SOLUTIONS

WEEKLY TEST-6

RBPA

(JEE ADVANCED PATTERN)

Test Date: 11-11-2017

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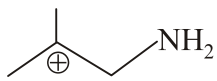
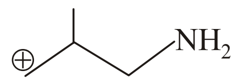
PHYSICS

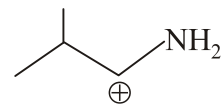
1. **(C)**

Anti Aromatic compounds having dimerization tendency

2. **(C)**


(I)
(8α - H)


(II)
(1α - H)


(III)
(+M.E of $-NH_2$, its resonating structure is more stable.)

3. **(B)**

4. **(B)**

Given, $H_2 + \frac{1}{2}O_2 \longrightarrow H_2O(\ell)$; $\Delta H = -285.85 \text{ kJ mol}^{-1}$ ....(i)

$C + O_2 \longrightarrow CO_2(g)$; $\Delta H = -293.7 \text{ kJ/mol}$ ....(ii)

$\therefore$ 1gm starch on combustion gives 17.49 kJ energy

$\therefore$ 162n gm starch on combustion gives = 17.49 × 162n kJ energy = 2833.38n kJ

Molecular weight of starch $(C_6H_{10}O_5)_n$ = 162n

Therefore,

$(C_6H_{10}O_5)_n + 6nO_2 \longrightarrow 6nCO_2 + 5nH_2O \quad \Delta H = -2833.38n \text{ kJ}$ ....(iii)

Multiplying equation (ii) by 6n and equation (i) by 5n, then add

$5nH_2 + \frac{5n}{2}O_2 + 6nC + 6nO_2 \longrightarrow 5nH_2O + 6nCO_2 \quad \Delta H = -3191.45\text{ n kJ}$ ....(iv)

Substracting equation (iii) from equation (iv)

$$6nC + 5nH_2 + \frac{5n}{2}O_2 \longrightarrow (C_6H_{10}O_5)_n \quad \Delta H = -358.07n \text{ kJ}$$

$\therefore$ Heat of formation for 162n gm starch = – 358.07n kJ

$\therefore$ Heat of formation for 1gm starch = $\frac{-358.07\text{ n}}{162\text{n}} = -2.21 \text{kJ}$

5. **(C)**

Given, $H_2 + \frac{1}{2}O_2 \longrightarrow H_2O; \Delta H = -241\text{KJ}$

$C_6H_{10} + \frac{17}{2}O_2 \longrightarrow 6CO_2 + 5H_2O; \Delta H = -3800 \text{ KJ}$

$C_6H_{10} + 9O_2 \longrightarrow 6\,CO_2 + 6\,H_2O;\ \Delta H = -3920\,KJ$

Equation (i) + (ii) – (iii)

$C_6H_{10} + H_2 \longrightarrow C_6H_{12} \quad \Delta H = -121\ \ KJ$

$\therefore$ Heat of hydrogenation of cyclohexene = – 121 KJ

6. (C)

$CH_3COOH(aq) \longrightarrow CH_3COO^-(aq) + H^+(aq) \quad \Delta H = 4\ kcal$

$NH_4OH(aq) \longrightarrow NH_4^+(aq) + OH^-(aq) \quad \Delta H = 6\ kcal\ \Delta H$

$H^+(aq) + OH^-(aq) \longrightarrow H_2O(\ell) \quad \Delta H = -13.7$

$CH_3COOH(aq) + NH_4OH(aq) \longrightarrow CH_3COONH_4(aq) + H_2O(\ell)$

$\Delta H = -3.7\ kcal$

7. (D)

PF_2Cl_3 – Non-planar molecule

$B_3N_3H_6$ – Planar molecule

$F_2C = C = CF_2$ – Non-planar molecule

8. (D)

$\xleftarrow[r]{Sc^{+3},\ Ti^{+3},\ V^{+2},\ Cr^{+3}}$

- Non-metals having gaint structure have high M.P. i.e. Si.
- Generally, M.P. $\propto$ no. of unpaired electrons

9. (B)

10. (A), (B), (D)

$$\ell nK = \frac{-\Delta H^0}{R.T} + \frac{\Delta S^0}{R}$$

11. (B,C,D)

I & II : Positional isomer

I & III : Geometrical isomer (cis & trans)

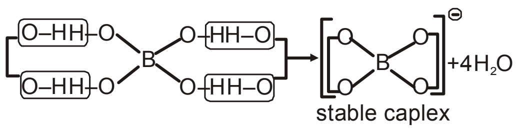
II and III : Positional isomers

12. (B,D)

(A) $H_3BO_3 + H-OH \rightleftharpoons B(OH)_4^- + H^+$

Weak mono basic lewi's acid

(B) Equalibrium (i) is shifted in forward direction by the addition of syn-diols like-ethylene cly col which form a stable complex with $(O+)_4^-$



(C) It ha a planer sheet like structure due to hydrogen bonding.

