SOLUTIONS

PROGRESS TEST-6

RBS-1801 & 1802

JEE ADVANCED PATTERN

Test Date: 25-11-2017

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Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd., Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

PHYSICS.

1. In the direction of electric field potential decreases.

If $a > b$ $V_B > V_A$

$a = b$ $V_B = V_A$

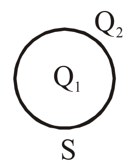
$a < b$ $V_A > V_B$

$\therefore$ **(B) (C) (D)**

2. **(A), (B) and (C)**

From Gauss Law

$$\phi = \oint E.dS = \frac{q_{\text{enclosed}}}{\epsilon_0}$$



It is obvious from Gauss Law that if Q_1 changes, E and ϕ both will change. So, choice (A) is correct.

If Q_2 changes, charge enclosed by Gaussian surface S will not change so ϕ will not change. But electric field at point under consideration is net electric field due to charges present inside and outside the surface. So E will change, hence choice (B) is correct.

If $Q_1 = 0$ then charge enclosed by Gaussian surface is zero so flux ϕ will be zero. But E can persist due to charge Q_2. So, choice (C) is correct.

Choice (D) is wrong. Since charge enclosed by Gaussian surface is Q1 (which is non-zero) so flux is non-zero. Flux has been defined as $\phi = \int E.dS = \frac{q_{enclosed}}{\varepsilon_0}$

If $\phi \neq 0$ then E must be non-zero.

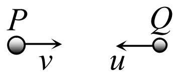
3. Area of a vs x gives $\frac{v^2 - u^2}{2} \Rightarrow$ Velocity at $x = 4$m is $4\sqrt{11}$ m/s

As $W_{\text{Conservation}} = -\Delta u$ also $\Delta KE = W_{\text{Conservation}} + W_{\text{External}}$

$\therefore$ **(A), (B), (C), (D)**

4. $5mu = 2mv - mu$

$v = 3u$



$$\frac{1}{2}m(5u)^2 + W = \frac{1}{2}mu^2 + \frac{1}{2}\times 2mv^2$$

$$W = -3mu^2$$

$\therefore$ **(B) and (D)**

5. Friction maximum = 24 N

So net applied force on *P* is less than f_{max}.

Hence acceleration is zero and T_A = 20 N, T_B = 40 N

Contact force $= \sqrt{N^2 + (f)^2} = \sqrt{(40)^2 + (20)^2} = 20\sqrt{5}$ N (g = 10m/s^2)

$\therefore$ **(A) (B) (C) and (D)**

6. For maxima path difference = $n\lambda$

If d = path difference between waves reaching point O = 7 λ

O will be maxima.

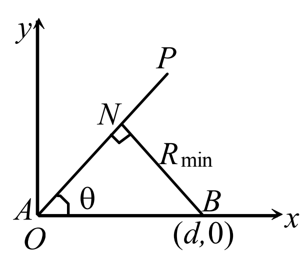
For $d = \lambda$ only one maxima at O is possible, the screen being finite.

$\therefore$ **(A), (B), (C) and (D)**

7. Upper part of lens L_3 behaves as lens L_1 and lower part of lens L_3 behaves as lens L_2.

$\therefore$ **(C) and (D)**

8. $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = v\hat{i} + 2v\hat{j}$



$$\tan\theta = \frac{2v}{v} = 2$$

$\overrightarrow{OP}$ gives the direction of $\vec{v}_{AB}$

$$R_{min} = d\sin\theta = \frac{2d}{\sqrt{5}}$$

$$T_0 = \frac{d}{5v}$$

$\therefore$ **(A) and (B)**

9. $d\vec{r} = dxi + dyj$, $dw = F.dr = -\alpha\, x\, y^2\, dy$

On the path $x = y$, so $= -\alpha y^3 dy$,–50.6

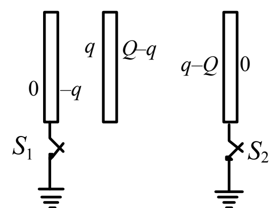
$\therefore$ **(C)**

10. w = $\int_{y_1=0}^{y_2=3} -\alpha\, x\, y^2 dy$ does not integrable without knowing relation between y & x show given force as non conservative in nature

