SOLUTIONS

PROGRESS TEST-4

RB-1810-1812 & RBK-1805

JEE MAIN PATTERN

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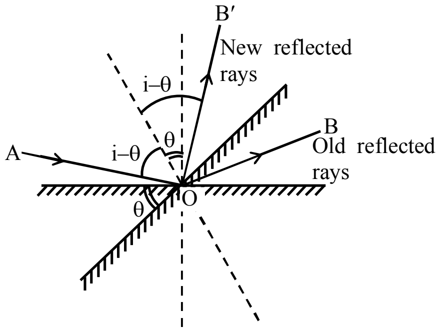
Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd.,
Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

PHYSICS.

1. [A, B, C, D]

(A)

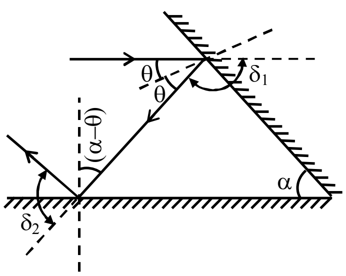


Angle BOB′ = $\angle$ AOB – $\angle$ AOB′

= 2 i – (2i – 2θ)

= 2θ

(B)



Total deviation $\delta = \delta_1 + \delta_2$

$= (180^o - 2\theta) + 180^o - 2(\alpha - \theta)$

$= 360^o - 2\alpha$

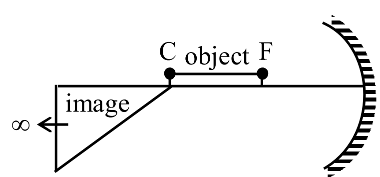
which is independent of angle of incidence.

(C) Power of a plane mirror is zero.

(D) Velocity of the image towards the object = v + v = 2v.

2. [A, C]

For concave mirror, if object is placed on the optic axis between focus & centre of curvature the image is formed between centre of curvature & infinity which is real, inverted & enlarged as well as elongated also.



3. [A,C,D]

$I_1 = I_2 = I$

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{I} + \sqrt{\frac{I}{2}}\right)^2 < 4I$

$I_{min} = \left(\sqrt{I} - \sqrt{\frac{I}{2}}\right)^2 > 0$

4. [A,B,D]

$\frac{dv}{dt} = -v^2 + 2v - 1$

Terminal velocity is attained when a = 0

$\Rightarrow \quad \frac{dv}{dt} = 0$

5. [A,B,C,D]

6. [A,C,D]

For same range, θ_1 and θ_2.

$\theta_2 = (90 - \theta_1) \Rightarrow \theta_1 + \theta_2 = 90^\circ$

$t_1 = \frac{2u\sin\theta_1}{g}$ and $t_2 = \frac{2u\sin\theta_2}{g}$

$\therefore \frac{t_1}{\sin\theta_1} = \frac{t_2}{\sin\theta_2}$ and $\frac{t_1}{t_2} = \frac{\sin\theta_1}{\sin\theta_2}$

$\Rightarrow \frac{t_1}{t_2} = \frac{\sin\theta_1}{\sin(90-\theta_1)}$

$\frac{t_1}{t_2} = \tan\theta_1$

7. [A,B,D]

8. [A,B]

9. [B]

10. [C]

11. [B]

12. [C]

13. [D]

14. **[D]**

15. **[B]**

16. **[B]**

17. **[A]**

18. **(A)**

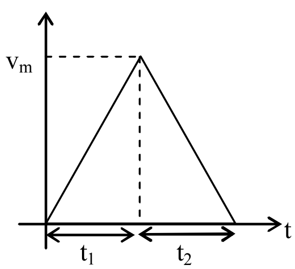
19. **(B)**

20. **(C)**

$\frac{v_m}{t_1} = \alpha$ and $\frac{v_m}{t_2} = \beta$

also $x + y = \frac{1}{2} \times v_m \times (t_1 + t_2)$

$\therefore (x + y) = \frac{1}{2} \times v_m \times \left(\frac{v_m}{\alpha} + \frac{v_m}{\beta}\right)$



