SOLUTIONS

PROGRESS TEST-6

CD-1802

(JEE ADVANCED PATTERN)

Test Date: 11-11-2017

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CHEMISTRY

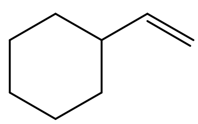
1. (D)

$$\text{Molar volume} = \frac{\text{Molar mass}}{\text{density}} = \frac{M}{\rho}$$

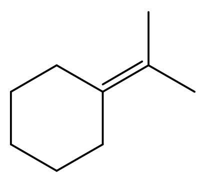
$$\because \quad \rho = \frac{Z.M.}{N_A \cdot a^3}$$

$$\therefore \quad \text{Molar volume} = \frac{N_A \cdot a^3}{Z} = \frac{6.022 \times 10^{23} \times (400 \times 10^{-10})^3}{4} = 9.64 \text{ ml}$$

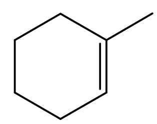
2. (C)



$\alpha_H = 1$

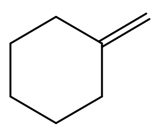


$\alpha_H = 10$



$\alpha_H = 7$

$\alpha_H = 4,$



3. (C)

II and III does not contain POS and COS.

4. (B)

Due to intramolecular H–bonding between two OH group present at C_1 and C_4 boat form be comes more stable.

5. (C)

(A) Complex is tetrahedral, hence no geometrical isomerism

(B) Complex is square planar, hence no optical isomerism

(C) Complex is having Fe(I) ion , contains unpaired electrons.

6. (D)

Ge

$\left.\begin{array}{l}\text{Sn}\\ \text{Pb}\end{array}\right\}$ (Exception) Lanthanide Contraction

$I.E_1$ = Ge > Pb > Sn

7. (C)

Cr^{2+} is reducing as its configuration changes from d^4 to d^3 the latter having a half filled t_2g level. On the other hand, the change from Mn^{3+} to Mn^{2+} results in d^5 configuration.

8. (A)

For B.C.C

$$4r = \sqrt{3}\,a \qquad \Rightarrow a = \frac{4r}{\sqrt{3}} = \frac{2\times 1.73}{\sqrt{3}} = 2\,\text{Å} = 200 \text{ pm}$$

9. (C,D)

$$\log_{10}^{k} = 5 - \frac{2000k}{T} \quad \text{............(i)}$$

also, $\log_{10}^{k} = \log_{10}^{A} - \frac{E_a}{2.303RT}$ ....(ii)

equating $\log_{10}^{A} = 5$

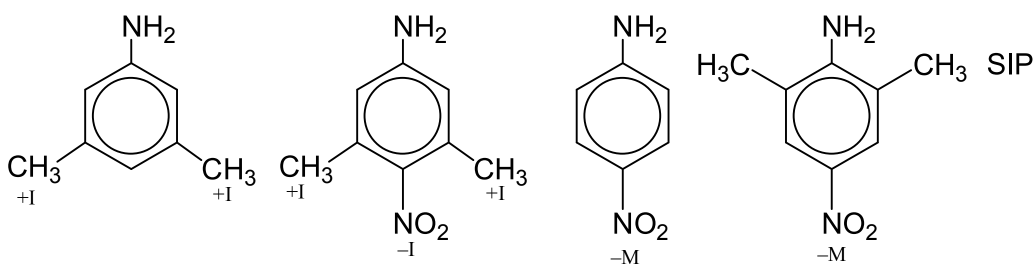
or, $A = 10^5$

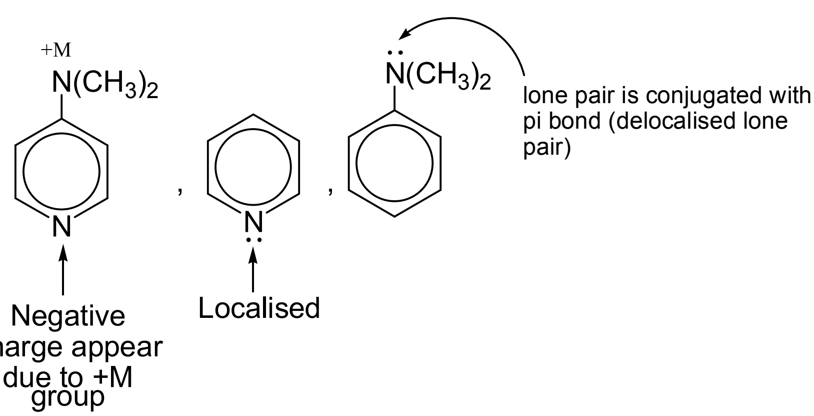
$$2000 = \frac{Ea}{2.303R}$$

or, $E_a = \frac{2000\times 2.30\times 3\times 2}{1000}\,k\,Cal$

$E_a = 9.212$

10. (A, C, D)