(D) H_3BO_3 is a weak electrolyte in water (due to pure water)

13. (A), (B), (C)

(A) $HA(aq) + BOH\,(aq.) \longrightarrow BA(aq.) + H_2O\ \Delta h.$

$$\Delta H = \Delta_{net}H + (\Delta H_{ionization})_{W.A} = -57.3 + 15$$
$$= -42.3\text{ kJ}$$

(B) $HA(g) \longrightarrow HA(aq.)\quad \Delta H = -70.7\text{ kJ/mol}$

$BOH(g) \longrightarrow BOH(aq.)\quad \Delta H = 20\text{ kJ/mol}$

$\underline{HA(g) + BON(aq) \longrightarrow BA(aq.) + H_2O\quad \Delta H = -42.3\text{ kJ/mol}}$

$HA(g) + BOH(g) \longrightarrow BA(aq.) + H_2O\quad \Delta H = -93\text{ kJ}$

14. (D)

or $\dfrac{T}{P^{2/5}} = \text{constant}$

or T = constant × $P^{2/5}$

$\therefore$ PV = R × constant × $P^{2/5}$

$\therefore\ \dfrac{P}{P^{2/3}} \times V = \text{constant}$

$P^{3/5} \times V = \text{constant}$

or $PV^{5/3} = \text{constant}\quad \left(\therefore\ \gamma = \dfrac{5}{3}\text{ for He}\right)$

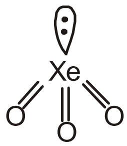
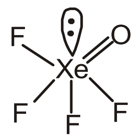
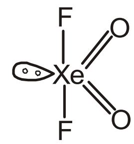
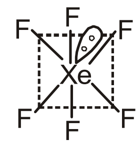
or $PV^{\gamma} = \text{constant}$

Thus, process is adiabatic $\therefore$ Q = 0

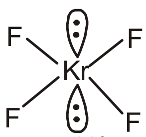
15. (A)

Adiabatic slopes are more steeper than isothermal.

16. (D)

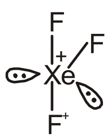
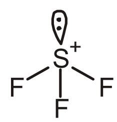
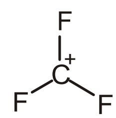
(A)  (B)  (C)  (D) 

17. (C)



$\theta = 90^\circ$

18. (B)

$XeF_3^+ =$  $SF_3^+ =$  $CF_3^+ =$ 

19. (6)

All statements are correct by their definition.

20. (2)

$$\log\left(\frac{P_2}{P_1}\right) = \frac{\Delta H}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\log\left(\frac{10}{1}\right) = \frac{460.6}{2.303 \times 2}\left[\frac{1}{50} - \frac{1}{T_2}\right] \quad ; \quad T_2 = 100 \text{ K}$$

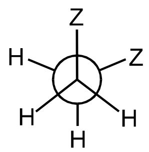
$$\frac{T_2}{T_1} = \frac{100}{50} = 2$$

21. (4)

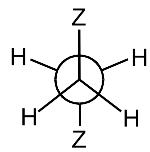
22. (7)

Compounds (iii), (vi), (vii), (viii), (ix), (xi) & (xiv) do not show geometrical isomerism

23. (6)



Gauche form



Anti form

$\mu_{obs} = 2D$

$\because X_{anti} = 0.7 \therefore \quad X_{gauche} = 1–0.7 = 0.3$

$\Rightarrow \mu_{obs} = \Sigma\mu_i x_i \qquad \Rightarrow \quad 2 = \mu_{gauche} \times 0.3 + 0.7 \times 0 \Rightarrow \mu_{gauche} = \dfrac{2}{0.3} = 6.67$ D

24. (6)

n = 3

No. of geometrical isomers = $2^{n-1} + 2^{\frac{n-1}{2}} = 6$

25. (6)

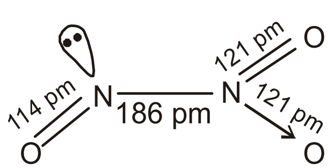
(II), (III), (IV), (VI), (VII), (X) are optically inactive due to presence of eighter plane of symmetry or centre of symmetry

(I), (V), (VIII) and (IX) are optically active due to presence of neighter plane of symmetry or centre of symmetry.

26. (3)

	MP (ºC)	**BP (ºC)**	**density (g/cm³)**
B	2076	3927	2.35
Al	660	2467	2.69
Ga	29.8	2237	5.9
In	157	2080	7.3
Tl	304	1457	11.8

27. (3)



28. (8)

Since in adiabatic process

q = 0 ; hence $\Delta U = W$

$W = nC_v(T_2 - T_1) = 1 \times \frac{3}{2} R \times \Delta T$

$\frac{3}{2} R \cdot \Delta T = 24$

$3 \cdot \Delta T = 24$ (R = 2)

$\Delta T = 8$

MATHEMATICS

29. (B)

$$f'(x) = \frac{1}{|\cos x|} \cos|x| \frac{|x|}{x} > 0$$

$\Rightarrow x\cos|x| > 0 \Rightarrow x\cos x > 0$

30. (B)

We have $f(x) = x^2 \sin \frac{1}{x} + x^3 \cos \frac{1}{2x}$; f '(x) has opposite signs is $\left[\frac{1}{2\pi}, \frac{1}{\pi}\right]$

$\Rightarrow$ f '(x) = 0 atleast once

Clearly f(x) is continuous as well as differentiable in $\left[\frac{1}{3\pi}, \frac{1}{\pi}\right]$

Also $f\left(\frac{1}{3\pi}\right) = 0 = f\left(\frac{1}{\pi}\right)$ (Only in this interval this will be true)

So, f(x) satisfies Rolle's theorem.

Hence there exist some $c \in \left(\frac{1}{3\pi}, \frac{1}{\pi}\right)$ such that f '(c) = 0.

