SOLUTIONS

PROGRESS TEST-7

CD-1801(A), CD-1801(B)

CDK-1801 & CDS-1801

(JEE MAIN PATTERN)

Test Date: 11-11-2017

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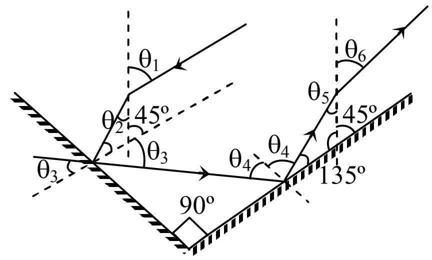
PHYSICS

1. (B)



$\theta_2 + \theta_3 + 135^\circ = 180^\circ$

$\theta_3 = 45^\circ - \theta_2$

$\theta_3 + \theta_4 = 90^\circ$

$45^\circ - \theta_2 + \theta_4 = 90^\circ$

$\theta_4 = 45^\circ + \theta_2$

$90^\circ - \theta_4 + \theta_5 + 135^\circ = 180^\circ$

$45^\circ - \theta_2 + \theta_5 + 135^\circ = 180^\circ$

$\theta_5 = \theta_2$

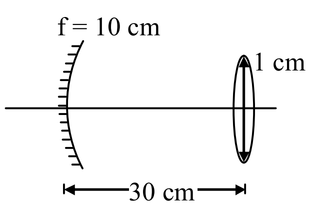
$\therefore \quad \theta_6 = \theta_1$ (By snell law)

$\therefore \quad \delta = 180^\circ$

2. (B)

At any instant velocity of particle can be resolved in two components, one parallel and other perpendicular to it. Parallel components of particle velocity and image velocity are identical and hence the path of light is straight line perpendicular to mirror.

3. (A)



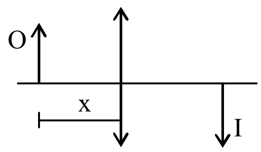
$$|m| = \left|\frac{f}{f-u}\right| = \left|\frac{-10}{-10+30}\right| = \frac{1}{2}$$

$$\frac{r_I}{r_o} = m = \frac{1}{2}$$

$\omega_o = \omega_I$

$$\therefore \frac{a_I}{a_o} = \frac{r_I}{r_o} = \frac{1}{2}$$

4. (C)



$m_1 m_2 = 1$

$m_2 = \frac{1}{m_1}$

$m_2 = \frac{1}{2}$

5. (B)

$\text{Sin } i_c = \frac{1}{{}_r\mu_d} = \frac{\mu_r}{\mu_d} = \frac{v_d}{v_r}$

$\sin 30° = \frac{V_d}{3\times10^8} \Rightarrow v_d = 1.5 \times 10^8$ m/s

6. (C)

7. (B)

$\cot A/2 = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2}$

$\frac{\cos A/2}{\sin A/2} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2}$

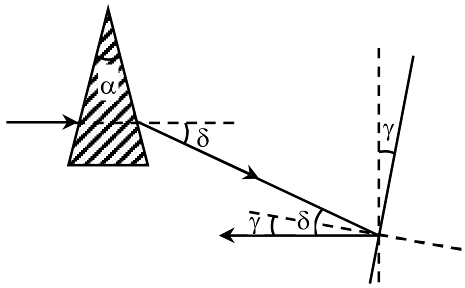
$\sin (90 - A/2) = \sin\left(\frac{A+\delta_m}{2}\right)$

so $\delta_m = 180° - 2A$

8. (A)

As the apex angle is very small ($\alpha = 4°$), the angle of deviation & can be obtained approximately:

$\delta = (n - 1)\alpha = (1.5 - 1) \times 4° = 2°$.

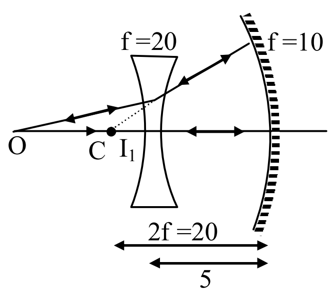


From figure, we see that if the reflected ray is to be horizontal, the mirror must be rotated clockwise through an angle γ given by

$$\gamma = \frac{\delta}{2} = 1^\circ$$

9. (B)

10. (B)



For lens f = –20, u = ?, v = – (20 – 5) = – 15

$$\frac{1}{-20} = \frac{1}{-15} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{1}{20} - \frac{1}{15}$$

u = – 60

11. (A)

$$\frac{f_o}{f_e} = 10$$

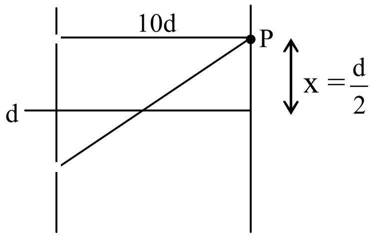
$f_o + f_e = 1.1$

$\therefore$ f_o = 100 cm and f_e = 10 cm

Final magnification = $f_o\left(\frac{1}{D} + \frac{1}{f_e}\right)$

$$= 100\left[\frac{1}{25} + \frac{1}{10}\right] = 14$$

12. (A)



Δx at P = $\frac{dx}{D}$ = $\frac{d^2}{2D}$ = $\frac{(5\lambda)^2}{2\times10\times d}$

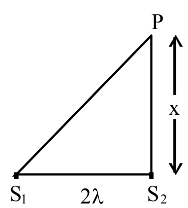
Δx = $\frac{(5\lambda)^2}{2\times10\times5\lambda}$ = $\frac{\lambda}{4}$

