SOLUTIONS

PROGRESS TEST-5

RB-1806 TO 1809

RBK-1804

(JEE ADVANCED PATTERN)

Test Date: 11-11-2017

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Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd., Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

CHEMISTRY

1. (B)

PV = nRT ....(1)

P = 3170 Pa, V = 1 dm^3 = 10^{-3} m^3,

R = 8.314 JK^{-1} mol^{-1}, T = 300 K

$\therefore$ From eqn. ....(1)

$$3170 \times 10^{-3} = n \times 8.314 \times 300$$

$$n = 1.27 \times 10^{-3}\ \text{mol}$$

2. (D)

3. (B)

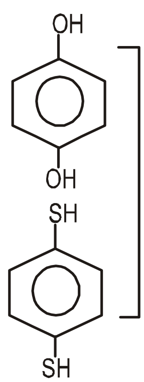
4. (C)

$2^3 = 8$.

5. (B)

$$\underset{I}{K_2CO_3} > \underset{III}{CaCO_3} > \underset{II}{MgCO_3} > \underset{IV}{BeCO_3}$$

6. (D)



$\mu \neq 0$ (exception)

7. (C)

If 2s – 2p mixing is not considered for Li_2 to N_2

$$\sigma_{1s}^2 \sigma_{1s}^{*2} \sigma_{2s}^2 \sigma_{2s}^{*2} \sigma_{2p_z}^2 \left(\pi_{2p_x}^1 = \pi_{2p_y}^1\right)$$

$$\left(\pi_{2p_x}^* = \pi_{2p_y}^*\right) \sigma 2p_z$$

$C_2 \rightarrow$ total e$^-$ = 12

8. (A)

$q_{cycle} + w_{cycle} = 0 \Rightarrow w_{cycle} = -5\,J$

$w_{AB} + w_{BC} + w_{CA} = -5$

$-10(2-1) + 0 + w_{CA} = -5$

$w_{CA} = +5\,J$

9. (A, B, C)

Cooling effect is observed when the real gas undergoes adiabatic expansion below the inversion tempeature.

10. (B), (C)

11. (A, C, D)

Equivalent weight = $\dfrac{\text{Molecular weight}}{n-\text{factor}}$

(A) n–factor of $H_2PO_3^-$ as acid = 1

as base = 1

In both case, eq. wt. of $H_2PO_3 = \dfrac{81}{1} \Rightarrow 81$

(B) n–factor of $H_2PO_4^-$ as acid = 1 or 2

as base = 1

(C) In acidic medium

Eq. wt. of $KMnO_4$ = $\dfrac{\text{Molecular weight}}{5}$

In basic medium

Eq. wt. of $KMnO_4 = \dfrac{\text{Molecular weight}}{1}$

In neutral/slightly basic medium

Eq. wt. of $KMnO_4 = \dfrac{\text{Molecular weight}}{3}$

(D) In MgH_2, O.S. of $H = -1$

In H_2O_2, O.S. of $H = +1$

12. (A,B,C)

(A) When (Na) sedium react with excess ammonia give a blue colour paramagnetic solution.

(B) $K + O_2 \longrightarrow KO_2$

$\downarrow$

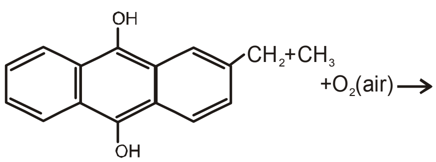
$K^+ + O_2^-$

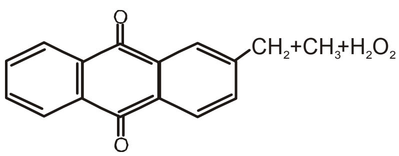
Paramagnetic

(C) $3Cu + 8HNO_3 \rightarrow 3Cu(NO_3)_2 + 2NO + 4H_2O$

(Dilute)

Where NO is paramagnetic

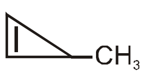
(D) 

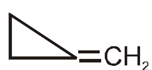


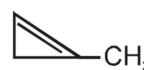
(2 - Ethyle anthraquinol)

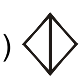
Where H_2O_2 is Diamagnetic

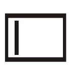
13. (9)

(1) 

(2) 

(3) 

(4) 

(5) 

(6) $CH_3–CH = C = CH_2$

(7) $CH_2 = CH – CH = CH_2$

(8) $CH_3C \equiv C–CH_3$

(9) $CH \equiv C–CH_2–CH_3$

14. (4)