31. (B)

$$\frac{dy}{dx} \text{at}(\alpha, \beta) = \frac{\alpha + \beta + 1}{\alpha + \beta - 1}$$

32. **(B)**

$$\lim_{x\to\infty} x\left(1-x\ell n\left(1+\frac{1}{x}\right)\right)$$

put x = 1/y

$$\lim_{y\to0^+}\left(\frac{1}{y}-\frac{\ell n(1+y)}{y^2}\right) = \lim_{y\to0^+}\left(\frac{y-\ln(1+y)}{y^2}\right)$$

$$=\lim_{y\to0^+}\frac{\left(y-\left(y-\frac{y^2}{2}+\frac{y^3}{3}....\right)\right)}{y^2}=\frac{1}{2}$$

33. **(D)**

$$\sum_{r=1}^{n}\tan^{-1}\left(\frac{(r+1)!-r!}{1+(r+1)!\cdot r!}\right)$$

$$=\sum_{r=1}^{n}\left(\tan^{-1}(r+1)!-\tan^{-1}(r!)\right)=\tan^{-1}(n+1)!-\frac{\pi}{4}$$

34. **(D)**

$$(2\cos\phi,\sin\phi)$$

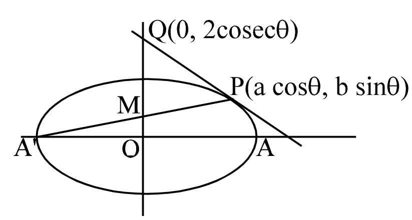
It lies inside of the circle

$$\therefore\quad x^2+y^2+4x+3<0$$

$$-2<\cos\phi<-\frac{2}{3}\Rightarrow\pi-\alpha<\phi<\pi+\alpha$$

35. **(C)**

a = 3 ; b = 2



Tangent : $\frac{x\cos\theta}{3}+\frac{y\sin\theta}{2}=1$

x = 0 ; y = 2 cosecθ

chord A'P : $y=\frac{2\sin\theta}{3(\cos\theta+1)}(x+3)$

put x = 0, $y=\frac{2\sin\theta}{1+\cos\theta}$ = OM

Now $OQ^2 – MQ^2 = OQ^2 – (OQ – OM)^2 = 2(OQ)(OM) – OM^2 = OM\{ 2(OQ) – (OM) \}$

$$= \frac{2\sin\theta}{1+\cos\theta}\left[\frac{2\times 2}{\sin\theta} - \frac{2\sin\theta}{1+\cos\theta}\right] = 4$$

36. (B)

Let equation of circle be $\left(\frac{x^2}{a^2}+y^2-1\right)+\lambda\left(\frac{x^2}{b^2}+y^2-1\right)=0$

$$x^2\left(\frac{1}{a^2}+\frac{\lambda}{b^2}\right)+y^2(1+\lambda)=1+\lambda$$

$$\Rightarrow \quad x^2\left(\frac{b^2+a^2\lambda}{a^2b^2(1+\lambda)}\right)+y^2=1$$

Clearly the circle is $x^2 + y^2 = 1$.

37. (A, D)

Let line is tangent at $(2t_1^3, 3t_1^2)$ and normal at $(2t_2^3, 3t_2^2), (t_1 \neq t_2)$

$$\because \left.\frac{dy}{dx}\right|_{(2t_1^3, 3t_1^2)} = \frac{1}{t_1}$$

and slope of normal at $(2t_2^3, 3t_2^2) = -t_2$

$$\Rightarrow \frac{1}{t_1} = -t_2 \Rightarrow t_2 = -\frac{1}{t_1}.$$

$$\Rightarrow \frac{1}{t_1} = \frac{3t_1^2 - \frac{3}{t_1^2}}{2t_1^3 + \frac{2}{t_1^3}} = \frac{3\left(t_1^2 - \frac{1}{t_1^2}\right)}{2\left(t_1^3 + \frac{1}{t_1^3}\right)} = \frac{3\cdot\left(t_1 - \frac{1}{t_1}\right)\left(t_1 + \frac{1}{t_1}\right)}{2\left(t_1 + \frac{1}{t_1}\right)\left(t_1^2 - 1 + \frac{1}{t_1^2}\right)} = \frac{3(t_1 - \frac{1}{t_1})}{2\left(t_1^2 - 1 + \frac{1}{t_1^2}\right)}$$

$$\Rightarrow 2t_1^2 - 2 + \frac{2}{t_1^2} = 3t_1^2 - 3$$

$$\Rightarrow 1 = t_1^2 - \frac{2}{t_1^2} \Rightarrow a - \frac{2}{a} = 1$$

(Put $t_1^2 = a$ (say) $(a > 0)$)

$\Rightarrow a^2 - a - 2 = 0 \Rightarrow (a-2)(a+1) = 0$

$\Rightarrow a = 2, -1 \because a \neq -1$

$\because a = 2, t_1^2 = 2, t_1 = \pm\sqrt{2}$

$\because$ equation of straight line are $x + \sqrt{2}(y-2) = 0$ and $x - \sqrt{2}(y-2) = 0$

38. (A, C)

$$\ell = \lim_{x\to 0} \frac{(1+qx) - (1+px)\sqrt{1+x}}{x^3\sqrt{1+x}(1+qx)}$$

$$= \lim_{x\to 0} \frac{(1+qx) - (1+px)\sqrt{1+x}}{x^3} = \lim_{x\to 0} \frac{(1+qx) - (1+px)\left(1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \ldots\right)}{x^3}$$

$$\lim_{x\to 0} \frac{qx - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16} - px - \frac{px^2}{2} + \frac{px^3}{8}}{x^3}$$

Now cofficient of x and x^2 must be 0 $\Rightarrow q - p = \frac{1}{2}$ & $\frac{p}{2} = \frac{1}{8} \Rightarrow p = \frac{1}{4}, q = \frac{3}{4}$

$\therefore \ell = -\frac{1}{32}$

39. (B, C)