$\Delta\phi$ = $\frac{2\pi}{\lambda}\times\Delta x$ = $\frac{\pi}{2}$

$I_0 = 4I \Rightarrow I = \frac{I_0}{4}$

$I_{net} = I + I + 2\sqrt{I}\sqrt{I}\cos\frac{\pi}{2} = 2I = \frac{I_0}{2}$

13. (A)



$S_1P - S_2P = \frac{3\lambda}{2}$

or $\sqrt{4\lambda^2 + x^2} - x = \frac{3\lambda}{2}$

On solving

$x = \frac{7\lambda}{12}$

14. (C)

$10\,\beta_1 = 10 \times \frac{\lambda D}{\mu d}$

in liquid

$\beta_2 = \frac{\lambda D}{d}$

$6\beta_2 = 10\beta_1$

$$\frac{6\lambda D}{d} = \frac{10\lambda D}{\mu d}$$

$$\mu = \frac{10}{6} = 1.67$$

15. (C)

$\sin\theta = \frac{\lambda}{a}$

$\Rightarrow \sin 30^\circ = \frac{6500}{a}$ Å

$\therefore \quad \frac{1}{2} = \frac{6500}{a}$ Å

$\therefore \quad a = 2 \times 6500$ Å $= 2 \times 6500 \times 10^{-10}$ m

$\therefore \quad a = 1.3 \times 10^{-6}$ m

16. (D)

Since, $\cos\theta = \frac{R}{Z} = \frac{IR}{IZ} = \frac{8}{10} = \frac{4}{5}$

($\cos\theta$ can never be greater than 1)

Also, $Ix_C > Ix_L \quad \Rightarrow \quad X_C > X_L$

Current will be leading

In a LCR circuit

$V = \sqrt{(V_L - V_C)^2 + V_R^2} \qquad = \sqrt{(6-12)^2 + 8^2}$

V = 10 ; which is less than voltage drop across capacitor .

17. (A)

From Kirchoff's current law,

$i_3 = i_1 + i_2 = 3 \sin \omega t + 4 \sin (\omega t + 90^\circ)$

$= \sqrt{3^2 + 4^2 + 2(3)(4)\cos 90^\circ} \; \sin(\omega t + \phi)$

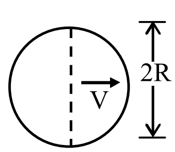
where $\tan \phi = \frac{4 \sin 90^\circ}{3 + 4\cos 90^\circ} = \frac{4}{3}$

$\therefore i_3 = 5\sin(\omega t + 53^\circ)$

18. (D)

If AC is the square wave then all these three options are possible

19. (C)



Considering a projected length 2R on the ring in vertical plane. This length will move at a speed v perpendicular to the field.

This results in an induced emf:

e = Bv (2R) in the ring

In Ring "A" : e_A = B(–V (2R)

In Ring "B": e_B = B(V) (2R)

$\Delta V = e_A - e_B = 4BVR$

20. (A)

U = $\frac{1}{2}$ LI_2

Rate = $\frac{dU}{dt}$ = LI $\left(\frac{dI}{dt}\right)$

At t = 0 , I = 0

$\therefore$ Rate = 0

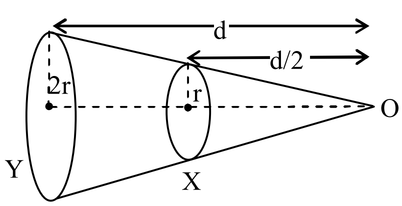
At t = ∞, I = I_0 but $\frac{dI}{dt}$ = 0, therefore, rate = 0

21. (D)

22. (B)

W = MB (cos 0º – cos 60º) = $\frac{MB}{2}$

23. (C)



As two coils subtend the same solid angle at O, hence area of coil, Y = 4 × area of coil X

$$\left[\text{Solid angle} = \frac{\text{area}}{(\perp \text{distance})^2}\right]$$

i.e. radius of coil Y = 2 × radius of coil X

$\therefore B_Y = \frac{\mu_0}{4\pi} \times \frac{2\pi I(2r)^2}{[(2r)^2 + (d^2)]^{3/2}}$

$B_X = \frac{\mu_0}{4\pi} \times \frac{2\pi I(r)^2}{\left[r^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}$

$\therefore \frac{B_Y}{B_X} = \frac{4}{(4r^2 + d^2)^{3/2}} \times \left[\frac{4r^2 + d^2}{4}\right]^{3/2}$

$= \frac{4}{(4)^{3/2}} = \frac{4}{8} = \frac{1}{2}$

24. (A)

K.E = QU

magnetic moment = i × Area

$= \frac{Q}{T} \times \pi R^2$

$\therefore T = \frac{2\pi m}{qB}$

$R = \sqrt{\frac{2mKE}{qB}} = \sqrt{\frac{2mUQ}{qB}}$

Magnetic moment = $\frac{Q^2 \times B}{2\pi m} \times \frac{\pi \times 2m \times UQ}{QB}$

Magnetic moment $= Q^2U$

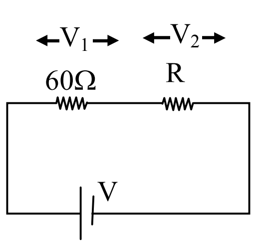
25. (D)

