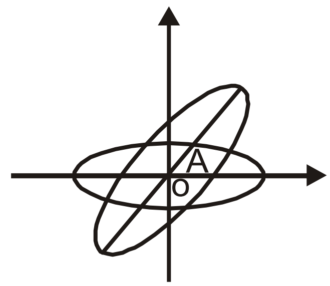
51. (A)

$$S_1 = \frac{x^2}{a^2} + \frac{y^2}{a^2\left(1-e^2\right)} = 1$$

So distance between foci = 2ae then OA = ae

Let slope of OA is $\tan\alpha$ then co-ordinate of

A (ae cos α, ae sin α) which lies on S_1



So, $e^2\cos^2\alpha + \dfrac{e^2\sin^2\alpha}{1-e^2} = 1$

$\Rightarrow\ e^2 - e^4\cos^2\alpha = 1 - e^2$

$\Rightarrow\ \cos^2\alpha = \dfrac{2e^2-1}{e^4}\ \Rightarrow\ 0 \le \dfrac{2e^2-1}{e^4} \le 1\ \Rightarrow\ 2e^2 - 1 \ge 0$ and $2e^2 - 1 \le e^4$

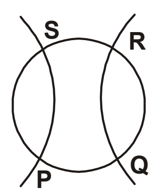
$\Rightarrow\ e^2 \ge \dfrac{1}{2}$ and $(e^2-1)^2 \ge 0\ \Rightarrow$ Minimum value of $e^2 = \dfrac{1}{2}$

52. (C)

Now $\tan\theta = \dfrac{a}{b}\tan\alpha = \dfrac{a}{a\sqrt{1-e^2}} \times \dfrac{1-e^2}{\sqrt{2e^2-1}} = \sqrt{\dfrac{1-e^2}{2e^2-1}}$ then $\sin\theta = \dfrac{\tan^2\theta}{1+\tan^2\theta} = \dfrac{\dfrac{1-e^2}{2e^2-1}}{\dfrac{e^2}{2e^2-1}} = \dfrac{1-e^2}{e^2}$

53. (A)

$y^2 = x^2 - 9a^2$ and $x^2 + y^2 + 2gx + 2fy + c = 0$



$\Rightarrow\ 2x^2 + 2gx + (c - 9a^2) = -2fy$

Squaring both sides,

$$4x^4 + 4g^2x^2 + (c-9a^2)^2 + 8gx^3 + 4gx(c-9a^2) + 4x^2(c-9a^2) - 4f^2x^2 + 36f^2a^2 = 0$$

x_1, x_2, x_3, x_4 are the roots of the above equation.

$\therefore\ x_1 + x_2 + x_3 + x_4 = \dfrac{-8g}{4}\ \Rightarrow\ x_1 + x_2 + x_3 = -2g - x_4$

Similarly, $y_1 + y_2 + y_3 = -2f - y_4$. $\therefore$ centroid of ΔPQR is $\left(\dfrac{-2g-x_4}{3}, \dfrac{-2f-y_4}{3}\right)$.

54. (A)

Let $x = \dfrac{-2g-x_4}{3}$ and $y = \dfrac{-2f-y_4}{3}$, then

$3x + 2g = -x_4$ and $3y + 2f = -y_4$

$x_4^2 - y_4^2 = 9a^2 \Rightarrow (3x+2g)^2 - (3y+2f)^2 = 9a^2$

55. (C)

$f''(c_2)f''(c_1) < 0$ and $f'(c_1) = f'(c_2) = 0$

$f''(c_1) - f''(c_2) > 0 \Rightarrow f''(c_1) > 0$ and $f''(c_2) < 0$

$\Rightarrow$ c_2 is point of local maximum and c_1 is point of local minimum for $f(x)$

$\Rightarrow$ $f'(x) = 0$ at least four times in $[c_1 - 1, c_2 + 1]$.

56. (B)

Here c_1 is local maximum and c_2 is local minimum

$\Rightarrow$ $f'(x) = 0$ has at least two roots in $[c_1 - 1, c_2 + 1]$.