$\sin^{-1}\left(a^2x^2 + b^2y^2\right) + \cos^{-1}|ax + by| = \pi$

$\Rightarrow a^2x^2 + b^2y^2 = 1$ and $ax + by = 0$

$\Rightarrow 2axby = -1$

40. (C, D)

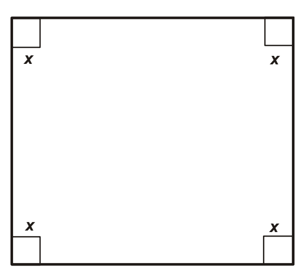
Suppose edge-length of the square sheet is ℓ then volume of the box V is

$V = (\ell - 2x)^2 x$

$= 4x^3 - 4x^2\ell + \ell^2 x$

$\frac{dV}{dx} = 12x^2 - 8\ell x + \ell^2 = 0$

$\Rightarrow x = \frac{\ell}{2}, \frac{\ell}{6}$



$$\frac{d^2V}{dx^2} = 24x - 8\ell$$

$$\frac{d^2V}{dx^2}(x = \frac{\ell}{2}) > 0$$

and $\frac{d^2V}{dx^2}\left(x = \frac{\ell}{6}\right) < 0$

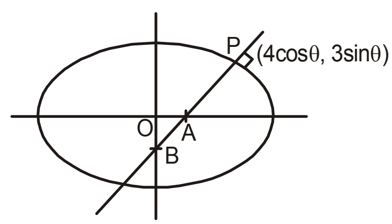
therefore for maximum volume, $x = \frac{\ell}{6}$, but x =10

$\Rightarrow \ \ell = 60$ units

Also maximum volume is = $40^2 \times 10$ = 16000 square units

41. (A, B, C, D)

m_N = 1 (slope of normal)



$\Rightarrow \ \tan\theta = \frac{3}{4}$

Hence equation of normal

$4\sec\theta \cdot x - 3\operatorname{cosec}\theta \cdot y = 16 - 9$ is 5x – 5y = 7

Point $A \equiv \left(\frac{7}{5}, 0\right)$ and $B \equiv \left(0, \frac{-7}{5}\right)$

Area $A = \frac{49}{50}$.

Hence [A] = 0.

Solution of Paragraph for Q. No. 42, 43, 44

42. (C)

43. (A)

44. (A)

OS_1 = ae = 6, OC = b

Also CS_1 = a $\Rightarrow$ Area of $\Delta OCS_1 = \frac{1}{2} OS_1 \times OC$

$\Rightarrow$ Semi - perimeter of $\Delta OCS_1 = \frac{1}{2}(OS_1 + OC + CS_1)$

$\frac{1}{2}(6 + a + b)$ .....(i)

$\Rightarrow$ In radius of $\Delta OCS_1 = 1$

$\Rightarrow \dfrac{3b}{\frac{1}{2}(6+a+b)} = 1 \Rightarrow$ 5b = 6 + a .....(ii)

Also $b^2 = a^2 - a^2e^2$

$= a^2 - 36$ .....(iii)

From (ii) we get

$25(a^2 - 36) = 36 + a^2 + 12a$

$\Rightarrow 2a^2 - a - 78 = 0 \Rightarrow a = \dfrac{13}{2}, -6$

$\Rightarrow$ a = 13/2 and b = 5/2

So Area of ellipse = π ab $= \dfrac{65\pi}{4}$ sq. units

Perimeter of $\Delta OCS_1 = 6 + a + b = 6 + \dfrac{13}{2} + \dfrac{5}{2}$ = 15 units

Equation of director circle is

$x^2 + y^2 = a^2 + b^2$

$\Rightarrow x^2 + y^2 = \dfrac{97}{2}$

$\Rightarrow x^2 + y^2 = 48.5$

45. (C)

a = 1

f (x) = $8x^3 + 4x^2 + 2bx + 1$

f ' (x) = $24x^2 + 8x + 2b = 2(12x^2 + 4x + b)$

for increasing function, $f'(x) \geq 0 \quad \forall \; x \in R$

$\therefore \quad D \leq 0 \quad \Rightarrow \quad 16 - 48b \leq 0 \quad \Rightarrow \quad b \geq \dfrac{1}{3}$

46. (B)

If b = 1

f (x) = $8x^3 + 4ax^2 + 2x + a$

f ' (x) = $24x^2 + 8ax + 2$ or $2(12x^2 + 4ax + 1)$

for non monotonic f ' (x) = 0 must have distinct roots

hence D > 0 i.e. $16a^2 - 48 > 0 \Rightarrow$ $a^2 > 3$; $\therefore$ $a > \sqrt{3}$ or $a < -\sqrt{3}$

$\therefore$ $a \in 2, 3, 4, ...$

sum = 5050 – 1 = 5049 Ans.

47. (3)

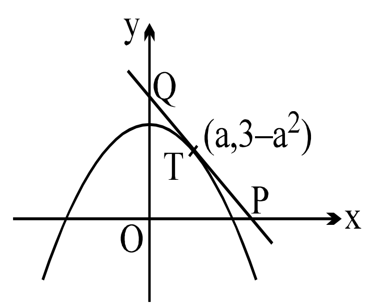
$$f'(x) = \begin{cases} 3ax^2 + 2bx + c \ ; x > 0 \\ -3ax^2 - 2bx - c \ ; x < 0 \end{cases}$$

Clearly, no. of critical points of the function f(x) is three.