$i = \frac{7V}{7\Omega} = 1$ A

Current flows in anticlockwise direction in the loop. Therefore 0 – 1 × 2 – 1 × 2 – 5 = V_1

V_1 = – 9V

26. (B)



$\frac{V_1}{V_2} = \frac{60}{R}$ ....(1)

$V_1 + V_2 = V$ ....(2)

from (1) & (2) $\Rightarrow V_2 = \frac{VR}{60+R}$ ....(3)

$\frac{V_2^2}{R}$ = 60 watt ....(4)

from (3) & (4) $V^2R = 60\,(60 + R)^2$

$V^2R = 60(3600 + R^2 + 120R)$

$60R^2 + \left(120\times 60 - V^2\right)R + 60\times 3600 = 0$

(for $D \geq 0$)

$\left(120\times 60 - V^2\right)^2 - 4\times 60\times 60\times 3600 \geq 0$

$\left(120\times 60 - V^2\right)^2 \geq 4\times 60\times 60\times 3600$

$V^2 \geq 2\times 2\times 60\times 60$

$V \geq 2\times 60$

$V \geq 120$

27. (C)

In case of a capacitor

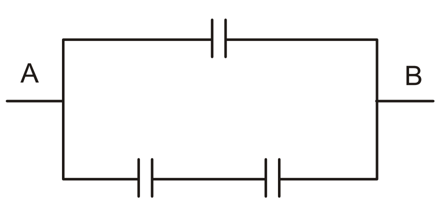
$q = CV$

$\therefore\ i = \frac{dq}{dt} = C\left(\frac{dV}{dt}\right)$

$\frac{dV}{dt} = \frac{4.0}{4.0}$ V/s = 1.0 V/s

Therefore, if C = 1 F then i = 1×1 = 1A (constant)

28. (B)



29. (B)

$E = -\frac{dV}{dx}$

30. (D)

CHEMISTRY

31. (A)

Fraction of total volume occupied by atom in a simple cube = $\dfrac{\frac{4}{3}\cdot\pi\cdot r^3}{a^3}$

Here, a = 2r

$\therefore$ Packing fraction = $\dfrac{\pi}{6}$

32. (D)

Molar volume = $\dfrac{\text{Molar mass}}{\text{density}} = \dfrac{M}{\rho}$

$\because \ \rho = \dfrac{Z.M.}{N_A \cdot a^3}$

$\therefore$ Molar volume $= \dfrac{N_A \cdot a^3}{Z} = \dfrac{6.022\times10^{23}\times(400\times10^{-10})^3}{4} = 9.64$ ml

33. (D)

No. of A atoms $= 7\times\dfrac{1}{8} = 7/8$

No. of B atoms $= 6\times\dfrac{1}{2} = 3$

Formula $A_{7/8}B_3 \equiv A_7B_{24}$

34. (A)

$\rho = \dfrac{Z.M.}{N_A \cdot a^3}$

$= \dfrac{2\times23}{6.022\times10^{23}\times\left(4.24\times10^{-8}\right)^3} = 1.002 \text{ g / cm}^3$

35. (C)

(I) $t_{1/2}$ = k (constant), 1st order

(II) $t_{1/2}$ = k $[A]^1$ or , $1 = 1-n \Rightarrow n = 0 \ [\because t_{1/2}\ \alpha\ [A]^{1-n}]$

(III) $t_{1/2} = k\dfrac{1}{[A]} = k[A]^{-1}$ or, –1 = 1 – n or n = 2

36. (A)

$NH_4NO_2(aq.) \rightarrow N_2(g)+2H_2O$

$$k = \frac{2.303}{t}\log_{10}\frac{V\infty}{V_\infty - V_t}$$

$$A_1 \quad k = \frac{2.303}{10}\log 10\left(\frac{33.05}{33.05-6.25}\right) = 0.02 = 2\times10^{-2}\ \text{min}^{-1}$$

37. (C)

$$\pi = \frac{5\times.0821\times300}{50\times1000} = 0.002463\,\text{atm}$$

$$= 0.002463\times760 = 1.872\ \text{mm of Hg}$$

heigh of sol = $\frac{1.872\times13.6}{.96} = 26.50\,\text{mm}$

38. (D)

A_2B_3 60% ionised

$$i = 1+\alpha(n-1)$$

$$i = 1+.6(5-1)$$

i = 3.4

$$\Delta T_b = i\times kb\times m$$

$$= 3.4\times.52\times1$$

$$= 1.76$$

$$T_b = 373.15+1.76$$

$$= 374.92\text{K}$$

39. (C)

$$i = 1+\alpha\left(\frac{1}{n-1}\right)$$

$$= 1+0.5\left(\frac{1}{2}-1\right)$$

$$= 1-.25$$

$$= .75$$

40. (A)