$\therefore$ **(A)**

11. **(C)**

12. $\frac{(Q-q)3d}{\varepsilon_0 A} = \frac{qd}{\varepsilon_0 A} \Rightarrow q = \frac{3Q}{4}$



$\therefore$ **(C)**

13. $L^2\left(-\frac{dy}{dt}\right) = A\sqrt{2gy}$ (*y* is height of liquid)

$$-\int_L^y y^{-\frac{1}{2}} dy = \frac{A}{L^2}\sqrt{2g}\int_0^t dt$$

$$y = \left[\sqrt{L} - \frac{A\sqrt{2g}\, t}{2L^2}\right]^2 = \left(3 - \frac{t}{9}\right)^2$$

$\therefore$ **(B)**

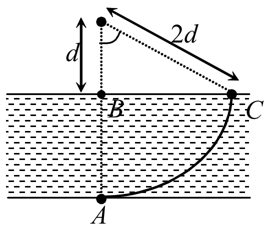
14. $C = \frac{\epsilon_0 (L-y)L}{L} + \frac{k \epsilon_0\, yL}{L}$ **=** $\epsilon_0 (L - y + ky)$ **=** $\frac{\epsilon_0}{9}(t^2 - 54t + 810)$

$\therefore$ **(B)**

15. Because resultant velocity is always perpendicular to line joining *C* and boat, so path is circular with center at *C*.

$\therefore$ **(C)**

16. (D)



$BC = \sqrt{4d^2 - d^2} = d\sqrt{3}$

17. (A)

If S_1 is closed, then $\dfrac{kQ_A}{a} + \dfrac{kQ}{2a} = 0$ $\quad Q_A = -\dfrac{Q}{2}$

If S_2 is closed, then $\dfrac{kQ_B}{2a} = 0$ $\quad Q_B = 0$

If S_3 is closed, then $\dfrac{kQ}{3a} + \dfrac{kQ_C}{3a} = 0$ $\quad Q_C = -Q$

If S_4 is closed, charge on shell B is $\underline{Q}$

$\therefore$ **(A) – 3, (B) – 1, (C) – 4, (D) – 2**

18. (A)

If maximum extension is x_m,

By conservation of energy,

$$\frac{1}{2}kx_m^2 = m_A g x_m$$

$$\frac{1}{2} \times 200 \times x_m = 10 \times 10$$

$\boldsymbol{x_m}$ **= 1 m**

Let extension in spring is x_1 when velocity of m_A is $\sqrt{10/3}$ m/s. The velocity of m_B will also be $\sqrt{\dfrac{10}{3}}$ m/s.

$$\frac{1}{2}m_A v^2 + \frac{1}{2}m_B v^2 + \frac{1}{2}kx_1^2 = m_A g x_1$$

$$\frac{1}{2} \times 10 \times \frac{10}{3} + \frac{1}{2} \times 5 \times \frac{10}{3} + \frac{1}{2} \times 200 x_1^2 = 10 \times 10 \times x_1$$

$25 + 100x_1^{\ 2} = 100x_1$

$4x_1^{\ 2} - 4x_1 + 1 = 0$

$(2x_1 - 1)^2 = 0$

$\boldsymbol{x_1 = \frac{1}{2} = 0.5\ \text{m}}$

If acceleration of m_A and m_B is a

$T - kx_1 = m_B a$

$T - 200 \times 0.5 = 5a$

$T = 5a + 100$ …(i)

$m_A g - T = m_A a$

$10 \times 10 - T = 10a$

$T = 100 - 10a$ …(ii)

(i) and (ii) $\Rightarrow$ $5a + 100 = 100 - 10a$

***a* = 0**

When acceleration m_B is 4 m/s^2, the acceleration of m_A will also be 4 m/s^2

$T - kx = m_B a$

$T - 200x = 20$ ...(i)

$m_A g - T = m_A a$

$100 - T = 40$ …(ii)

(i) + (ii) $\Rightarrow$ 100 – 200 x = 60

$x = \frac{40}{200} = \frac{1}{5}$ m

$U = \frac{1}{2}kx^2 = \frac{1}{2} \times 200 \times \frac{1}{25}$ = 4 J

19. (C)