(A) 

(C) 

(D) $CH_2=\ddot{N}—CH_3$ (Localised) , $\underbrace{CH_2=CH}_{-M}—\ddot{N}H_2$, $CH_3—\underset{\uparrow\,-M}{\overset{O}{\overset{\|}{C}}}—\ddot{N}H_2$

$—\overset{O}{\overset{\|}{C}}—$ has stronger –M effect than $CH_2 = CH$ — group so in $CH_3—\overset{O}{\overset{\|}{C}}—\ddot{N}H_2$ l.p. of Nitrogen is more delocalised as compare to $CH_2 = CH — \ddot{N}H_2$

11. (B,C,D)

$t_{1/2}$ independent for first order reaction

pH = 3 or $[H^+] = 10^{-3}$

pH = 2 or $[H^+] = 10^{-2}$

increasing $[H^+]$ 10 times increases rate 100 times.

So, rate = $k[Zn]\,[H^+]^2$

12. (A, B, C, D)

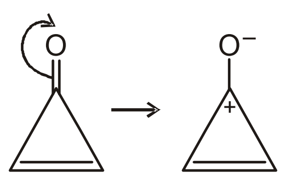
(A) $\underset{\text{Blue}}{4CrO_5} + 12H^+ \longrightarrow \underset{\text{green}}{4Cr^{+3}} + 7O_2 + 6H_2O$

(C) $Cr_2O_7^{-2} + 2H^+ + 4H_2O_2 \longrightarrow CrO_5 + 5H_2O$

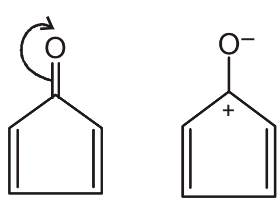
or $CrO_4^{-2} + 2H^+ + 2H_2O_2 \longrightarrow CrO_5 + 3H_2O$

(D) $I^- + 2MnO_4^- + H_2O \longrightarrow IO_3^- + 2MnO_2 + 2\bar{O}H$

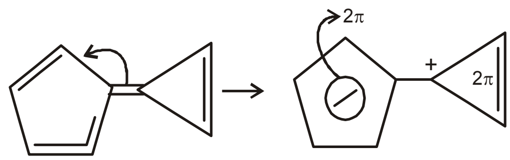
13. (3)



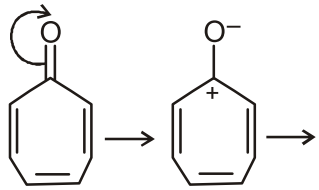
Aromaticion



⟶ Anti aromaticion



both rings are Aromatic



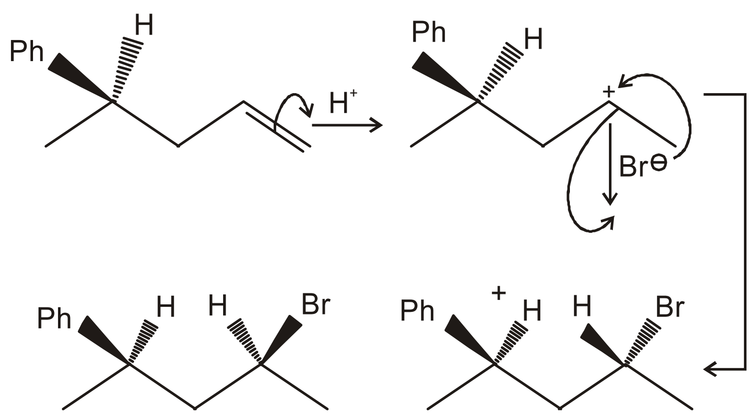
Aromaticicion

14. (3)

If substrate contain already onechiral centre, and due to electrophilic addition reaction.