(i) $Cu^{2+} + 2e^- \longrightarrow Cu \quad E^o = 0.337\ V$

$\Delta G^o_1 = -n_1FE^o_1$ $(n_1 = 2)$

(ii) $Cu^{2+} + e^- \longrightarrow Cu^+$ $E^o = 0.153$ V

$\Delta G_2^0 = -n_2FE^o_2$ $(n_2 = 1)$

Target $Cu^+ + e^- \longrightarrow Cu$ $E^o_3 = ?$

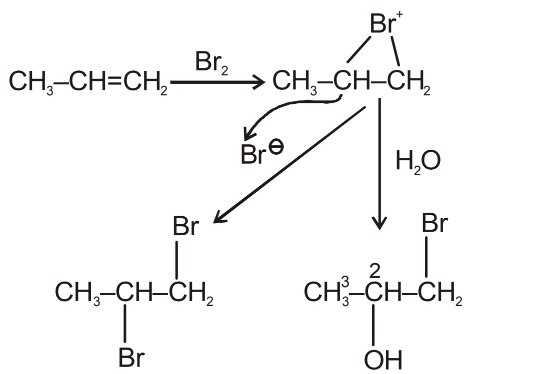
$\Delta G^o_3 = -n_2FE^o_3$ $(n_3 = 1)$

$\Delta G^o_3 = \Delta G^o_1 - \Delta G^o_2 = -2F \times 0.337 + F \times 0.153$

$-FE^o_3 = -2F \times 0.337 + F \times 0.153$

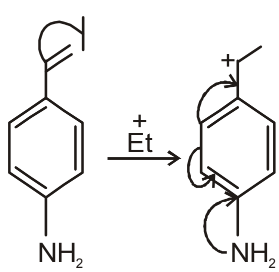
$\therefore$ $E^o_3 = 0.521$ V

41. (D)



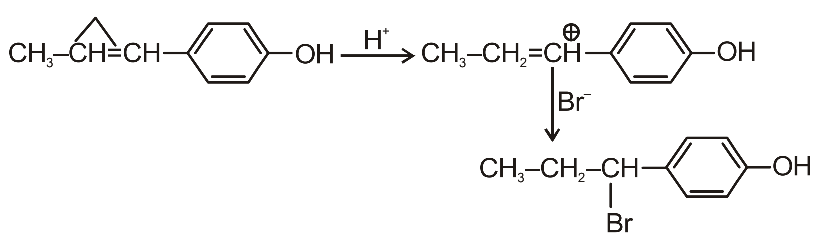
1–Bromo Propan–2–ol

42. (D)



This carbocation is more stable due to more activating powr of NH_2

43. (B)



44. (A)

About both chiral centre configuration is changed.

45. (A)

Molecule belongs to symmetrical nature having chiral centre odd.

Total No. of optical active isomers

n = 3 $A = 2^{n-1} - 2^{\frac{n-1}{2}}$

$= 2^{3-1} - 2^{\frac{(3-1)}{2}}$

$= 2^2 - 2^1 = 4 - 2 = 2.$

46. (C)

Total no. of stereo genic unit ingiven molecule = 4

and molecule is unsymmetrical

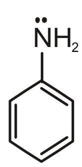
Total no. of optical is active isomers = 2^4 = 16.

47. (B)

$\overset{\ominus}{C}Cl_3$ is more stable than $\overset{\ominus}{C}F_3$ due to dorbital resonance deu to presence of vacant Cl orbital of Cl atom.

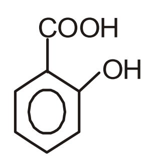
48. (B)

lone pair present on hitrogen atom is localised on nitrogenatom.



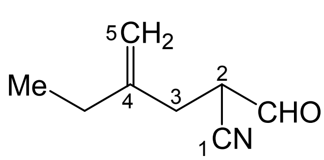
(lpis not involved in res non Cl)

49. (B)

Extent of chelation is very high in case of 

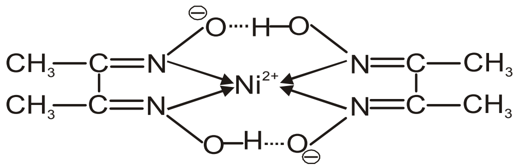
so it is more acidic.

50. (D)



51. (C)

The complex is $[Ni(DMG)_2]$' a non ionic complex.



Square planar (dsp^2)

52. (A)

In aqueous solution Cu^{+1} undergoes disporoportionation to form a more stable Cu^{+2} ion.

$$2Cu^{+}(aq) \rightarrow 2Cu^{+2}(aq) + Cu(s)$$

The higher stability of Cu^{+2} in aqueous solution may be attributed to its greater negative standard change in hydration energy. Cr^{+2} is reducing as its conifiguration changes from d^4 to d^3, a more stable half filled t_2g configuration, while Mn^{+3} is oxidising as Mn^{+3} to Mn^{+2} change results in a more stable half filled d^5 configuration.

53. (C)

$Ni^{+2} \longrightarrow 3d^8 \longrightarrow$ No effect of S.F.L

So, $sp^3, d^2, \quad n = 2$

$\mu = \sqrt{n(n+2)}$ B.M

$= \sqrt{8}$ B.M

54. (D)

Among B_2, C_2, N_2 $\overset{b}{\pi}$ orbitals filled before $\overset{b}{\sigma}$, indicating s–p intermixing.

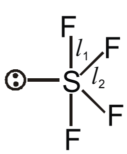
55. (D)

$Na_2Cr_2O_7 > K_2Cr_2O_7$
More soluble

56. (A)

$IF \rightarrow IF_5 + I_2$

57. (A)

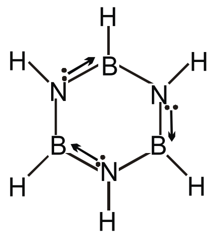


$l_1 > l_2$

Hint : $PCl_5(s) \longrightarrow \underset{sp^3}{PCl_4^+} + \underset{sp^3d^2}{PCl_6^-}$

all bonds are identical bond length

58. (C)



$\left.\begin{array}{l} sp^2 \\ \mu = 0 \end{array}\right\}$ Planar as well as non-polar

59. (B)

$KMnO_4$ has no unpaired electron present in its configuration.