$\Rightarrow v_m = \sqrt{\frac{2(x+y).\alpha\beta}{(\alpha+\beta)}}$

Now, $v_{av} = \frac{x+y}{t_1+t_2} = \frac{x+y}{\frac{v_m}{\alpha}+\frac{v_m}{\beta}} = \frac{\alpha\beta}{\alpha+\beta} . \frac{x+y}{v_m}$

CHEMISTRY

21. (B)

22. (A,B,D)

23. (B, D)

$\because$ both are liquid CH_3OH is solute (less amount)

Mass of CH_3OH = 30 × 0.8 = 24 g,

Mass of C_2H_5OH = 60 × 0.92 = 55.2 g

Mass of solution = 24 + 55.2 = 79.2 g

Volume of solution = $\frac{79.2}{0.88} = 90\ \text{mL}$.

Molarity = $\frac{nCH_3OH}{V(L)} = \frac{24/32}{90} \times 1000$

= 8.33 mol L^{-1}

Molality = $\frac{n_{Solute}}{w_{Solvent}(kg)} = \frac{24/32}{55.2} \times 1000$

= 13.59

Mole fraction of solute = $\frac{\frac{24}{32}}{\frac{24}{32} + \frac{55.2}{46}} = 0.385$

Mole fraction of solvent = 1 – 0.385 = 0.615

24. (B,C)

25. (A,B,C,D)

Bridge head carbocation & cabanion are unstable.

26. (A,B,C,D)

27. (A,B,C)

28. (A, B, C)

29. (D)

30. (A)

31. (A)

At low pressure, (V–b) = V

$$\left(P+\frac{a}{V^2}\right)V=RT$$

$$PV+\frac{a}{V}=RT$$

$$PV=RT-\frac{a}{V}$$

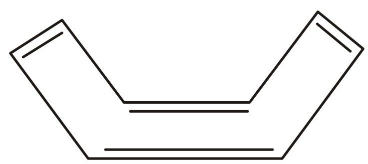
$$\frac{PV}{RT}=1-\frac{a}{VRT}$$

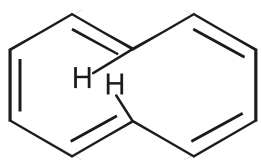
32. (D)

Greater is the value of van der Waals' constant 'b', lesser is the compressibility of gas.

33. (C)

34. (A)

COT is tub like structure 

[10] annulene : 

Non-planar

35. (D)

36. (D)

37. (A)

Use % by moles = $\frac{M_{avg}-M_1}{M_2-M_1}\times 100$

% by mass = % by moles × $\frac{M_2}{M_{avg}}$

38. (C)

Internal energy may be equal to $\frac{3}{2}RT$ or 3.716 kJ/mol or may be equal to $\frac{5}{2}RT$.

Translational kinetic energy = $\frac{3}{2}RT = 3.716\,kJ\ mol^{-1}$

At (–273°C or 0 K), molecular motion, i.e., translational motion stops and at this temperature gas molecules have no heat.

39. (A)

40. (A)

Acid dissociation constant$(K_a) \propto \frac{1}{P^{K_a}}$

$\because \quad P^{K_a} = -\log K_a$

MATHEMATICS

41. (A, B, C, D)

Solving the two equations $\alpha x^2 + \alpha x + \frac{1}{24} - y = \alpha y^2 + \alpha y + \frac{1}{24} - x$

$(x - y)\{\alpha(x + y) + \alpha + 1\} = 0$

$x = y \;\Rightarrow\; \alpha x^2 + (\alpha - 1)x + \frac{1}{24} = 0$

If the curves touch each other, then

D = 0 $\Rightarrow\; \alpha = \frac{2}{3}$ or $\frac{3}{2}$. Also $\alpha(x + y) + \alpha + 1 = 0 \;\Rightarrow\; y = -x - 1 - \frac{1}{\alpha}$

$\Rightarrow \quad \alpha x^2 + (\alpha + 1)x + \frac{25}{24} + \frac{1}{\alpha} = 0$

$\therefore\; D = 0 \Rightarrow \alpha = \frac{13 \pm \sqrt{601}}{12}$

42. (A, B)

Only h(x) is periodic function because

domain of f = [–1, 1]

domain of g = $(-\infty, -1] \cup [1, \infty)$

domain of h = R

43. **(A, C)**

The bases of all triangles PAB (A, B fixed P moving indeed A, B, P lie on a circle) will be fixed

$\Rightarrow$ area = $\frac{1}{2}$ × base × height

Thus if area is maximum then height must be maximum which will be true if P lies on perpendicular to bisector of AB.