$w_g = \vec{F}.\vec{S} = (mg\sin\theta)S = 2 \times 10 \times \frac{1}{2} \times \frac{20}{100} = 2\,\text{J}$

$$w_s = -\frac{1}{2}kx^2 = -\frac{1}{2}(1000)\left(\frac{20}{100}\right)^2 = -20\text{ J}$$

$w_N = 0$

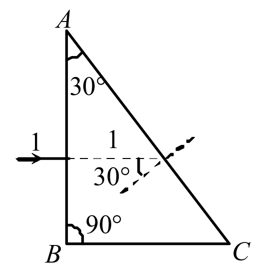
From work energy theorem $w_g + w_{sp} + w_N + w_{ex} = \Delta k$ or $2 - 20 + 0 + w_{ex} = 0$

$w_{ex} = 18$ J

∴ **(A) – 4, (B) – 3, (C) – 2, (D) – 1**

20. (C)

(A) Refractive index of the prism is the minimum value required for ray (1) to undergo total internal reflection at face *AC*. Ray (1) falls on face *AC* at an angle of incidence 30°



∴ $30° > i_C$

$\sin 30° > \sin i_C$

∴ $\mu > 2$

Minimum value of μ can be taken as 2.

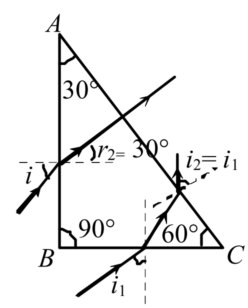
(B) For ray 2, refractive angle of prism is 30°.

Apply Snell's law for refraction at face *AB*.

$1 \sin i = \mu \sin r$

$i = 90°$

(C) Using the relation $i_1 + i_2 = A + \delta$ for ray 2.



$90° + 0° = 30° + \delta$

$\delta = 60°$

(D) $\mu = \dfrac{\sin\left(\dfrac{A + \delta m}{2}\right)}{\sin\dfrac{A}{2}} \Rightarrow \delta m = 120°$

∴ **(A) –3; (B)–2; (C)–4; (D)–1**

CHEMISTRY

21. (B, D)

$\because$ both are liquid CH_3OH is solute (less amount)

Mass of CH_3OH = 30 × 0.8 = 24 g,

Mass of C_2H_5OH = 60 × 0.92 = 55.2 g

Mass of solution = 24 + 55.2 = 79.2 g

Volume of solution = $\frac{79.2}{0.88} = 90\ \text{mL}$.

Molarity = $\frac{nCH_3OH}{V(L)} = \frac{24/32}{90} \times 1000$ = 8.33 mol L^{-1}

Molality = $\frac{n_{Solute}}{w_{Solvent}(kg)} = \frac{24/32}{55.2} \times 1000$ = 13.59

Mole fraction of solute = $\frac{\frac{24}{32}}{\frac{24}{32} + \frac{55.2}{46}} = 0.385$

Mole fraction of solvent = – 1 – 0.385 = 0.615

22. (A), (C)

23. (A, B, C)

24. (A,B,C)

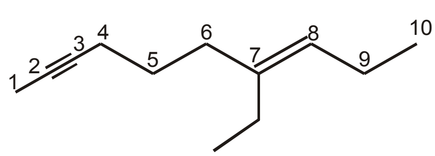
$$Pb(NO_3)_2 \xrightarrow{\Delta} PbO + 2NO_2 + \frac{1}{2}O_2$$

$$2LiNO_3 \xrightarrow{\Delta} Li_2O + 2NO_2 + \frac{1}{2}O_2$$

$$2NaNO_3 \xrightarrow{\Delta} 2NaNO_2 + O_2$$

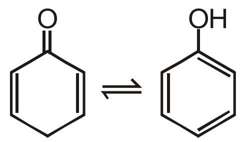
(Both O_2 & NO_2 are paramagnetic)

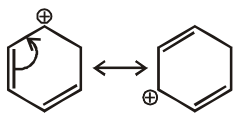
25. (A,B,D)