60. (D)

H_2O_2 reduces Cr^{+6} in $K_2Cr_2O_7$ to Cr^{+3} ion, and color changes from orange to green.

MATHEMATICS

61. (C)

$y^2 = kx \qquad 2y.\frac{dy}{dx} = k = \frac{y^2}{x} \qquad \Rightarrow \frac{dy}{dx} = \frac{y}{2x}$

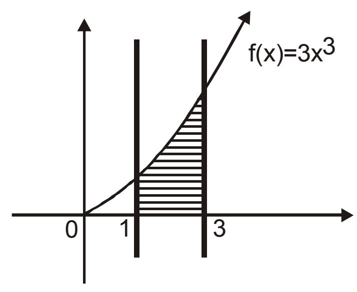
now, $\tan\frac{\pi}{4} = \left|\frac{\frac{dy}{dx} - \frac{y}{2x}}{1 + \frac{y}{2x}.\frac{dy}{dx}}\right| \Rightarrow \frac{dy}{dx} = \frac{2x+y}{2x-y}$ or $\frac{y-2x}{y+2x}$

62. (A)

$f(x) = ax^3 + bx$

Now $\lim_{x \to 0} \frac{f(x)}{x} = 0 \Rightarrow b = 0$

$\lim_{x \to 0} \left(\frac{f(x)}{x} + 1\right)^{1/x^2} = e^3 \Rightarrow e^a = e^3 \Rightarrow a = 3$



$\therefore f(x) = 3x^3$

$A = \int_1^3 3x^3 dx = 60$

63. (C)

$$x\,dy = y\,dx + \sqrt{x^2+y^2}\,dx \Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sqrt{1+(y/x)^2}$$

Put y/x = v $\qquad \frac{dy}{dx} = x\frac{dv}{dx} + v$

$$\therefore \int \frac{1}{\sqrt{1+v^2}}\,dv = \int \frac{dx}{x} \Rightarrow \ell n(v+\sqrt{1+v^2}) = \log x + \log c \Rightarrow y + \sqrt{x^2+y^2} = cx^2$$

64. (D)

$$xy(x) = x^2y'(x) + y(x).2x$$

$$xy(x) + x^2y'(x) = 0$$

$$x\frac{dy}{dx} + y = 0$$

x y = 6

65. (B)

$$\frac{dx}{dy} = \frac{x}{y} + 2y^2$$

$$\frac{dx}{dy} - \frac{x}{y} = 2y^2$$

$$\frac{x}{y} = \int 2y\,dy$$

$$\frac{x}{y} = y^2 + c$$

66. (C)

In a skew symmetric matrix,
diagonal elements are zero.
Also $a_{ij} + a_{ji} = 0$
Hence number of matrices = 2 × 2 × 2 = 8

$$\begin{bmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{bmatrix}$$

67. (C)

we have $2a^2 + a + 1 > 3a^2 - 4a + 1 \Rightarrow a^2 - 5a < 0 \Rightarrow 0 < a < 5$ ...(A)

Also $3a^2 - 4a + 1 = (3a-1)(a-1) > 0 \Rightarrow a \in (-\infty, 1/3) \cup (1, \infty)$ ...(B)

Intersection of (A) and (B) yields $a \in (0, 1/3) \cup (1, 5)$

68. (A)

$3 \le |[x]| + |[y]| \le 6$

$|[x]| = 0 \;\&\; 3 \le |[y]| \le 6 \;\Rightarrow\; 0 \le x < 1 \;\&\; 3 \le y < 7,\ -6 \le y < -2$

Hence area = 1 × 8 = 8.

$|[x]| = 1 \;\&\; 2 \le |[y]| \le 5 \;\Rightarrow\; 1 \le x < 2, -1 \le x < 0 \;\&\; 2 \le y < 6,\ -5 \le y < -1$

Area = 2 × 8

$|[x]| = 2 \;\&\; 1 \le |[y]| \le 4 \;\Rightarrow\; 2 \le x < 3, -2 \le x < -1 \,\&\, 1 \le y < 5,\ -4 \le y < 0$

Area = 2 × 8

For $|[x]| = 3,\ 0 \le |[y]| \le 3,\ \text{Area} = 2 \times 7$

For $|[x]| = 4,\ |[y]| \le 2,\ \text{Area} = 2 \times 5$

For $|[x]| = 5,\ |[y]| \le 1,\ \text{Area} = 2 \times 3$

For $|[x]| = 6,\ |[y]| = 0,\ \text{Area} = 2 \times 1$

Hence total area = 8 + 16 + 16 + 14 + 10 + 6 + 2 = 72

$\Rightarrow\ \alpha = 7,\ \beta = 2$

Now, $I = \int_{\beta}^{\alpha} \frac{\sqrt{9-x}}{\sqrt{x}+\sqrt{9-x}}\,dx = \int_{2}^{7} \frac{\sqrt{9-x}}{\sqrt{x}+\sqrt{9-x}}\,dx = \int_{2}^{7} \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}\,dx$

Hence, $2I = \int_{2}^{7} dx = 5 \;\Rightarrow\; I = \frac{5}{2}$

69. (A)

$I_n = 2\int_{0}^{\pi} \left(\frac{\pi}{2} - x\right)^{n+1} \cos nx\,dx$

$= 2\int_{0}^{\pi} \left(\frac{\pi}{2} - (\pi - x)\right)^{n+1} \cos(n(\pi - x))\,dx$