48. (4)

$y = 3 - x^2$

$$\left.\frac{dy}{dx}\right|_T = -2x = -2a$$



equation of tangent at T is

$y - (3 - a^2) = -2a(x - a)$

$2ax + y = 2a^2 + 3 - a^2 = a^2 + 3$

$y = 0, x = \dfrac{a^2+3}{2a}$; $x = 0$, $y = a^2 + 3$

Area of OPQ = $\dfrac{1}{2} \cdot \dfrac{(a^2+3)^2}{2a}$; Let f (a) = $\dfrac{(a^2+3)^2}{4a}$

$$f'(a) = \frac{1}{4}\left[\frac{2a^2 \cdot 2(a^2+3) - (a^2+3)^2}{a^2}\right] = 0$$

$(a^2 + 3)(4a^2 - a^2 - 3) = 0$

$a^2 = 1 \Rightarrow a = 1$ or -1

$\therefore$ $A_{min} = \dfrac{16}{4} = 4$ sq. units Ans.]

49. (3)

$$\lim_{x \to 0}\left[\frac{\tan x}{x}\right] = 1$$

$$f(x) = \frac{\tan x}{x} = 1 + \frac{x^2}{3} + \frac{2}{15}x^4 + ...$$

$$\{f(x)\} = \frac{x^2}{3} + \frac{2}{15}x^4 + ...$$

$$\therefore \quad \ell n\left(\lim_{x\to 0}\left([f(x)] + x^2\right)^{\frac{1}{\{f(x)\}}}\right) = \ell n\left(\lim_{x\to 0}\left(1 + x^2\right)^{\frac{1}{\{f(x)\}}}\right)$$

$$= \ell n\left(e^{\lim_{x\to 0} x^2 \cdot \frac{1}{\{f(x)\}}}\right) \quad = \lim_{x\to 0}\frac{x^2}{\frac{x^2}{3} + \frac{2}{15}x^4 + ...} = 3$$

50. (2)

$x_{n+1} = \sqrt{2 + x_n}$

or $\lim_{n\to\infty} x_{n+1} = \sqrt{2 + \lim_{n\to\infty} x_n}$

or $t = \sqrt{2+t}$ $\qquad \left(\because \lim_{x\to\infty} x_{n+1} = \lim_{x\to\infty} x_n = t\right)$

or $t^2 - t - 2 = 0$

or (t – 2) (t + 1) = 0 $\qquad$ or t = 2 $\qquad (\because x_n > o \forall n, t > 0)$

51. (6)

$$\tan^{-1}\frac{1}{\sqrt{2}} - \left(\tan^{-1}\sqrt{3} - \tan^{-1}\sqrt{2}\right)$$

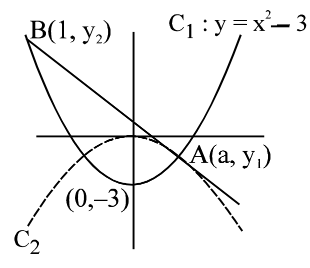
$$\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$$

52. (3)

Point $A(a, y_1)$ lies on C_1 and C_2

hence $y_1 = a^2 - 3$ and $y_2 = ka^2$

$\Rightarrow a^2 - 3 = ka^2 \quad ....(1)$



now $y = kx^2 \Rightarrow \frac{dy}{dx} = 2kx$

$\therefore \left.\frac{dy}{dx}\right]_{(a, y_1)} = 2ka = \frac{y_2 - y_1}{1-a}$ (But $y_2 = 1 - 3 = -2$)

$= \frac{-2-(a^2-3)}{1-a} \quad \Rightarrow 2ka = \frac{1-a^2}{1-a} = 1 + a$

2ka = 1 + a .....(2)

Substituting $k = \frac{a^2-3}{a^2}$ from (1) in (2) we get $\frac{2a(a^2-3)}{a^2} = 1 + a$ $\Rightarrow$ $2a^2 - 6 = a + a^2$

$\Rightarrow$ $a^2 - a - 6 = 0$ $\Rightarrow$ a = +3 , a = – 2 (rejected)]

53. (5)

Put x = cos θ, $y = \frac{1}{3}\sin\theta$

Let $u = 3x^2 - 27y^2 + 24xy$

$u = 3\cos2\theta + 4\sin2\theta$

$-5 \le u \le 5.$

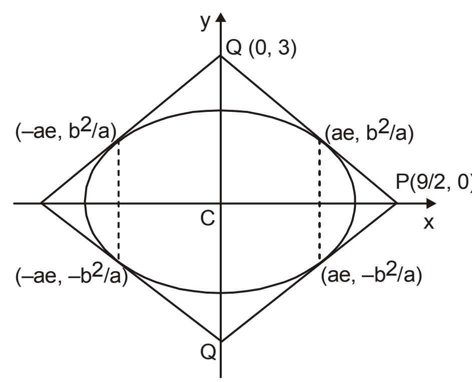
54. (9)

$\frac{x^2}{9} + \frac{y^2}{5} = 1$ $\Rightarrow$ $e^2 = 1 - \frac{5}{9} = \frac{4}{9}$ $\Rightarrow$ $e = \frac{2}{3}$

One end of latus rectum is (2, 5/3)

Equation of tangent at (2, 5/3) is $\frac{2x}{9} + \frac{y}{3} = 1$.

F and F′ be foci.



Area of ΔCPQ = $\frac{1}{2} \times \frac{9}{2} \times 3$ = $\frac{27}{4}$ sq, units

$\Rightarrow$ Area of quadrilateral = λ = 4 × $\frac{27}{4}$ = 27

$\Rightarrow$ $\frac{\lambda}{3} = 9$

55. (1)

The chord of contact of tangents from (h, k) to $y^2 = 4x$ is ky = 2(x + h). If it is a tangent to the hyperbola $x^2 - y^2 = 1$, then $4h^2 + K^2 = 4$, therefore, locus is $\frac{x^2}{1} + \frac{y^2}{4} = 1$.