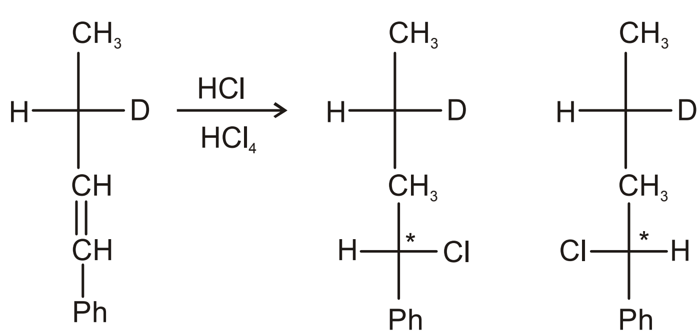
One new chiral centres is create about which two arrangement is possible diastereomers are obtained.

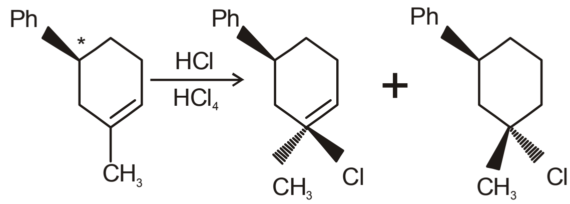
(B)



both are diastweames

(D)



(F) 

only about one centre conf is changed so both are diasteveamers

15. **(6)**

$4\,Fe\,Cr_2O_4 + 16\,NaOH + 7O_2 \rightarrow 8\,Na_2CrO_4 + 2Fe_2O_3 + 8H_2O$

Oxidation state of Cr : 6

16. **(8)**

Reduction half reaction :

$$MnO_4^{-1} + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$$

$$\frac{-d[MnO_4^-]}{dt} = -\frac{1}{8}\frac{d[H^+]}{dt}$$

So, $\frac{r_{H^+}}{r_{MnO_4^-}} = 8$

17. **(4)**

$$x \rightarrow y \qquad \Delta G^o = -193\,kJ$$

$$M^+ \rightarrow M^{+3} + x \qquad \Delta G^o = -2 \times 0.25F$$

$$\text{For } x \text{ mole } M^+ \qquad \Delta G^o = -x \times 0.5F$$

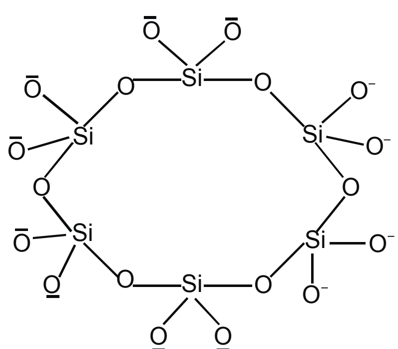
$$\text{Or } -x \times 0.5F = -193kJ$$

$$x = 4$$

18. **(6)**

General formula : $\left[SiO_3\right]_n^{-2n}$

n = 6, $Si_6O_{18}^{-12}$



19. **(A - q, s, t) ; (B - p, r, t) ; (C - p, t) ; (D - p, r, t)**

20. **(A - p, q) ; (B - p, r) ; (C - s) ; (D - p, s)**

PHYSICS

21. (A)

The given lens is a convex lens. Let the magnification be m, then for real image

$$\frac{1}{mx} + \frac{1}{x} = \frac{1}{f} \qquad \text{... (i)}$$

and for virtual image $\frac{1}{-my} + \frac{1}{y} = \frac{1}{f}$ ... (ii)

From Eq. (i) and Eq. (ii), we get

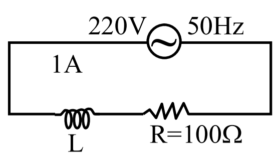
$$f = \frac{x+y}{2}$$

22. (B)

23. (D)

From the rating of the bulb, the resistance of the bulb can be calculated.

$$R = \frac{{V_{rms}}^2}{P} = 100\Omega$$



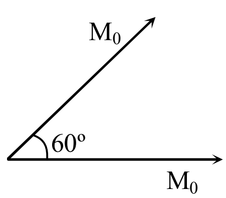
For the full to be operated at its rated value the rms current through it should be 1A

Also,

$$_{rms} = \frac{V_{rms}}{Z} \therefore 1 = \frac{200}{\sqrt{100^2 + (2\pi 50L)^2}} \Rightarrow L = \frac{\sqrt{3}}{\pi}\text{ H}$$

24. (D)

25. (C)