$= 2\int_{0}^{\pi} (-1)^{n+1} \left(\frac{\pi}{2} - x\right)^{n+1} (-1)^n \cos nx\,dx = -I_n$

$\therefore\ I_n = 0\ \forall\, n \in N$

70. (C)

$$\vec{a}\cdot\vec{c} = |\vec{a}||\vec{c}|\cos\left(\cos^{-1}\frac{1}{4}\right) \Rightarrow \vec{a}\cdot\vec{c} = \frac{1}{4}$$

$$\vec{b} - 2\vec{c} = \lambda\vec{a}$$

$$\vec{a}\cdot\vec{b} - 2(\vec{a}\cdot\vec{c}) = \lambda$$

$$\vec{a}\cdot\vec{b} = \lambda + \frac{1}{2} \quad \text{...(i)}$$

Similarly, $\vec{b}\cdot\vec{c} = 8 - \frac{\lambda}{4} - \frac{\lambda^2}{2}$ ...(ii)

and $\vec{b}\cdot\vec{c} - 2 = \lambda(\vec{a}\cdot\vec{c})$ ...(iii)

$$\therefore\ 8 - \frac{\lambda}{4} - \frac{\lambda^2}{2} - 2 = \lambda\left(\frac{1}{4}\right) \Rightarrow \lambda = -4 \text{ or } 3$$

71. (B)

Using LMVT, $\dfrac{\tan^{-1}\beta - \tan^{-1}\alpha}{\beta - \alpha} = \dfrac{1}{1+c^2}$

where, $0 < \alpha < c < \beta < \sqrt{3}$

So, $\dfrac{1}{4} < \dfrac{1}{1+c^2} < 1$

$$\Rightarrow \frac{1}{4} < \frac{\tan^{-1}\beta - \tan^{-1}\alpha}{\beta - \alpha} < 1 \Rightarrow \frac{1}{4} < \frac{\tan^{-1}\left(\frac{\beta-\alpha}{1+\alpha\beta}\right)}{\beta - \alpha} < 1 \Rightarrow 1 < \frac{\beta - \alpha}{\cot^{-1}\left(\frac{1+\alpha\beta}{\beta-\alpha}\right)} < 4$$

72. (C)

Given $\displaystyle\int_0^4 f(x)\,dx - \int_0^4 g(x)\,dx = 10$

$(A_1 + A_3 + A_4) - (A_2 + A_3 + A_4) = 10$

$A_1 - A_2 = 10$ ....(i)

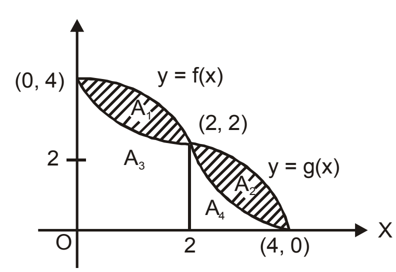
Again $\displaystyle\int_2^4 g(x)\,dx - \int_2^4 f(x)\,dx = 5$

$(A_2 + A_4) - A_4 = 5$ ....(ii)

$A_2 = 5$

$\therefore$ (i) + (ii)

$A_1 = 15$



73. (A)

$$I = \int_{\pi/4}^{\pi/2} 2\sin t\cos t\,dt + \underbrace{\int_{\pi/2}^{\pi} (-\sin t\cos t)+(\sin t\cos t)dt}_{\text{zero}} + \int_{\pi}^{5\pi/4} -2\sin t\cos t\,dt$$

$$= \int_{\pi/4}^{\pi/2} \sin 2t\,dt - \int_{\pi}^{5\pi/4} \sin 2t\,dt = 0$$

74. (A)

$$u = \int_0^{\pi/2} \cos\left(\frac{2\pi}{3}\sin^2 x\right)dx \Rightarrow u = \int_0^{\pi/2} \cos\left(\frac{2\pi}{3}\cos^2 x\right)dx$$

$$\Rightarrow 2u = \int_0^{\pi/2} \left(\cos\left(\frac{2\pi}{3}\sin^2 x\right) + \cos\left(\frac{2\pi}{3}\cdot\cos^2 x\right)\right)dx$$

$$\Rightarrow 2u = \int_0^{\pi/2} 2\cos\frac{\pi}{3}\cdot\cos\left(\frac{\pi}{3}\cos 2x\right)dx = \frac{1}{2}\int_0^{\pi} \cos\left(\frac{\pi}{3}\cos t\right)dt \qquad \text{[Put } 2x = t\text{]}$$

$$= \int_0^{\pi/2} \cos\left(\frac{\pi}{3}\cos t\right)dt = \int_0^{\pi/2} \cos\left(\frac{\pi}{3}\sin t\right)dt = v$$

75. (C)

$$A^2 = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix},\ A^3 = \begin{bmatrix} \cos 3\alpha & \sin 3\alpha \\ -\sin 3\alpha & \cos 3\alpha \end{bmatrix},\ A^4 = \begin{bmatrix} \cos 4\alpha & \sin 4\alpha \\ -\sin 4\alpha & \cos 4\alpha \end{bmatrix}$$

Now, $\cos\alpha + \cos 2\alpha + \cos 3\alpha + \cos 4\alpha = \cos\alpha + \cos 2\alpha + \cos(\pi - 2\alpha) + \cos(\pi - \alpha) = 0$