Thus (A) and (C) are connect choices.

44. **(A, B , C, D)**

Equation of line through A (4, 3) is

$$\frac{x-4}{\cos\theta} = \frac{y-3}{\sin\theta} = r \qquad .....(i)$$

A $\equiv$ (4 + rcosθ, 3 + rsinθ).

4 + rcosθ = 8 $\Rightarrow$ r = 4 sec θ.

$\therefore$ AB = 4 secθ.

Similarly AC = 3 cosec θ

$$\frac{16}{AB^2} + \frac{9}{AC^2} = \cos^2\theta + \sin^2\theta = 1$$

So $AB + AC = \frac{4}{\cos\theta} + \frac{3}{\sin\theta} = \frac{2(4\sin\theta + 3\cos\theta)}{\sin 2\theta}$

45. **(A, C)**

$$k = f(1) = \lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^{2^{32}} - 2^{32}x + 4^{16} - 1}{(x-1)^2} = \frac{2^{32}(2^{32}-1)}{2} = 2^{63} - 2^{31}$$

46. **(A, C)**

cos x + cos y = a

$\cos^2 x + \cos^2 y + 2\cos x \cos y = a^2$ .....(1)

cos2x + cos2y = b

$\therefore \quad \cos^2 x + \cos^2 y = \frac{b+2}{2}$ .....(2)

$\therefore$ From (1) and (2), $\cos x \cos y = \frac{a^2}{2} - \frac{b+2}{4}$

Now, cos 3x + cos 3y = c

$\therefore \quad 4\cos^3 x - 3\cos x + 4\cos^3 y - 3\cos y = c$

$4(\cos x + \cos y)(\cos^2 x + \cos^2 y - \cos x \cos y) - 3(\cos x + \cos y) = c$

$\Rightarrow \quad 2a^3 + c = 3a(1 + b)$.

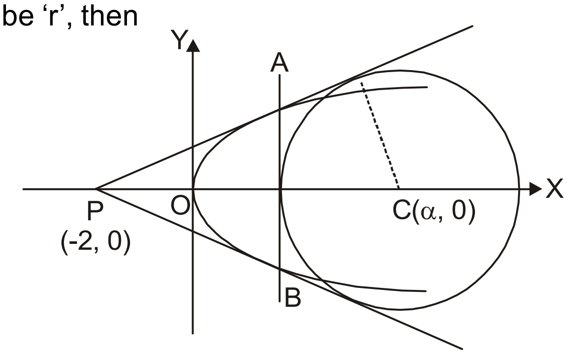
47. (A, B)

Point 'P' clearly lies on the directrix of $y^2 = 8x$. Thus slope of PA and PB and 1 and – 1 respectively.

Equation of PA is $y = x + 2$ and equation of PB is $y = -x - 2$

Equation of AB is $x = 2$

Let centre of circle be $C(\alpha, 0)$ and radius be 'r', then



$$\frac{|\alpha+2|}{\sqrt{2}} = \frac{|\alpha-2|}{1} = r$$

$\Rightarrow \quad \alpha^2 + 4 + 4\alpha = 2(\alpha^2 + 4 - 4\alpha)$

$\Rightarrow \quad \alpha = \frac{12 \pm 8\sqrt{2}}{2} = 6 \pm 4\sqrt{2}$

$\Rightarrow \quad r = |\alpha - 2| = 4(\sqrt{2}-1)$ or $4(\sqrt{2}+1)$

48. (A, B)

$f(x+y) = 3^y f(x) + 2^x f(y)$

put $x = 1$

$f(1+y) = 3^y + 2f(y)$

put $y = 1$

$f(x+1) = 3f(x) + 2^x$

$\therefore 3^x + 2f(x) = 3f(x) + 2^x$

$f(x) = 3^x - 2^x$

49. (A)

$x - 1 = 3\cos\theta$

$y - 2 = 4\sin\theta$

$x + y = 3 + 3\cos\theta + 4\sin\theta$

maximum value = 3 + 5 = 8

50. (C)

Since range of $f(x) = \sin^{2n} x + \cos^{2n} x$ where $n \in N$ is

$\left[\frac{1}{2^{n-1}}, 1\right]$ so range of $\sin^8 x + \cos^8 x$ is

$$\left[\frac{1}{2^{4-1}},1\right]=\left[\frac{1}{8},1\right] \because n=4$$

51. (C)

$P''(x)=P''(0)=a$ constant

$\therefore$ P(x) is a quadratic polynomial.