(1) $CH_3—CH_2—CH_2—CH_2—CH_2—OH$

(2) $CH_3 — \underset{}{\overset{CH_3}{\overset{|}{C}H}} — CH_2 — CH_2 — OH$

(3) $CH_3 — CH_2 — \overset{CH_3}{\overset{|}{C}H} — CH_2 — OH$

(4) $CH_3 — \underset{\underset{CH_3}{|}}{\overset{CH_3}{\overset{|}{C}}} — CH_2 — OH$

15. (2)

$$\left.\begin{array}{l}[Cr(H_2O)_6]Cl_3\\ [Cr(H_2O)_4Cl_2]Cl.2H_2O\\ [Cr(H_2O)_3Cl_3].3H_2O\\ [Cr(H_2O)_4Cl_2].2H_2O\end{array}\right\}\text{are not affected by conc.}H_2SO_4$$

16. (3)

$$\Delta H = nC\Delta T$$

$$= 8 \times \frac{3}{2} \times 8.31 \times 30 = 3000\ J = 3\ kJ$$

17. (4)

$$\underset{0.220g}{M_2O_x}\ (\text{Metallic oxide}) + H_2 \longrightarrow M(s) + \underset{0.045g}{H_2O(\ell)}$$

$\Rightarrow$ POAC on O

$$\Rightarrow \left(\frac{0.220}{2M + 16x}\right)x = \left(\frac{0.045}{18} \times 1\right)$$

$$\Rightarrow 3.96x = 0.09\,M + 0.72\,x$$

$$\Rightarrow E = \frac{M}{x} = 36$$

$$\frac{E}{9} = 4$$

18. (5)

$\ddot{N}H_2 - \ddot{N}H_2$ is a monodontate ligands due to formation of unstable chelation and all other compounds are chelating ligands.

19. (A - q, s) ; (B - r) ; (C - p, t) ; (D - t)

20. (A – r) ; (B – s) ; (C – p) ; (D – q)

PHYSICS

21. Applying Snell's law between the points *O* and *P*, we have

$$2\times \sin 60^0 = (\sin 90^0)\times \frac{2}{(1+H^2)}, \qquad 2\times\frac{\sqrt{3}}{2} = 1\times\frac{2}{(1+H^2)}$$

$$(1+H^2) = \frac{2}{\sqrt{3}}, \qquad H = \sqrt{\left(\frac{2}{\sqrt{3}}-1\right)}$$

(A)

22. Apparent position of the object w.r.t lens.

$$u = \left(\frac{10}{1}+\frac{10}{2}\right) = 15\,\text{cm}$$

$$\frac{1}{15}+\frac{1}{v} = \frac{2(1.5-1)}{R}$$

v = 7.5 cm

∴ **(A)**

23. **(B)**

Take the mass m as a point mass. At the instant when the pendulum collides with the nail, m has a velocity v = $\sqrt{2g\ell}$. The angular momentum of the mass with respect to the point at which the nail locates is conserved during the collision. Then the velocity of the mass is still **v** at the instant after the collision and the motion thereafter is such that the mass is constrained to rotate around the nail. Under the critical condition that the mass can just swing completely round in a circle, the gravitational force when the mass is at the top of the circle. Let the velocity of the mass at this instant be $\mathbf{v_1}$, and we have

$$\frac{mv_1^2}{\ell - d} = mg,$$

or $v_1^2 = (\ell - d)g$

The energy equation

$$\frac{mv^2}{2} = \frac{mv_1^2}{2} + 2mg(\ell - d),$$

or $2g\ell = (\ell - d)g + 4(\ell - d)g$

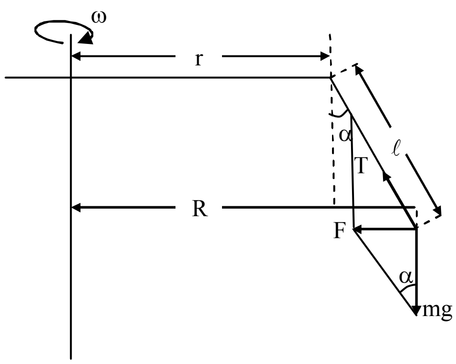
then gives the minimum distance as

$$d = \frac{3\ell}{5}$$

24. (A)

The plumb-line is so set up that the resultant of its weight mg and the tension in the thread T produces a centripetal force F = $m\omega^2R$ (fig.). Clearly $R = r + \ell \sin \alpha$. Therefore,

$$\omega^2 = \frac{g \tan \alpha}{r + \ell \sin \alpha}, \ \omega = \sqrt{\frac{g \tan \alpha}{r + \ell \sin \alpha}}.$$



25 As $F_1 - F_2 < 2\mu Mg$, so system will not accelerate. Again here $F_1 > F_2$, so block A is the driving block and block *B* is driven block. So friction on block *A* acts towards left but in the block B it may act left or right.