$\sin\alpha + \sin 2\alpha + \sin 3\alpha + \sin 4\alpha$

$= \sin\alpha + \sin 2\alpha + \sin(\pi - 2\alpha) + \sin(\pi - \alpha) = 2(\sin\alpha + \sin 2\alpha)$

$$= 4\sin\frac{3\alpha}{2}\cos\frac{\alpha}{2} = 4\sin\frac{3\pi}{10}\cos\frac{\pi}{10} = a$$

$$\therefore B = \begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$$

76. (B)

$g(x) = \lambda \sin x$

where $\lambda = 1 - 2\int_0^{\pi/2} (\cos t)g(t)dt$

$$\Rightarrow \lambda = 1 - 2\int_0^{\pi/2} \cos t.\lambda \sin t\, dt$$

$$\Rightarrow \lambda = 1 - \lambda\int_0^{\pi/2} \sin 2t\, dt \qquad \Rightarrow \lambda = 1 - 2\lambda\int_0^{\pi/4} \sin 2t\, dt$$

$$\Rightarrow \lambda = 1 + 2\lambda\left[\frac{\cos 2t}{2}\right]_0^{\pi/4} \Rightarrow \lambda = 1 + 2\lambda\cdot\left(-\frac{1}{2}\right) \quad \Rightarrow \quad 2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$g'(x) = \lambda \cos x \Rightarrow g'(\pi/4) = \frac{\lambda}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

77. (B)

$$I = \int \frac{dx}{x^{29}\left(1 - \frac{6}{x^7}\right)}$$

Let $\left(1 - \frac{6}{x^7}\right) = p \Rightarrow \frac{42}{x^8}dx = dp$ and $x^7 = \left(\frac{6}{1-p}\right)$

$$I = \frac{1}{42}\int \frac{(1-p)^3}{(6)^3 p}dp = \frac{1}{(42)(216)}\int \frac{1 - p^3 - 3p + 3p^2}{p}$$

$$= \frac{1}{54432}\ [\ln p^6 + 9p^2 - 2p^3 - 18p] + c$$

78. (A)

Equation of tangent at $P(x_1, f(x_1))$

$$y - f(x_1) = f'(x_1)(x - x_1) \quad \Rightarrow B \equiv (x_1, 0),\ A \equiv \left(x_1 - \frac{f(x_1)}{f'(x_1)}, 0\right)$$

According to equation, $2x_1 - \frac{f(x_1)}{f'(x_1)} = 4$

on generalising, $f'(x) = \frac{f(x)}{2(x-2)}$

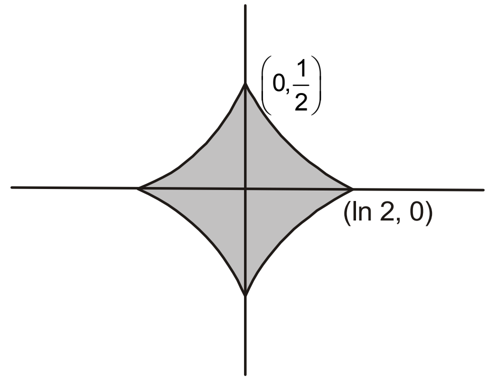
$$\frac{dy}{dx} = \frac{y}{2(x-2)} \quad \Rightarrow \int 2\frac{dy}{y} = \int \frac{dx}{x-2} \Rightarrow 2\ln y = \ln(x-2) + k \Rightarrow y^2 = c(x-2)$$

79. (D)

$|y| \le e^{-|x|} - \frac{1}{2}$

Graph is symmetric about x-axis and y-axis,

and $e^{-|x|} - \frac{1}{2} \ge 0$



$$\text{Area} = 4\int_0^{\ln 2}\left(e^{-x} - \frac{1}{2}\right)dx$$

$= 2 - 2\ln 2$

80. (A)

$$\left(\frac{dy}{dx} - e^{-x}\right)\left(\frac{dy}{dx} - e^{x}\right) = 0$$

$\Rightarrow \frac{dy}{dx} = e^{-x}$ or $\frac{dy}{dx} = e^{x}$

$\Rightarrow y = -e^{-x} + c$ or $y = e^{x} + c$

$\Rightarrow y + e^{-x} = c$ or $y = e^{x} + c$

81. (A)

Case –1 : Let $\frac{1}{2} \le x < 1$

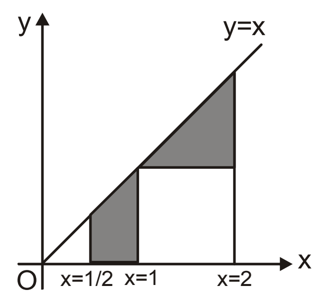
$\Rightarrow 0 < \log_y x \le 1$

$\Rightarrow 1 < x \le y$, if y > 1(not possible) and $1 > x \ge y$, if 0<y<1

Case –2 : Let $1 < x \le 2$

$\Rightarrow \log_y x \ge 1 \Rightarrow x \ge y$, if y > 1

and $x \le y$, if y < 1(not possible)



So, the possibilities are $x \ge y$, if $\frac{1}{2} \le x < 1$ and 0<y<1 and $x \ge y$, if $1 < x \le 2$ and y > 1

So, required area is 7/8.