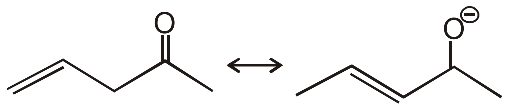


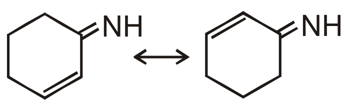
7-Ethyldec-7-en-2-yne

26. (A,C,D)

(A)  (tautomers)

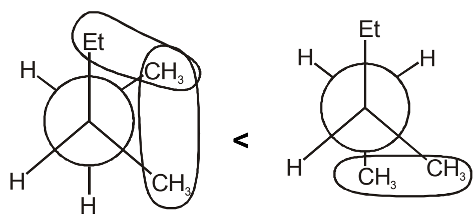
(B)  (Resonating structure)

(C)  (Not resonating structure)

(D)  (Identical)

27. (A,B,D)

Stability order:



more gauche intraction (repulsion) ⇓ Less stability

less gauche intraction (repulsion) ⇓ More stability

28. (A, C, D)

$V_{strength} = 28;$

$\therefore \quad M = \dfrac{28}{11.2} = 2.5$

$\therefore$ 1 L contain 2.5 moles of H_2O_2

or 2.5 × 34 = 85 g H_2O_2

Mass of 1 litre solution = 265g

$(\because \; d = 265 \text{ g/L})$

$\therefore \quad w_{H_2O} = 180$ g or moles of H_2O = 10

$$x_{H_2O_2} = \frac{2.5}{2.5+10} = 0.2$$

$$\% \frac{w}{v} = \frac{2.5 \times 34}{1000} \times 100 = 8.5$$

$$m = \frac{2.5}{180} \times 1000 = 13.88$$

29. (D)

Due to synergic bond.

30. (C)

Bond order $\propto$ bond energy

31. (A)

At low pressure, (V–b) = V

$$\left(P + \frac{a}{V^2}\right)V = RT$$

$$PV + \frac{a}{V} = RT$$

$$PV = RT - \frac{a}{V}$$

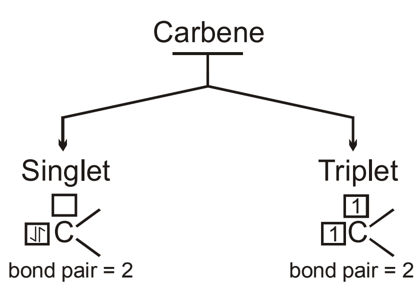
$$\frac{PV}{RT} = 1 - \frac{a}{VRT}$$

32. (D)

Greater is the value of van der Waals' constant 'b', lesser is the compressibility of gas.

33. (B)

34. (C)



35. (A)

Due to intermolecular hydrogen bond HF is a weak electrolyte.

36. (B)

According to Bent rule.

37. (B)

A → P,R; B → P; C → Q; D → Q,S

(A) $\overset{0}{Br_2} + OH^- \longrightarrow \overset{+5}{BrO_3^-} + \overset{-1}{Br^-}$

Intramolecular, disproportionation Redox Reaction

(B) $\overset{+2}{Fe}Cl_2 + K\overset{+7}{Mn}O_4 + HCl \longrightarrow \overset{+2}{Mn}Cl_2 + \overset{+3}{Fe}Cl_3$

Intermolecular Redox Reaction

(C) $\overset{+1}{Ag_2}\overset{-2}{O} \longrightarrow \overset{0}{Ag} + \frac{1^\circ}{2}O_2$

Intramolecular Redox

(D) $\overset{-3}{N}H_4\overset{+5}{N}O_3 \longrightarrow \overset{+1}{N_2}O + H_2O$

Intramolecular comproportionation Redox Reaction.

38. (A)

(A–P, R), (B–P, Q, R), (C–P, R, S), (D–P, R, S)

$N^{3-} \longrightarrow$ 1 mol of it will have 10 mol electrons i.e., 6.023×10^{24} electrons

$O^{2-} \longrightarrow$ 1 mol of it will have 10 mol electrons i.e., 6.023×10^{24} electrons.

It has 8 mole protons in the nucleus also.