$\Rightarrow$ $M_{net} = \sqrt{3}\ M_0$

26. (B)

$$C_K = \frac{\varepsilon_0 A}{d - b + \frac{b}{K}}$$

We set b = 0

$C_K = \frac{\varepsilon_0 A}{d} = C$ if $C_K = 2C$

Then, $\frac{\varepsilon_0 A}{d - b + \frac{b}{K}} = \frac{2\varepsilon_0 A}{d} \Rightarrow K = \frac{2b}{2b - d}$

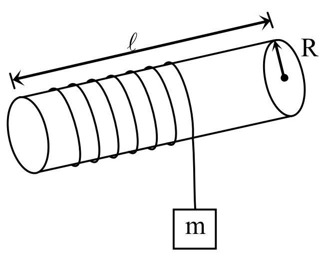
$\therefore$ $K > 0$ & $b \le d$

$\therefore K = \frac{2b}{2b - d}$ and $2b - d > 0$

$\therefore \frac{d}{2} < b \le d$ $\qquad\qquad$ $\therefore b > \frac{d}{2}$

27. (B)

As mass come down cylinder will rotate about it axis. Thus charge on cylinder also rotate due to which electric current is produced electrical current will depend on angular speed of cylinder.



$mg - T = ma$

$v = at$

$\omega = \frac{v}{R} = \frac{at}{R}$

ν = frequency of revolution $= \frac{\omega}{2\pi} = \frac{at}{2\pi R}$

Effective current $i = Q\nu = \frac{Qat}{2\pi R}$

Magnetic field due to i on the axis = $\mu_0 ni$

$$B = \frac{\mu_0 \times 1}{\ell} \frac{Qat}{2\pi R}$$

$$\frac{dB}{dt} = \frac{\mu_0 Q a}{2\pi R \ell}$$

Electric field due to time varying magnetic field

$$E = \frac{R}{2}\frac{dB}{dt} = \frac{\mu_0 Q a}{4\pi\ell}$$

Torque (τ) due to electric field = qER

$$= \frac{\mu_0 Q^2 Ra}{4\pi\ell}$$

Moment of inertia of cylinder is zero

$\therefore$ Net toque on it should be zero

$\therefore$ Torque due to tension of the string = Torque of electric field

$$TR = \frac{\mu_0 Q^2 Ra}{4\pi\ell}$$

$$T = \frac{\mu_0 Q^2 a}{4\pi\ell}$$

$$mg - T = ma$$

$$ma = mg - \frac{\mu_0 Q^2 a}{4\pi\ell}$$

$$a = \frac{g}{1 + \frac{\mu_0 Q^2}{4\pi m\ell}}$$

28. (C)

Since 2Q is the bigger charge hence electric field will be zero near the smaller charge. Hence graph (A) and (B) are wrong. On both the charges the electric field will be infinite. Hence (D) is wrong and correct answer is (C).

29. (A, B, C, D)

When light is incident normally on the boundary, then i = 0. So, according to Snell's law $\mu_1 \sin i = \mu_2 \sin r$, r is also zero, i.e., there is no refraction.

If $\mu_1 = \mu_2$, then boundary will not be visible and r = i, i.e., there will be no refraction.

If i < r and $i > \sin^{-1}(\mu_R/\mu_D)$ or $i > i_c$ (critical angle), then also there will be no refraction of light and light will be totally internally reflected

30. (C, D)

From fig. in question, $r + r' = 90^\circ$

$\therefore r' = 90^\circ - r = 90^\circ - i$

When light travels from denser medium to rarer medium,

$$\frac{1}{\mu} = \frac{\sin i}{\sin r'} = \frac{\sin i}{\sin(90^\circ - i)} = \frac{\sin i}{\cos i}$$

$= \tan i$

Also, $\sin i_c = \frac{1}{\mu}$

(if angle of incidence = critical angle)

$\therefore \sin i_c = \tan i = \tan r$

$\therefore i_c = \sin^{-1}(\tan i)$

$= \sin^{-1}(\tan r)$

31. (A,C,D)

$V_1 = V_2 \Rightarrow X_L = X_C \quad \Rightarrow f = \frac{1}{2\pi\sqrt{LC}} = 125\text{Hz}$