82. (D)

Let $g(x)=f^{-1}(x)$; $f\left(\frac{\pi}{2}\right)=\pi \Rightarrow f^{-1}(\pi)=\frac{\pi}{2}$

$f'(x)=6(2x-\pi)^2+2+\sin x \Rightarrow f'\left(\frac{\pi}{2}\right)=3$

Also $g(\pi)=\frac{\pi}{2}$

Now f(g(x)) = x $\Rightarrow f'(g(x)).g'(x)=1$

$\Rightarrow f'(g(\pi)).g'(\pi)=1 \Rightarrow f'\left(\frac{\pi}{2}\right)\cdot g'(\pi)=1 \Rightarrow 3g'(\pi)=1 \Rightarrow \quad g'(\pi)=\frac{1}{3}$

83. (B)

(i) AB is symmetric $(AB)^T=B^TA^T=AB \Rightarrow BA=AB$

(II) $\left(B^TAB\right)^T=B^TA^T\left(B^T\right)^T=B^TA^TB=B^TAB$

(iii) and (iv)

Let A be skew symmetric, then $A^T=-A$

and $\left(A^n\right)^T=\left(A^T\right)^n, \forall n \in N$

$$\Rightarrow\left(A^n\right)^T=\begin{cases} A^n & \text{If n is even} \\ -A^n & \text{If n is odd}\end{cases}$$

Hence A^n is symmetric if n is even

Hence Answer is B.

84. (D)

Rearranging the given differential eqn.

$$xdx+\frac{ydx-xdy}{y^4}=0 \Rightarrow x^3dx+\frac{x^2}{y^2}.\frac{ydx-xdy}{y^2}=0 \Rightarrow x^3dx+\left(\frac{x}{y}\right)^2.d\left(\frac{x}{y}\right)=0$$

on integration, $\frac{x^4}{4}+\frac{1}{3}\left(\frac{x}{y}\right)^3=c$

85. (C)

$$x^2(4ydx + xdy) = \frac{xdy - 2ydx}{x^4 + y^2}$$

$$4x^2ydx + x^3dy = \frac{xdy - 2ydx}{x^4 + y^2}$$

$$4x^3ydx + x^4dy = \frac{x^2dy - 2xydx}{x^4 + y^2}$$

$$d(x^4y) = d\left(\tan^{-1}\frac{y}{x^2}\right)$$

$$\Rightarrow\ x^4y = \tan^{-1}\frac{y}{x^2} + c$$

86. (D)

Without loss of generality, we may assume that

$\vec{p} = \hat{i},\ \vec{q} = \hat{j},\ \hat{r} = \hat{k}$ and $\vec{a} = x\hat{i} + y\hat{j} + 2\hat{k}$

$\therefore\quad (\vec{a}\times\vec{p})^2 + (\vec{a}\times\vec{q})^2 + (\vec{a}\times\vec{r})^2 = (z\hat{j} - y\hat{k})^2 + (x\hat{k} - z\hat{i})^2 + (y\hat{i} - x\hat{j})$

$$= 2(x^2 + y^2 + z^2)$$

and $(\vec{a}\cdot\vec{p})^2 + (\vec{a}.\vec{q})^2 + (\vec{a}.\vec{r})^2 = (\vec{a}\cdot\hat{i})^2 + (\vec{a}.\hat{j})^2 + (\vec{a}.\hat{k})^2$

$$= x^2 + y^2 + z^2$$

$\therefore\quad \lambda = 2\cdot$

87. (C)

$$xy^2dx - y(x^2 - y^2)^2\,dx = y^3\,dy - x(x^2 - y^2)^2\,dy$$

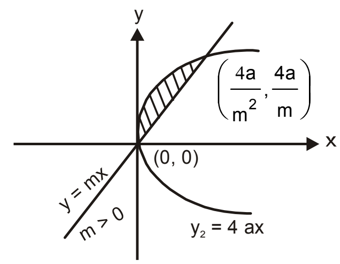
$$y^2(x\,dx - ydy) = (x^2 - y^2)^2(ydx - xdy)$$

$$\frac{d(x^2 - y^2)}{(x^2 - y^2)^2} = 2d\left(\frac{x}{y}\right)$$

$$\frac{2x}{y} + \frac{1}{x^2 - y^2} = c$$

88. **(B)**

Required area



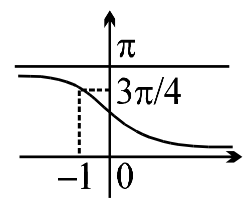
$$= \int_0^{4a/m^2} [y(\text{parabola})\,dx - y(\text{line})]\,dx$$

$$= \int_0^{4a/m^2} \left(2\sqrt{a}\sqrt{x} - mx\right) dx$$

$$= \frac{4}{3}\sqrt{a}\left(\frac{4a}{m^2}\right)^{3/2} - \frac{m}{2}\left(\frac{4a}{m^2}\right)^2$$

$$= \frac{32}{3}\frac{a^2}{m^3} - \frac{8a^2}{m^3} = \frac{8a^2}{3m^3}$$

89. **(B)**



$y = (x^2 - 1)^2 + 2 \Rightarrow \quad y_{min} = 2$

$\Rightarrow \log_{0.5}(x^4 - 2x^2 + 3) \le -1 \Rightarrow$ range $\left[\frac{3\pi}{4}, \pi\right)$

90. **(B)**

$$\lim_{x \to 0^-}\left[\frac{4f(x) - 12}{\tan(2f(x) - 6)}\right] = \lim_{x \to 0^-}\left[\frac{2(2f(x) - 6)}{\tan(2f(x) - 6)}\right] = 1$$