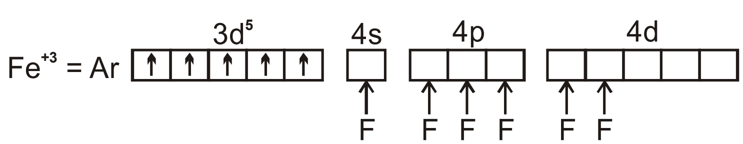
CH_4 and $H_2O \longrightarrow$ 1 mol of these compounds will have 10 mole electrons i.e., 6.023×10^{24} electrons. 1 mol of these compouds will contain 10 mol protons in the nuclei.

39. (B)

(a) — (q) ; (b) — (p) ; (c) — (r) ; (d) — (s)

(i) $[FeF_6]^{3-}$

$_{26}Fe^{+3} \Rightarrow [Ar]\ 3d^5 4s^0$



$\mu = \sqrt{n(n+2)}$

$\mu = \sqrt{5(5+2)} = 5.93$ BM

(ii) $[Ti(H_2O)_6]^{3+}$

$_{22}Ti^{+3} = 3d^1 4s^0$

$\mu = 1.73$ BM

(iii) $[Cr(NH_3)_6]^{3+}$

$_{24}Cr^{+3} \Rightarrow 3d^3 4s^0$

$\mu = 3.88$ BM

(iv) $[Ni(H_2O)_6]^{2+}$

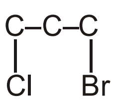
$_{28}Ni^{+2} \Rightarrow 3d^8 4s^0$

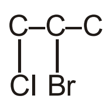
$\mu = 2.83$ BM

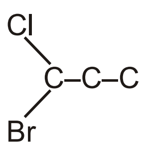
40. (C)

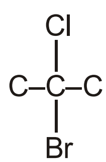
A-q; B-s; C-p; D-r

(A) M.F = C_3H_6ClBr (U.F = O)

(1) 

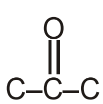
(2) 

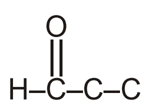
(3) 

(4) 

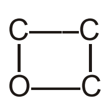
(B) M.F = C_3H_6O (U.F = 1)

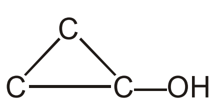
No. of structural isomers are

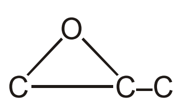
(1) 

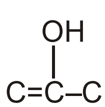
(2) 

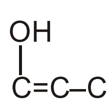
(3) C=C–O–C

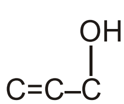
(4) 

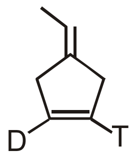
(5) 

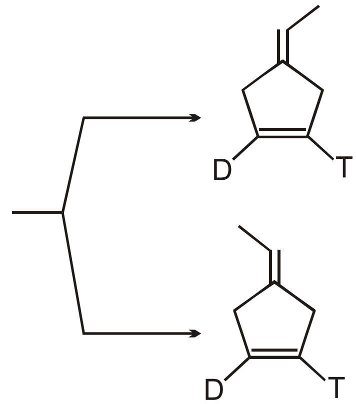
(6) 

(7) 

(8) 

(9) 

(C)  Number of geometrical isomers = 2



(D) CH_3–CH=N–CH=CH–N=N–CH_3,

Number of geometrical isomers = 2^n (n = 3)

$= 2^3 = 8$

MATHEMATICS

41. (B, D)

Equation of tangents are

$$y = \frac{8}{9}x \pm \sqrt{\frac{1}{4}\left(\frac{8}{9}\right)^2 + 1/9}$$

$$8x - 9y - 5 = 0; \quad 8x - 9y + 5 = 0$$

42. (B, C)

$$e = \frac{\sqrt{3}}{2}; \; a = \frac{\sqrt{3}}{2}$$

equation of parabolas are

$$2\sqrt{3}\left(y - \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}\right)\right) = x^2$$

$$-2\sqrt{3}\left(y - \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\right)\right) = x^2$$

43. (B, C)

$$t = x^2; \quad \frac{1}{2}\int \frac{dt}{t^2+t+1} = \frac{1}{2}\int \frac{dt}{(t+\frac{1}{2})^2+\frac{3}{4}}$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\frac{2x^2+2}{\sqrt{3}} + C$$

44. (A, B, D)

Integrate both sides we get.