$I_0 = \frac{V_0}{R} = \frac{200}{100} \qquad \because X = 0 \therefore Z = R$

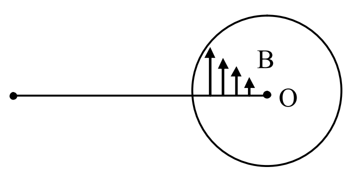
$I_0 = 2\text{A}$

$V_1 = V_2 = I = I\omega L = 2 \times 2\pi \times 125 \times \frac{2}{\pi}$

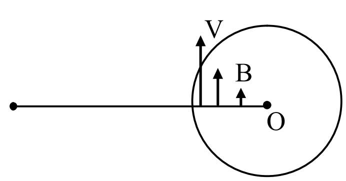
$= 1000$ volt

32. (B,C,D)

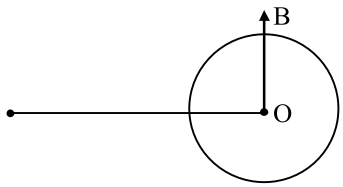
Top view



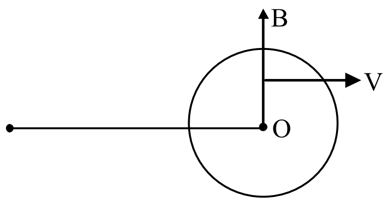
Flux is maximum



B & V are parallel so no induced emf



Flux is zero



Angle between B & V is 90°. So max. induced emf

33. (1)

$\therefore$ Net magnetic field at centre is zero

$\therefore$ $B_1 = B_2$

$\therefore$ $\frac{B_1}{B_2} = 1$]

34. (3)

For image formed by lens

$$\frac{1}{v_1} - \frac{1}{-15} = \frac{1}{+10}$$

$\Rightarrow v_1 = +30$ cm

i.e. 20 cm behind mirror

For mirror

$$\frac{1}{v_2} + \frac{1}{20} = \frac{1}{-20}$$

$\Rightarrow v_2 = -10$ cm

Overall magnification = $\left(\frac{30}{-15}\right) \times \left(\frac{10}{20}\right) = -1$

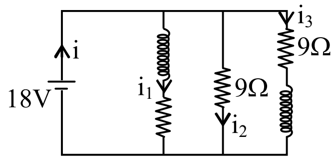
Length of image = 1 × 3 = 3 mm

35. (3)

Apply $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\mu_2 = 1$, $\mu_1 = \frac{3}{2}$, R = – 10 cm, u = – 4 cm

36. (2)



Just after closing of switch S

i_1 and i_3

(current through inductor is zero)

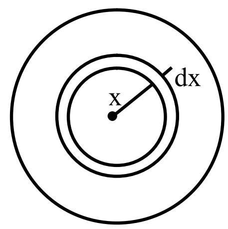
$\therefore$ $i = i_2$

$i_2 = \frac{18}{9} = 2$ amp

37. (3)

Consider a ring of thickness dx

Torque on this ring = QE × x



$E \times 2\pi x = \pi x^2 \times \frac{dB}{dt}$

$E = \frac{x}{2} \times 2Kxt - Kx^2t$

charge on ring = $\frac{Q}{\pi R^2} \times 2\pi x dx$

Torque on ring = $\frac{2Q}{R^2} x \times K x^2 t \times x dx = \frac{2KQ}{R^2} x^4 t\, dx$

Total torque = $\int_0^R \frac{2KQ}{R^2} x^4 t\, dx = \left[\frac{2KQtx^5}{R^2 \times 5}\right]_0^R$

$= \frac{2KQR^3t}{5} = 3$ N- m

38. (2)

As in case of discharging of a capacitor through a resistance

$q = q_0\, e^{-t/CR}$

$i = -\frac{dq}{dt} = \frac{q_0}{CR} e^{-t/CR}$

Here, $CR = \left(\frac{\in_0 KA}{d}\right)\left(\rho \frac{d}{A}\right) = \frac{\in_0 K}{\sigma}$ [as $\rho = 1/\sigma$]

i.e., CR = $\frac{8.846\times10^{-12}\times5}{7.4\times10^{-12}}$ = 6

So, i = $\frac{8.85\times10^{-6}}{6}$ $e^{-12/6}$

= $\frac{8.85\times10^{-6}}{6\times7.39}$ [As e = 2.718, e^2 = 7.39]