56. (2)

$(x-y)^2 - (x+y+1) = 0$

$\Rightarrow (x-y+\lambda)^2 - x - y - 1 - \lambda^2 - 2\lambda(x-y) = 0$

$\Rightarrow (x-y+\lambda)^2 - x(1+2\lambda) - y(1-2\lambda) - \lambda^2 - 1 = 0$

We take value of λ such that $x - y + \lambda = 0$ and $-x(1+2\lambda) - y(1-2\lambda) - \lambda^2 - 1 = 0$ may be perpendicular.

$\therefore -(1+2\lambda) + (1-2\lambda) = 0 \therefore \lambda = 0$

$$\therefore \left(\frac{x-y}{\sqrt{2}}\right)^2 = \frac{\sqrt{2}}{2}\left(\frac{x+y+1}{}\sqrt{2}\right) = \frac{1}{\sqrt{2}}\left(\frac{x+y+1}{\sqrt{2}}\right)$$

Therefore, length of the latus rectum = $\frac{1}{\sqrt{2}}$

$$\therefore 4\ell^2 = 4 \cdot \frac{1}{2} = 2$$

Alter

The given equation is a parabola since $h^2 = 1,\ ab = 1 \quad \therefore h^2 = ab$

$\therefore$ x – y = 0 is perpendicular to x + y + 1 = 0

So, x – y = 0 is the equation of axis and x + y + 1 = 0 is equation of tangent at the vertex.

$$\therefore \left(\frac{x-y}{\sqrt{2}}\right)^2 = \frac{1}{\sqrt{2}}\left(\frac{x+y+1}{\sqrt{2}}\right)$$

$\therefore$ Length of the latus rectum (ℓ) = $\frac{1}{\sqrt{2}}$

$$\therefore 4\ell^2 = 4 \times \frac{1}{2} = 2$$

PHYSICS

57. (C)

$H = \frac{1}{2} g (2t)^2 = 2gt^2$ .... (1)

$h = H - \frac{1}{2} gt^2$ .... (2)

By (1) and (2)

$h = H - \frac{H}{4} = \frac{3H}{4}$

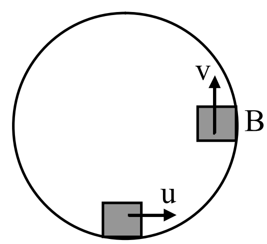
58. (B)

$R = u \sqrt{\frac{2h}{g}} = 12 \sqrt{\frac{10}{10}} = 12 \text{ m}$

$\therefore S = \sqrt{R^2 + r^2} = 13 \text{ m}$

59. (C)

60. (A)



$u^2 = 5gR$

$\therefore v^2 = u^2 - 2gR$

$= 5gR - 2gR = 3gR$

Tangential acceleration at B is

$a_t = g$ (downwards)

Centripetal acceleration at B is

$a_C = \frac{v^2}{R} = 3g$

$\therefore$ Total acceleration will be

$a = \sqrt{a_C^2 + a_t^2} = g\sqrt{10}$

61. (A)

$x = 2t \Rightarrow V_x = \frac{dx}{dt} = 2$

$y = 2t^2 \Rightarrow v_y = \frac{dy}{dt} = 4t$

$\therefore \tan\theta = \frac{v_y}{v_x} = \frac{4t}{2} = 2t$

Differentiating with respect to time we get,

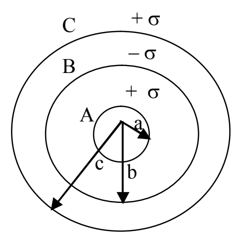
$(\sec^2\theta)\ \frac{d\theta}{dt} = 2$

or $(1 + \tan^2\theta)\ \frac{d\theta}{dt} = 2$ or $(1 + 4t^2)\ \frac{d\theta}{dt} = 2$ or $\frac{d\theta}{dt} = \frac{2}{1+4t^2}$

$\frac{d\theta}{dt}$ at t = 2 s is $\frac{d\theta}{dt} = \frac{2}{1+4(2)^2} = \frac{2}{17}$ rad/s

62. (D)

Potential of shell A is,



$$V_A = \frac{1}{4\pi\epsilon_0}\left(\frac{4\pi a^2\sigma}{a}\ \frac{-4\pi b^2\sigma}{b}\ \frac{+4\pi c^2\sigma}{c}\right)$$

$$= \frac{\sigma}{\epsilon_0}\ (a - b + c)$$

Potential of shell C is,

$$V_C = \frac{1}{4\pi\epsilon_0}\left(\frac{4\pi a^2\sigma}{c}\ \frac{-4\pi b^2\sigma}{c}\ \frac{+4\pi c^2\sigma}{c}\right)$$

$$= \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$$

As $V_A = V_C$

$$\therefore\ \frac{\sigma}{\epsilon_0}\ (a - b + c) = \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{c}\ \frac{-b^2}{c} + c\right)$$

or $a - b = \frac{(a-b)(a+b)}{c}$ or a + b = c

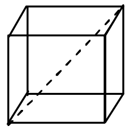
63. (A)

$$E_x = -\frac{\partial V}{\partial x}\hat{i} = -2x\hat{i}$$

$E_y = -\frac{\partial V}{\partial y}\hat{j} = +2y\hat{j}\ \therefore\ E = 2\,(-x\hat{i} + y\hat{j})$

Hence electric field lines should be straight line in x-y plane.