$f(t) = e^t.\cos^2 t$

45. (A, B , C, D)

Equation of line through A (4, 3) is

$$\frac{x-4}{\cos\theta} = \frac{y-3}{\sin\theta} = r \qquad .....(i)$$

$A \equiv (4 + r\cos\theta, 3 + r\sin\theta)$.

$4 + r\cos\theta = 8 \Rightarrow r = 4 \sec \theta$.

$\therefore$ AB = 4 $\sec\theta$.

Similarly AC = 3 $\text{cosec}\ \theta$

$$\frac{16}{AB^2} + \frac{9}{AC^2} = \cos^2\theta + \sin^2\theta = 1$$

$$\text{So } AB + AC = \frac{4}{\cos\theta} + \frac{3}{\sin\theta} = \frac{2(4\sin\theta + 3\cos\theta)}{\sin 2\theta}$$

46. (B, D)

$$\lim_{x\to 0^+}(x^x) = 1; \ \lim_{x\to 0^+} x^x = 0$$

$$-\frac{1}{2}\lim_{x\to 0^+} x^2 \ln x = 0$$

$$\lim_{x\to 0}\frac{\frac{5^x-1}{x}\cdot\frac{2^x-1}{x}\cdot x}{1+\frac{\tan x}{x}} = 0.$$

47. (A, B)

$$\sqrt{\sin x} > \sin^2 x$$

$\sqrt{\cos x} > \cos^2 x$

Also $f_2(x) < f_3(x)$

$f_3(x) < f_5(x)$

48. (A, B)

$\alpha x - \beta y = 8$ divides the area of the region enclosed by the curve $x^2 + y^2 - 4x + 2y - 5 = 0$.

$\Rightarrow\ 2\alpha + \beta = 8$

Also, $\dfrac{2\alpha+\beta}{2} \ge \sqrt{2\alpha\beta}$

$4 \ge \sqrt{2\alpha\beta} \Rightarrow \alpha\beta \le 8$

49. (C)

50. (A)

51. (D)

52. (B)

Sol. for Q.No. (51-52)

f(1) = 1

$x f'(xy) = f'(y) + x - 1$

y = 1 ‘ $f'(x) = \dfrac{3}{x} + 1;\ f(x) = 3\ \ln x + x + c$

$\int x^3 e^x\, dx = e^x (ax^3 + bx^2 + cx + d) + \lambda$

Differentiate both sides we get

53. (B)

54. (C)

Sol. for Q.No. (53 - 54)

$f(10) = \sin^{-1} \sin 10 + \cos^{-1} \cos 10$

$= 3\pi - 10 + 4\pi - 10$

$$\sin^{-1}\sin x = \begin{cases} x; & x \in \left[0, \dfrac{\pi}{2}\right] \\ \pi - x; & x \in \left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right] \end{cases}$$

$\cos^{-1}\cos x = x$.

55. (D)

f(0) = –3, f(1) = 1, f(2) = –1 and f(3) = 3.

So f(x) = 0 has one root in each of the intervals (0, 1), (1, 2) and (2, 3) and hence no root in (3, 4).

56. (C)

Let $\alpha \le \beta \le \gamma$, then

$\alpha \in (0, 1)$, $\beta \in (1, 2)$, $\gamma \in (2, 3)$

$\therefore$ $[\alpha] = 0$, $[\beta] = 1$ and $[\gamma] = 2$

$\therefore$ $\{\alpha\} + \{\beta\} + \{\gamma\} = (\alpha + \beta + \gamma) - ([\alpha] + [\beta] + [\gamma]) = \frac{9}{2} - (0+1+2) = \frac{3}{2}$.