= 0.20 μ A

39. (**A – r) ; (B – q) ; (C – p) ; (D – p)**

(A) Velocity of fish in air = 8 × $\frac{3}{4}$ = 6 $\uparrow$

Velocity of fish w.r.t. bird = 6 + 6 = 12 $\uparrow$

(B) Velocity of image of fish after reflection

from mirror in air = $8\times\frac{3}{4}$ = 6 $\downarrow$

w.r.t. bird = – 6 + 6 = 0

(C) Velocity of bird as seen from water = 6 × $\frac{4}{3}$

= 8 $\downarrow$ w.r.t. fish = 8 + 8 = 16 $\downarrow$

(D) Velocity of bird in water after reflection

from mirror = 8 $\downarrow$

w.r.t. fish = 8 – 8 = 0

40. (A - q, r) ; (B - q, r) ; (C - q, r) ; (D - q, s)

For A, B and C : Magnetic field at location of 1 due to 2 is parallel or anti parallel to current in 1, so force experienced by 1 due to magnetic field of 2 is zero.

For D : Force experienced by upper half of 2 due to 1 is along left, while on lower half it is towards right.

So, net force of interaction between the two is zero.

Direction of magnetic field at P can be found by using right hand palm rule no. 1.

MATHEMATICS

41. (C)

$\vec{a}\cdot\vec{b}<0$

$\Rightarrow\ cx^2-12+6cx<0,\ \forall x\in R$

$\Rightarrow\ c<0$ and $36c^2+48c<0$

$\frac{-4}{3}<c<0$

42. (C)

Clearly k > 0 ; $kx^2 = e^x$

$f'(x)=g'(x)\,;e^x=2kx=kx^2$

x = 2

43. (B)

(i) AB is symmetric $(AB)^T=B^TA^T=AB\Rightarrow BA=AB$

(II) $\left(B^TAB\right)^T=B^TA^T\left(B^T\right)^T=B^TA^TB=B^TAB$

(iii) and (iv)

Let A be skew symmetric, then $A^T=-A$

and $\left(A^n\right)^T=\left(A^T\right)^n,\forall n\in N$

$$\Rightarrow\left(A^n\right)^T=\begin{cases}A^n & \text{If n is even}\\ -A^n & \text{If n is odd}\end{cases}$$

Hence A^n is symmetric if n is even

Hence Answer is B.

44. (B)

A is orthogonal $AA^T=I_3$

$\det\left(AA^T\right)=\det\left(I_3\right)$

$\det A\det A^T=1$

$(\det A)^2=1,\ (\det A)=\pm1$

$B = 5A^5$, $(\det B) = 5^3(\det A)^5$

$= 125(\pm 1)^5 = \pm 125$

45. (A)

$$f(x) = \sqrt{1+x\sqrt{1+(x+1)(x+3)}} = \sqrt{1+x(x+2)} = (x+1) \quad \therefore f'(x) = 1$$

46. (B)

$$I = \int_0^1 (1-x^4)^7\,dx = \left[x(1-x^4)^7\right]_0^1 + 7\times 4\int_0^1 x(1-x^4)^6.x^3dx$$

$$I = -28\int_0^1 (1-x^4)^7 + 28\int_0^1 (1-x^4)^6\,dx$$

$$29\,I = 28\int_0^1 (1-x^4)^6 dx$$

$$\frac{\int_0^1 (1-x^4)^7}{\int_0^1 (1-x^4)^6\,dx} = \frac{28}{29}$$

47. (A)

Domain of f is [–1, 1]

$$f'(x) = \cos x - \sin x + \sec^2 x + \frac{1}{1+x^2} > 0$$

$$(f(x))^{\min} = f(-1) = -\sin 1 + \cos 1 - \tan 1 - \frac{\pi}{2} + \pi - \frac{\pi}{4} = m$$

$$(f(x))^{\max} = f(1) = \sin 1 + \cos 1 + \tan 1 + \frac{\pi}{2} + \frac{\pi}{4} = M$$

48. (C)

$$\lim_{n\to\infty} \cos(2\pi\sqrt{n^2+1}) = \lim_{n\to\infty} \cos(2\pi\sqrt{n^2+1} - 2n\pi)$$

$$= \lim_{n\to\infty} \cos\left(\frac{2\pi}{\sqrt{n^2+1}+n}\right) = 1$$

49. (A,B,C)