64. (D)



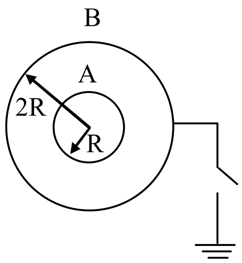
max string through the cube= $\sqrt{3}$ a

$\phi. = \frac{\Sigma q}{\epsilon_0} = \frac{\sqrt{3}\lambda a}{\epsilon_0}$

65. (C,D)

66. (A, D)

Before switching



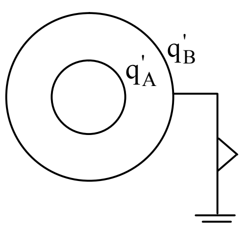
$V_A = \frac{kq_A}{R} + \frac{kq_B}{2R}$ = 2V ... (i)

$V_B = \frac{kq_B}{2R} + \frac{kq_A}{2R} = \frac{3}{2}V$ ... (ii)

By the help of these two equation $\frac{q_A}{q_B}$ can be find out.

After switching

Shell B become earthed and its potential become zero.



$V_B = \frac{kq_B'}{2R} + \frac{kq_A'}{2R} = 0$

$q_A' = -q_B'$

$\frac{q_A'}{q_B'} = \frac{-1}{1}$

Charge on A remain conserved

$\therefore\ q_A' = q_A$

$q_B' = -q_A$

$V_A = \frac{kq_A}{R} - \frac{kq_A}{2R} = \frac{kq_A}{2R}$

Using (1) and (2) we can find out V_A.

67. (A,B,C)

Potential of innermost shell is zero.

$\therefore \frac{q_1}{r} + \frac{q_2}{2r} + \frac{q_3}{3r} = 0$

or, $6q_1 + 3q_2 + 2q_3 = 0$ …(1)

Similarly, potential on outermost shell is also zero.

$\therefore \frac{q_1}{3r} + \frac{q_2}{3r} + \frac{q_3}{3r} = 0$

or, $q_1 + q_3 = -q_2$ …(2)

Solving equations (1) and (2), we get

$q_1 = -\frac{q_2}{4}, \frac{q_3}{q_1} = 3$

and, $\frac{q_3}{q_2} = -\frac{3}{4}$

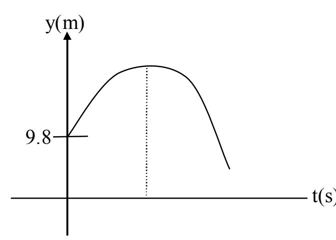
$\therefore$ The options (A), (B) and (C) are correct.

68. (C, D)

69. (A,B,C)

70. (C)

$$\left. \begin{array}{l} y = y_0 + u_y t + \frac{1}{2} a_y t^2 \\ y = y_0 t v_0 \sin\theta t - \frac{1}{2} g t^2 \end{array} \right| \text{ where } y_0 = 9.8 \text{ m.}$$



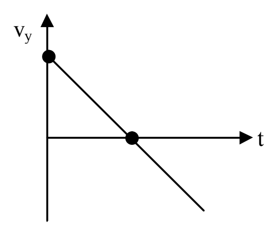
This is parabola opening downward

71. (D)

$v_y = u_y + a_y t$

$v_y = u \sin\theta - gt$

This is straight line with negative slope



72. (D)

$y = y_0 + u_y t + \frac{1}{2} a_y t^2$

$2 = 9.8 + (5gm\ 30^o)t - \frac{1}{2} gt^2$

$-7.8 = \frac{5}{2}t - \frac{9.8}{2}t^2$

$9.8t^2 - 5t - 15.6 = 0$

$t = \frac{5 \pm \sqrt{25 + 4 \times 15.6 \times 9.8}}{19.6}$

$t = \frac{5 + \sqrt{25 + 62.4 \times 9.8}}{19.6}$

$t = \frac{5 + \sqrt{87.4 \times 9.8}}{19.6}$

t = 1.54 sec.

73. (C)

74. (D)

75. (5)

Assume given sphere is solid, potential V_1 at P is to be calculated. But in cavity there is no charge therefore potential V_2 due to charge assumed in cavity must be subtracted from V_1.

Charge on solid sphere = $\frac{4}{3}\pi R^3 \times \rho$

$= \frac{5}{3} \times 10^{-10}$ cb

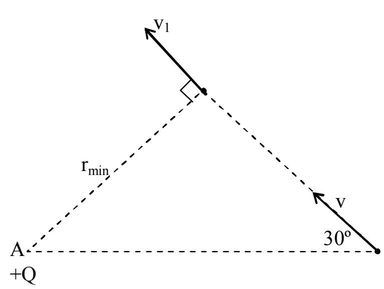
Potential at P can be calculated say V_1

V_2 = Potential due to cavity sphere

$= \frac{\frac{4}{3}\pi r^3 \rho}{4\pi\varepsilon_0 a} = 0.24V$

Potential at P = $V_1 - V_2$ = 35.16 volt

76. (9)



Angular momentum about A is conserved as $\tau_{external} = 0$

$mv\sin30^\circ \times R = v_1 \times m \times r_{min}$

$\frac{Rv}{2} = \frac{1}{\sqrt{3}} \times r_{min}$

$r_{min} = \frac{\sqrt{3}R}{2} = \frac{\sqrt{3} \times 6\sqrt{3}}{2} = 9$ cm

77. (5)

Force on a charge –q in an electric field

$\vec{F} = -q\vec{E}$

This force acts in a direction opposite to $\vec{E}$. Therefore the particle, initially placed at rest, will move opposite to $\vec{E}$ under the action of force. Obviously, direction of $\vec{V}$ will be opposite to $\vec{E}$.