57. (A)

(P) $f'(x) = 3ax^2 - 18x + 9 = 3(ax^2 - 6x + 3)$

As f (x) is strictly increasing on R, so

a > 0 and $D \le 0 \Rightarrow a \ge 3$

(Q) Put $\cos x = t, t \in [-1,1]\ \forall x \in R$

Let $g(t) = t^3 - 6t^2 + 11t - 6 = (t-1)(t-2)(t-3), t \in [-1,1]$

$\therefore$ range of g(t) = [g(–1), g(1)] = [–24, 0]

(R) $x^3 - y^2 = 0$

$$\left.\frac{dy}{dx}\right]_{P(4m^2, 8m^3)} = \frac{3 \times 16m^4}{16m^3} = 3m$$

Let Q be $(4m_1^2, 8m_1^3)$

$\therefore$ Slope of normal at Q $= \frac{-1}{3m_1}$

$\therefore$ $3m = \frac{-1}{3m_1} \Rightarrow m_1 = \frac{-1}{9m}$ ... (1)

Also, slope of PQ $= \frac{8(m^3 - m_1^3)}{4(m^2 - m_1^2)} = \frac{2(m^2 + m_1^2 + mm_1)}{m + m_1} = 3m$

$\Rightarrow (2m_1 + m)(m_1 - m) = 0$

$\Rightarrow\ 2\left(\frac{-1}{9m}\right)+m=0 \quad \Rightarrow\ m^2=\frac{2}{9}$

(S) $(\sin\theta-\cos\theta)(\tan\theta+\cot\theta)=2$

$\Rightarrow\ \frac{\sin\theta-\cos\theta}{\sin\theta\cos\theta}=2$

Let $y=\sin\theta-\cos\theta$

$\therefore y=1-y^2 \Rightarrow y^2+y-1=0 \Rightarrow y=\frac{-1\pm\sqrt{5}}{2}$

$\therefore y=\sin\theta-\cos\theta=\left(\frac{\sqrt{5}-1}{2}\right)$

$(\sin\theta+\cos\theta)(\tan\theta-\cot\theta)=(\sin\theta+\cos\theta)\frac{(\sin^2\theta-\cos^2\theta)}{\sin\theta\cos\theta}=\frac{(\sin\theta+\cos\theta)^2}{\sin\theta\cos\theta}.y$

$=\frac{(1+1-y^2)y}{\frac{1-y^2}{2}}=\frac{2y(1+y)}{y}$ = $\sqrt{5}+1$

58. (B)

P. Let $x=4\cos\theta, y=3\sin\theta,$ than x + y = $4\cos\theta+3\sin\theta\le 5$

$\therefore\ \log_5(x+y)\le 1$

Q. $\because\ 2^{\sqrt{\log_2 3}}=\left(2^{\log_2 3}\right)^{\frac{1}{\sqrt{\log_2 3}}}=3^{\sqrt{\log_3 2}}$ and $3^{\log_3 2}-2^{\log_2 3}=2-3=-1$

$\therefore\ \alpha+\beta=3 \text{ and } \alpha\beta=2$

$\Rightarrow\ \alpha^2+\alpha\beta+\beta^2=(\alpha+\beta)^2-\alpha\beta=7$

R. $3\log_{18}96\,\log_{12}3+\log_{18}96+3\log_{12}3$

$=\left(\log_{18}96+1\right)\left(3\log_{12}3+1\right)-1$

$=\log_{18}(96\times 18).\ \log_{12}(27\times 12)-1$

$=\left(3\log_{18}12\right)\left(2\log_{12}18\right)-1=6-1=5$

S. $2^{\log_x 3} = y^{\log_5 y} \Rightarrow \log_x 3\ \ln 2 = \log_5 y \ln y$

$\Rightarrow (\ln x)\,(\ln y)^2 = \ln 2 \ln 3 \ln 5$

similarly, $3^{\log_y 5} = x^{\log_2 x} \Rightarrow (\ln x)^2(\ln y) = \ln 2 \ln 3 \ln 5$

$\therefore\ (\ln x)(\ln y)^2 = (\ln x)^2(\ln y) \Rightarrow \ln y = \ln x \Rightarrow x = y$

$$\therefore\ \frac{\left(x^{\log_y x} + y^{\log_x y}\right)^2}{\left(x^{\log_x y}\right)^2 + \left(y^{\log_y x}\right)^2} = \frac{(x+x)^2}{x^2+x^2} = 2$$

59. **(C)**

(P) $f(x) = \sin^{-1} x$

$$\lim_{x \to \frac{1}{2}^+} f(3x - 4x^3) = \ell - 3\left(\lim_{x \to \frac{1}{2}} f(x)\right)$$