Let $P(x) = ax^2 + bx + c$, then

$P(1) – P(0) = 2 \quad \Rightarrow a + b = 2$ ... (i)

$P'(1)-P'(0)=2 \quad \Rightarrow 2a = 2$ ... (ii)

$\therefore$ a = b = 1

$\therefore$ P(2) – P(1) = 3a + b = 4

52. (D)

$P(x) = 0 \quad \Rightarrow x^2 + x + c = 0$

roots are real

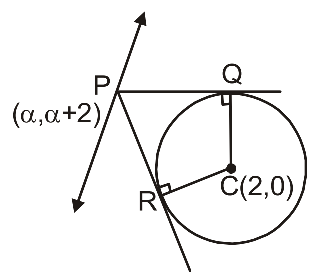
$$\Rightarrow D \geq 0 \quad \Rightarrow 1 – 4c \geq 0 \quad \Rightarrow c \leq \frac{1}{4}$$

53. (A)

Circumcircle of ΔPQR passes through the centre C of the circle with PC as a diameter.

Let S(h, k) be the circumcentre, then

S is mid point of PC



$$\Rightarrow (h,k)=\left(\frac{\alpha+2}{2},\frac{\alpha+2}{2}\right)$$

$\Rightarrow h=k$

$\therefore$ Locus is y = x.

54. (B)

Orthocentre of ΔPQR is the image of C about QR. Let $P \equiv (\alpha, \alpha+2)$

and H(h, k) be the orthocentre.

Equation of chord of contact QR is

$$x\alpha + y(\alpha + 2) – \frac{4(x+\alpha)}{2} = 0$$

i.e., $(\alpha – 2)x + (\alpha + 2)y = 2\alpha$

$$\therefore \frac{h-2}{\alpha-2}=\frac{k-0}{\alpha+2}=\frac{-2(2(\alpha-2)-2\alpha)}{(\alpha-2)^2+(\alpha+2)^2}$$

$\Rightarrow h-2=\dfrac{8(\alpha-2)}{(\alpha-2)^2+(\alpha+2)^2},\ k=\dfrac{8(\alpha+2)}{(\alpha-2)^2+(\alpha+2)^2}$

$\Rightarrow (h-2)^2+k^2=\dfrac{64}{(\alpha-2)^2+(\alpha+2)^2}$ and $h-k-2=\dfrac{-32}{(\alpha-2)^2+(\alpha+2)^2}$

$\Rightarrow (h-2)^2+k^2=-2(h-k-2)$

$\Rightarrow h^2 + k^2 - 2h - 2k = 0$

$\therefore$ Locus is $x^2 + y^2 - 2x - 2y = 0$

55. (B)

The circumcircle always passes through the vertex of the parabola.

56. (D)

2 : 1

orthocentre (α, β) — centroid $\left(0,\dfrac{2}{3}(h-2a)\right)$ — circumcentre $\left(\dfrac{k}{4},a+\dfrac{h}{2}\right)$

$$\left(0,\frac{2}{3}(h-2a)\right)\equiv\left(\frac{\alpha+\frac{k}{2}}{3},\frac{\beta+2a+h}{3}\right)\Rightarrow(\alpha,\beta)\equiv\left(-\frac{k}{2},h-6a\right).$$

57. (A)

(P) f(x) is continuous $\forall x \in R$ but not differentiable at x = 2

(Q) g(x) is discontinuous and non-differentiable at x = I

(R) h(x) is continuous and differentiable for $\forall x \in R$

(S) $\Psi(x)$ is continuous everywhere but not differentiable at x = 1

58. (A)