(A) $\vec{r} = (\vec{r}\cdot\hat{i})\hat{i} + (\vec{r}\cdot\hat{j})\hat{j} + (\vec{r}\cdot\hat{k})\hat{k}$ put $\vec{r} = (\vec{a}\times\vec{b})$ $\Rightarrow$ **(A) is correct**

(B) put $\vec{r} = \vec{a}$ and $\vec{r} = \vec{b}$ respectively and take $\vec{a}\cdot\vec{b}$ $\Rightarrow$ **(B) is correct**

(C) $\vec{u} = \hat{b}\times(\hat{a}\times\hat{b})$ $|\vec{u}| = |\vec{a}\times\vec{b}| = |\vec{v}|$ $\Rightarrow$ **(C) is correct**

(D) is obviously wrong.

50. (A,B,C,D)

(A) $\sin\left(\frac{11\pi}{12}\right)\cdot\sin\left(\frac{5\pi}{12}\right)=\sin\left(\frac{\pi}{12}\right)\cdot\cos\left(\frac{\pi}{12}\right)=\frac{1}{2}\sin\left(\frac{\pi}{6}\right)=\frac{1}{4}\in Q$

(B)

$$\operatorname{cosec}\left(\frac{9\pi}{10}\right)\cdot\sec\left(\frac{4\pi}{5}\right)=-\operatorname{cosec}\left(\frac{\pi}{10}\right)\cdot\sec\left(\frac{\pi}{5}\right)=\frac{1}{\sin 18^\circ\cdot\cos 36^\circ}=\frac{-16}{(\sqrt{5}-1)(\sqrt{5}+1)}=-4\in Q$$

(C) $\sin^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{\pi}{8}\right)=1-\frac{1}{2}\sin^2\left(\frac{\pi}{4}\right)=1-\frac{1}{4}=\frac{3}{4}\in Q$

(D) $2\cos^2\frac{\pi}{9}\cdot 2\cos^2\frac{2\pi}{9}\cdot 2\cos^2\frac{4\pi}{9}=8(\cos 20^\circ\cdot\cos 40^\circ\cdot\cos 80^\circ)^2 = \frac{1}{8}\in Q$

51. (A, B, C, D)

$f'(x) > 0$ in [-1, 2] $\Rightarrow$ f (x) increases in [-1,2]

$f(2^-) = f(2) = f(2^+) = 35 \Rightarrow f(x)$ is continuous

$f'(2^-) = 24$, $f'(2^+) = -1 \Rightarrow f'(2)$ does not exist

f(2) = 35 is the maximum value.

52. (A, C)

We have $AB = A$

$\Rightarrow A(BA) = A^2$ $[\because BA = B]$

$\Rightarrow A(B) = A^2$

$\Rightarrow AB = A^2$ $[\because AB = A]$

$\Rightarrow A = A^2$

Again

$\Rightarrow BA = B$

$\Rightarrow B(AB) = B^2$

$\Rightarrow B(A) = B^2$

$B = B^2$

53. (4)

$\int_{-3}^{b} -(x+3)(x-1)^2(x-4)\,dx$ is maximum when f(x) = – (x + 3) (x – 1)^2 (x – 4) is above x-axis

$\Rightarrow x \in [-3,4]$ so b = 4

54. (2)

$(\vec{a}+x\vec{b})\cdot(\vec{a}-\vec{b})=0 \;\Rightarrow |\vec{a}|^2 - x|\vec{b}|^2 + (x-1)(\vec{a}\cdot\vec{b})=0$

$\Rightarrow |\vec{a}|^2 - 4x|\vec{a}|^2 + (x-1)\left(2|\vec{a}|^2\right)\cos\frac{2\pi}{3}=0 \;\Rightarrow 1-4x-(x-1)=0 \Rightarrow x=\frac{2}{5} \quad \Rightarrow 5x=2$

55. (5)

$A\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}$ ....(1) ; $A^2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$ ....(2)

Let $A=\begin{bmatrix}a & b\\c & d\end{bmatrix}$.

The first equation gives

a – b = – 1 ....(3) and c – d = 2 ....(4)

For second equation, $A^2\begin{bmatrix}1\\-1\end{bmatrix} = A\left(A\begin{bmatrix}1\\-1\end{bmatrix}\right) = A\left(\begin{bmatrix}-1\\2\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$.