$\Rightarrow\ \ell = \pi\ \ [\ell] = 3$

(Q) $\sin\left(\frac{1}{2}\left(\tan^{-1} x\left(\frac{2x}{1-x^2}\right)\right) - \tan^{-1} x\right)$ $\qquad$ $\tan^{-1} x = \theta;\ x = \tan\theta$

$\sin\left(\frac{1}{2}\left(\tan^{-1}(\tan 2\theta)\right) - \theta\right)$ $\qquad$ $\sin(\theta - \pi/2 - \theta) = \sin \pi/2 = -1$

(R) Domain of given question is x = –1 and 1 and x = 1 satisfy the equation

(S) $\tan^{-1} x + \tan^{-1}\frac{1}{x} = -\frac{\pi}{2}$ when x < 0

60. **(A)**

P. Eqn. of tangent at $(2\cos\theta, \sqrt{3}\sin\theta)$ is $\frac{x}{2}\cos\theta + \frac{y}{\sqrt{3}}\sin\theta = 1$. Mid point (h,k) of portion of tangent between coordinate axes is $(\sec\theta, \frac{\sqrt{3}}{2}\operatorname{cosec}\theta)$

$\therefore\ h = \sec\theta,\ k = \frac{\sqrt{3}}{2}\operatorname{cosec}\theta$

$$\Rightarrow\ \frac{1}{h^2} + \left(\frac{\sqrt{3}}{2k}\right)^2 = 1 \Rightarrow \frac{4}{h^2} + \frac{3}{k^2} = 4$$

Q. Eqn. of chord having mid point (x_1, y_1) is $\frac{x\,x_1}{4}+\frac{y\,y_1}{3}=\frac{x_1^2}{4}+\frac{y_1^2}{3}$

Eqn of pair of lines joining origin to extremities of the chord is

$$\frac{x^2}{4}+\frac{y^2}{3}=1\left(\frac{\frac{x\,x_1}{4}+\frac{y\,y_1}{3}}{\frac{x_1^2}{4}+\frac{y_1^2}{3}}\right)^2$$

The lines are mutually perpendicular

$\therefore$ coeff. of x^2 + coeff of $y^2 = 0$

$$\Rightarrow \frac{1}{4}\left(\frac{x_1^2}{4}+\frac{y_1^2}{3}\right)^2-\frac{x_1^2}{16}+\frac{1}{3}\left(\frac{x_1^2}{4}+\frac{y_1^2}{3}\right)^2-\frac{y_1^2}{9}=0$$

$$\Rightarrow 7\left(\frac{x_1^2}{4}+\frac{y_1^2}{3}\right)^2=12\left(\frac{x_1^2}{16}+\frac{y_1^2}{9}\right)$$

R. Eqn. of chord of contact from (h,k) is $\frac{hx}{4}+\frac{ky}{3}=1$ which passes through the focus (1, 0) or (-1, 0)

$\therefore \pm\frac{h}{4}=1 \Rightarrow h^2=16$

S. Eqn. of normal at $(2\cos\theta, \sqrt{3}\sin\theta)$ is $2x\sec\theta-\sqrt{3}y\,\mathrm{cosec}\theta = 2^2-\sqrt{3}^2=1$ .... (1)

Eqn. of chord with mid point (x_1, y_1) is $\frac{x\,x_1}{4}+\frac{y\,y_1}{3}=\frac{x_1^2}{4}+\frac{y_1^2}{3}$ .... (2)

Comparing (1) and (2), $\frac{2\sec\theta}{x_1/4}=-\frac{\sqrt{3}\,\mathrm{cosec}\theta}{y_1/3}=\frac{1}{\frac{x_1^2}{4}+\frac{y_1^2}{3}}$

$$\Rightarrow \cos\theta=\frac{8}{x_1}\left(\frac{x_1^2}{4}+\frac{y_1^2}{3}\right),\ \sin\theta=\frac{3\sqrt{3}}{y_1}\left(\frac{x_1^2}{4}+\frac{y_1^2}{3}\right)$$

$$\Rightarrow \left(\frac{64}{x_1^2}+\frac{27}{y_1^2}\right)\left(\frac{x_1^2}{4}+\frac{y_1^2}{3}\right)^2=1$$