(P) $\left(\dfrac{1}{100}\right)^{\log\frac{1}{5}-\frac{1}{2}} = \dfrac{1}{(100)^{\log_{10}\frac{1}{5}}}\cdot\dfrac{1}{100^{\frac{-1}{2}}} = \dfrac{10}{10^{\log_{10}\frac{1}{25}}} = 250 \Rightarrow$ P,S

(Q) $\log_3\left(\sqrt{73}-8\right) = \log_3\left(\dfrac{73-64}{\sqrt{73}+8}\right) = \log_3\left(\dfrac{9}{\sqrt{73}+8}\right) = 2-\log_3\left(\sqrt{73}+8\right)$

negaitve and irrational $\Rightarrow$ Q,R

(R) $\log_{10}(\log 10) = 0 \Rightarrow$ S

(S) $\left(\dfrac{1}{3}\right)^{\log_9 2-3} = 27\cdot\dfrac{1}{3^{\log_9 2}} = \dfrac{27}{3^{\log_3\sqrt{2}}} = \dfrac{27}{\sqrt{2}} \Rightarrow$ P, R]

59. (C)

(P) A(0,0), B(t), C(–t)

BC = 12t, $y^2 = 4ax$, a = 3

D is mid-point of BC and $\frac{BD}{AD} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{2}{t} = \frac{1}{\sqrt{3}}$.

$\therefore \quad BC = 4at = 24\sqrt{3}.$

(Q) D is mid-point of chord BC; $\angle BAD = 45°$

AB = ?

$B(at^2, 2at); \frac{2}{t} = \tan 45° \Rightarrow t = 2.$

Now, $(AB)^2 = 4a^2t^2 + a^2t^4$

$= 9(4 \times 4 + 16) = 9 \times 32$

$\Rightarrow \quad AB = 12\sqrt{2}.$

(R) Normal at $P(t_1)$ cuts curve at $Q(t_2)$; S(a, 0)

Also, $t_2 = -t_1 - \frac{2}{t_1}$

Now, $PS \perp QS$

$\Rightarrow \quad \frac{2at_1}{a\left(t_1^2 - 1\right)} \times \frac{2at_2}{a\left(t_2^2 - 1\right)} = -1 \quad \Rightarrow -4t_1t_2 = (t_1^2 - 1)(t_2^2 - 1)$

$\Rightarrow \quad 4\left(t_1 + \frac{1}{t_1}\right)^2 = (t_1^2 + 1)^2 \quad \Rightarrow \quad t_1 = \pm 2$

$\therefore \quad (PQ)^2 = a^2(t_1 - t_2)^2 \times [4 + (t_1 + t_2)^2] \quad \Rightarrow PQ = 15\sqrt{5}.$

(S) If PQ is a normal chord, $P(t_1)$, $Q(t_2)$, $\angle QOP = \frac{\pi}{2}$

then $t_1t_2 = -4$ and $t_2 = -t_1 - \frac{2}{t_1} \quad \Rightarrow t_1^2 = 2 \quad \Rightarrow t_1 = \pm\sqrt{2}$

$\Rightarrow \quad t_2 = \mp 2\sqrt{2}$

$\therefore \quad (PQ)^2 = a^2(t_1 - t_2)^2[4 + (t_1 + t_2)^2]$

$= 9(3\sqrt{2})^2(4 + (\sqrt{2})^2) = 81 \times 2 \times 6$

$\Rightarrow \quad PQ = 18\sqrt{3}.$

60. (C)

(P) $f(x) = \sin^{-1} x$

$\lim_{x \to \frac{1}{2}^+} f(3x - 4x^3) = \ell - 3\left(\lim_{x \to \frac{1}{2}} f(x)\right)$

$\Rightarrow \ell = \pi \quad [\ell] = 3$

(Q) $\sin\left(\frac{1}{2}\left(\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right) - \tan^{-1} x\right)$ $\qquad \tan^{-1} x = \theta;\ x = \tan\theta$

$\sin\left(\frac{1}{2}\left(\tan^{-1}(\tan 2\theta)\right) - \theta\right)$ $\qquad \sin(\theta - \pi/2 - \theta) = \sin\frac{\pi}{2} = -1$

(R) Domain of given question is x = –1 and 1 and x = 1 satisfy the equation

(S) $\tan^{-1} x + \tan^{-1}\frac{1}{x} = -\frac{\pi}{2}$ when $x < 0$