Now $\vec{V} = 10\hat{i} - 10\hat{j}$ m/s (given)

unit vector in the direction of $\vec{V}$,

$$\vec{V} = \frac{10\hat{i} - 10\hat{j}}{\sqrt{(10)^2 + (-10)^2}}$$

$$= \frac{10\hat{i} - 10\hat{j}}{10\sqrt{2}}$$

$$\therefore \ \vec{V} = \frac{i}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$

So unit vector opposite to $\vec{V}$, i.e. in the direction of

$$\vec{E} = -\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}$$

Magnitude of $\vec{E}$ is 10 N/C (given)

Therefore $\vec{E} = 10\left[-\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}\right)$ ….(1)

The surface of area Am^2 has been placed in the x-z plane so that its area vector can be expressed as,

$\vec{A} = A\hat{j}(\vec{A}$ being normal to x-z plane, will be along y-axis)

Electric flux, in case of a uniform electric field,

$$\phi = \vec{E}.\vec{A} = 10\left[-\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}\right].\ A\hat{j}$$

$$= \frac{10A}{\sqrt{2}} = 5\sqrt{2} \ A\ Nm^2/C$$

78. (4)

$qV_a = qV_b + \frac{1}{2}mv^2$

$2.0 \times 10^{-9} \times 9 \times 10^9\left[\frac{3\times10^{-9}}{1}-\frac{3\times10^{-9}}{2}\right] \times 100$

$= 2.0 \times 10^{-9} \times 9 \times 10^9\left[-\frac{3\times10^{-9}}{1}+\frac{3\times10^{-9}}{2}\right]\times 100 + \frac{1}{2} \times 5.0 \times 10^{-9}\, v^2$

$10^{-9} \times 1800\left[\frac{3}{2}\right]\times 100 = 18 \times 10^{-9} \times 100\left[-\frac{3}{2}\right] + \frac{1}{2} \times 5.0 \times 10^{-9}v^2$

$1800\left[\frac{3}{2}+\frac{3}{2}\right] = \frac{1}{2} \times 5.0 \times v^2$

$\frac{1800\times6}{5} = v^2$

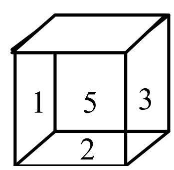
$360 \times 6 = v^2$

$6 \times 6 \times 10 \times 6 = v^2$

$12\sqrt{15} = v$

79. (2)

Flux through cube = $\frac{q}{8\varepsilon_0}$



Flux through face (1) is zero

flux through each

(2) (3) (5) each is $\frac{1}{3}\frac{q}{8\varepsilon_0} = \frac{q}{24\varepsilon_0}$

Flux (2) and (3) combine is = $2 \times \frac{q}{24\varepsilon_0} = \frac{q}{12\varepsilon_0}$

80. (5)

$v_C^2 = v_A^2 + 2a_tS$

Tangential acceleration :

$a_t = \frac{v_C^2 - v_A^2}{2s}$ = 1.447 m/s^2

Normal acceleration at C is

$a_n = \frac{v^2}{r_C}$ = 2.41 m/s^2

Tangential force at C

$F_t = ma_t$ = 1500 × 1.447 = 2170 N

Normal force at C

$F_n = ma_n$

= 1500 (2.41) = 3620 N

Total force at C

$F = \sqrt{F_n^2 + F_t^2}$

$F = \sqrt{(2170)^2 + (3620)^2}$

F = 4220 N

81. (6)

$F_E = qE = 11$ N

$F_g = mg = 5$N

So Net force = F = 6N upward

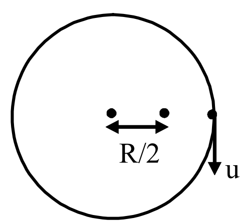
$g_{eff} = \frac{F}{m} = \frac{6}{0.5}$ 12 m/s^2

so $V_{min} = \sqrt{5g_{eff}\ell} = \sqrt{5\times12\times(60\times10^{-2})}$

so V_{min} = 6 m/sec

82. (8)

$U = -\int \vec{F}.\vec{dr}$



$U = -\frac{km}{r}$

$K_i + U_i = K_f + U_f$

$\frac{1}{2}mv^2 - \frac{Km}{\left(\frac{R}{2}\right)} = 0 - \frac{Km}{3R/2}$

$\frac{mv^2}{2} = \frac{2Km}{R} - \frac{-2Km}{3R}$

$\frac{mv^2}{2} = \frac{4Km}{3R}$

$V = \sqrt{\frac{8K}{3R}}$,V = 8 m/s

83. (4)

Area under P – x graph = $\int P dx$ = $\int \left(\frac{mdN}{dt}\right)$vdx

= $\int_1^v mv^2 dv$ = $\left[\frac{mv^3}{3}\right]_1^V = \frac{10}{7\times3}(v^3-1)$ …(1)

from graph, area = $\frac{1}{2}$ × (2 + 4) × 10 = 30 …(2)

from (1) & (2)

$\frac{10}{7\times3}(v^3-1) = 30$

$\Rightarrow$ v = 4m/s

84. (0)

$W_{net} = \Delta K$

$\Rightarrow$ (F sin θ. ℓ – mgℓ (1 – cosθ) = $\frac{1}{2}$ mv^2

where θ = 37º , F = $\frac{mg}{3}$

$\Rightarrow$ v = $\left\{\frac{2\ell}{5m}(3F-mg)\right\}^{\frac{1}{2}}$ = 0