57. (C)

(P) $f'(x) = -2\sin x + a \geq 0 \forall x$

(Q) $f'(x) = a^2 - \sin x \geq 0$

$|a| \geq 1 \Rightarrow a \in (-\infty, -1] \cup [1, \infty)$

(R) Product of slopes is (– 1)

$a^2 = 2$

$a = \pm\sqrt{2}$

(S) $\lim_{x\to 0}(1 + a\sin x)^{\operatorname{cosec} x} = \frac{1}{\sqrt{e}}$

$= e^a = \frac{1}{\sqrt{e}}$

$\Rightarrow a = -\frac{1}{2}$

58. (A)

(P) $f'(x) = 3ax^2 - 18x + 9 = 3(ax^2 - 6x + 3)$

As f (x) is strictly increasing on R, so

a > 0 and $D \leq 0 \Rightarrow a \geq 3$

(Q) Put $\cos x = t, t \in [-1,1]\ \forall x \in R$

Let $g(t) = t^3 - 6t^2 + 11t - 6 = (t-1)(t-2)(t-3), t \in [-1,1]$

$\therefore$ range of g(t) = [g(–1), g(1)] = [–24, 0]

(R) $x^3 - y^2 = 0$

$$\left.\frac{dy}{dx}\right]_{P(4m^2, 8m^3)} = \frac{3 \times 16m^4}{16m^3} = 3m$$

Let Q be $(4m_1^2, 8m_1^3)$

$\therefore$ Slope of normal at Q $= \dfrac{-1}{3m_1}$

$\therefore\ 3m = \dfrac{-1}{3m_1} \Rightarrow m_1 = \dfrac{-1}{9m}$ ... (1)

Also, slope of PQ $= \dfrac{8(m^3 - m_1^3)}{4(m^2 - m_1^2)} = \dfrac{2(m^2 + m_1^2 + mm_1)}{m + m_1} = 3m$

$\Rightarrow (2m_1 + m)(m_1 - m) = 0$

$\Rightarrow 2\left(\dfrac{-1}{9m}\right) + m = 0 \quad \Rightarrow\ m^2 = \dfrac{2}{9}$

(S) $(\sin\theta - \cos\theta)(\tan\theta + \cot\theta) = 2$

$\Rightarrow \dfrac{\sin\theta - \cos\theta}{\sin\theta\cos\theta} = 2$

Let $y = \sin\theta - \cos\theta$

$\therefore y = 1 - y^2 \Rightarrow y^2 + y - 1 = 0 \Rightarrow y = \dfrac{-1 \pm \sqrt{5}}{2}$

59. (B)

(P) Common tangent $y = x + 2\sqrt{2}$ point of contact with two hyperbola are $\left(\dfrac{-9}{2\sqrt{2}}, \dfrac{-1}{2\sqrt{2}}\right)$ and $\left(\dfrac{1}{2\sqrt{2}}, \dfrac{9}{2\sqrt{2}}\right)$ hence length = 5

(Q) For hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

S is (ae, 0) equation of line $y=\frac{b}{a}(x-ae)$, $P\left(\frac{a^2(1+e^2)}{2ae},\frac{-ab(e^2-1)}{2ae}\right)$.

$\therefore\ SP=\frac{b^2}{2a}=\frac{9}{8}$

(R) Equation of tangent $\frac{x\cos\theta}{2\sqrt{3}}+\frac{y\sin\theta}{18}=1$

Sum of intercept is $f(\theta)=2\sqrt{3}\sec\theta+18\text{cosec}\theta$

$\Rightarrow\ f'(\theta)=\frac{2\sqrt{3}\sin^3\theta-18\cos^3\theta}{\sin^2\theta\cos^2\theta}$

For max./min. $f'(\theta)=0$

$\tan^3\theta=3\sqrt{3}\ \Rightarrow\ \tan\theta=\sqrt{3}$

(S) Equation of tangent at (x_1, y_1) is $xx_1 - 2yy_1 = 4$ which is same as $2x+\sqrt{6}y=2$

Hence $\frac{x_1}{2}=\frac{-2y_1}{\sqrt{6}}=\frac{4}{2}$

$x_1 = 4$, $y_1 = -\sqrt{6}$

m = slope of line = $\frac{-\sqrt{6}}{4}\ \Rightarrow\ 4m=-\sqrt{6}$

60. (A)

(P) $\frac{x-4}{5}=\frac{y+13}{1}=\frac{-2(20-13+6)}{26}$

$x=-1, y=-14$

(Q) $|a|\in(1,\sqrt{2})$

(R) Equation of circle passes though origin and touching the line y = x is $x^2 + y^2 + \lambda(y - x) = 0$ therefore according to quesiton equation of common chord will be $(6x + 8y - 7) + \lambda(x - y) = 0$ and this common chord always passes though the point (1/2, 1/2)

(S) The point at shortest distance form the line and lying on the circle is (2, 1)