This gives – a + 2b = 1 ....(5) and – c + 2d = 0 ....(6)

(3) + (5) $\Rightarrow$ b = 0 and a = – 1

(4) + (6) $\Rightarrow$ d = 2 and c = 4

so the sum a + b + c + d = 5

56. (1)

$1+\frac{dy}{dx}-\frac{1}{(x+y)}\left(1+\frac{dy}{dx}\right)=2 \;\Rightarrow \left.\frac{dy}{dx}\right|_{\alpha,\beta}=\frac{\alpha+\beta+1}{\alpha+\beta-1} \;\Rightarrow \alpha+\beta=1$

57. (3)

Let $L=\lim_{x\to 0}\frac{(\cos x-1)\left(\cos x-e^x\right)}{x^k}$

$=-\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)\left(\cos x-e^x\right)}{(1+\cos x)x^k}=-\lim_{x\to 0}\frac{\left(\frac{\sin x}{x}\right)^2\left(\frac{1-\cos x}{x}+\frac{e^x-1}{x}\right)}{x^{k-3}}\cdot\frac{1}{1+\cos x}$

For L to be finite non-zero, k = 3.

58. **(1)**

$$AB=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}a+2c & b+2d\\ 3a+4c & 3b+4d\end{bmatrix};\ BA=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}=\begin{bmatrix}a+3b & 2a+4b\\ c+3d & 2c+4d\end{bmatrix}$$

If AB = BA, then a + 2c = a + 3b $\Rightarrow 2c=3b\Rightarrow b\neq 0$

$$b+2d=2a+4b\ \Rightarrow 2d-2a=3b\ \therefore\ \frac{d-a}{3b-c}=\frac{\frac{3}{2}b}{3b-\frac{3}{2}b}=1$$

59. **A $\to$(r); B $\to$(r,s,t); C $\to$(q) ; D $\to$(q)**

(A) $$\int_{\sqrt{2}-1}^{\sqrt{2}+1}\frac{(x^2+1)^2-(x^2-1)}{(x^2+1)^2}dx=\int_{\sqrt{2}-1}^{\sqrt{2}+1}\left(1-\frac{x^2-1}{(x^2+1)^2}\right)dx$$

$$2-\int_{\sqrt{2}-1}^{\sqrt{2}+1}\frac{x^2-1}{(x^2+1)^2}dx$$

Now $\mathrm{I}_1=\int_{1/a}^{a}\frac{x^2-1}{(x^2+1)^2}dx$ where $a=\sqrt{2}+1$ Putting $x=\frac{1}{t};dx=-\frac{1}{t^2}dt$

$$\int_{a}^{1/a}\frac{\frac{1}{t}-1}{\left(\frac{1}{t^2}+1\right)^2}\left(-\frac{1}{t^2}\right)dt=-\int_{a}^{1/a}\frac{1-t^2}{(t^2+1)^2}dt\ =-\int_{1/a}^{a}\frac{t^2-1}{(t^2+1)^2}dt=-\mathrm{I}_1 \qquad \therefore \mathrm{I}_1=0$$

(B) $\ell nf(x)=1\Rightarrow f(x)=e$ .... constant function and $D_f=(0,1)\cup(1,\infty)$

(C) $f'(x)=\frac{2^x}{x}+2^x(\ell n2)(\ell nx)$; $g'(x)=x^{2x}\left(2x\times\frac{1}{x}+2\ell nx\right)$

Common point is (1, 0)

(D) $3y^2\frac{dy}{dx}-3y-3x\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=\frac{y}{y^2-x}$

$$\frac{dy}{dx}=0\Rightarrow y=0\Rightarrow \text{no real x.}$$

$$\frac{dx}{dy}=0\Rightarrow y^2=x\Rightarrow y^3=1\Rightarrow y=1$$

60. (A)$\rightarrow$(r) ; (B)$\rightarrow$(t) ; (C)$\rightarrow$(q) ; (D)$\rightarrow$(t)

(A) $f(x)=\tan\frac{3\pi}{7}[x]$

$f(x+T)=\tan\frac{3\pi}{7}[x+1]$

$\frac{3\pi}{7}T=\pi \Rightarrow \; T=\frac{7}{3}$

(B) $x^2+5x=t$

$(t+4)(t+6)+2\geq 1$

(C) $-1\leq \sin x\leq 1$

$\frac{\pi}{4}\leq 2^{\sin x}\frac{\pi}{2}\leq \pi$

(D) $e^x = x^2$

$x = 2\ell n|x|$