$\therefore$ **(B)**

26. (A)

27. The equivalent capacitance

$$C = \frac{4\pi\varepsilon_0(3a \times 4a)}{(4a - 3a)} + 4\pi\varepsilon_0(4a)$$

$$= 64\pi\varepsilon_0 a$$

$\therefore$ **(A)**

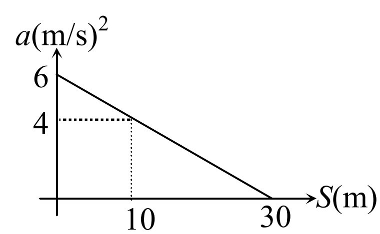
28. Work done to rotate the ring is equal to work done to return the charge at its initial position.

$\therefore$ **(B)**

29. (C,D)

30. (B,D)

31. Area $= \frac{1}{2} \times 10 \times (6 + 4) = \frac{v^2}{2}$



v = 10 m/s

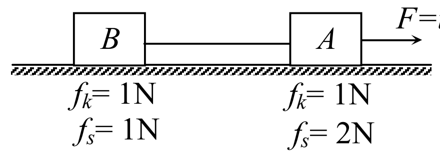
Area upto 30 m $= \frac{1}{2} \times 30 \times 6 = \frac{v^2}{2}$

$v^2 = 180$

$v_{max} = \sqrt{180} < 14$

$\therefore$ **(B) and (C)**

32. When t = 3s block just about to move and acceleration of block given by $a=\frac{t-2}{1}$ $t > 3$



$$\int_0^v dv=\int_3^{10}(t-2)dt$$

$$v=\frac{t^2}{2}-2t\Big|_3^{10}=(50-20)-\left(\frac{9}{2}-6\right)$$

$=30+1.5=31.5$ m/s

$\therefore$ **(A) (B) and (C)**

33. (5)

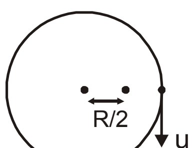
If C doesn't move then $a_A = 4a_B$ … (i)

$P - T - \mu mg = m\,4a_B$ …(ii)

$4T - mg = ma_B$ …(iii)

$\therefore \frac{p}{mg} = 5$

34. (8)



$U=-\int \vec{F}.\vec{dr}$

$U=-\frac{km}{r}$

$K_i + U_i = K_f + U_f$

$$\frac{1}{2}mv^2-\frac{Km}{\left(\frac{R}{2}\right)}=0-\frac{Km}{3R/2}$$

$$\frac{mv^2}{2}=\frac{2Km}{R}-\frac{-2Km}{3R}$$

$$\frac{mv^2}{2}=\frac{4Km}{3R}$$

$V=\sqrt{\frac{8K}{3R}}$,V = 8 m/s

35. (6)

$F_E = qE = 11$ N

$F_g = mg = 5N$

So Net force = F = 6N upward

$g_{eff} = \frac{F}{m} = \frac{6}{0.5}$ 12 m/s^2

so $V_{min} = \sqrt{5g_{eff}\ell} = \sqrt{5 \times 12 \times (60 \times 10^{-2})}$

so V_{min} = 6 m/sec

36. (2)

$|\Phi| = \frac{q}{\varepsilon_0}\left(1 - \frac{1}{\sqrt{1+(R/l)^2}}\right)$. The sign of Φ depends on how the direction of the normal to the circle is chosen.

37. (4)

$qV_a = qV_b + \frac{1}{2}mv^2$

$2.0 \times 10^{-9} \times 9 \times 10^9\left[\frac{3\times10^{-9}}{1} - \frac{3\times10^{-9}}{2}\right] \times 100$

$= 2.0 \times 10^{-9} \times 9 \times 10^9\left[-\frac{3\times10^{-9}}{1} + \frac{3\times10^{-9}}{2}\right] \times 100$

$+ \frac{1}{2} \times 5.0 \times 10^{-9}\, v^2$

$10^{-9} \times 18\left[\frac{3}{2}\right] \times 100 = 18 \times 10^{-9} \times 100\left[-\frac{3}{2}\right]$

$+ \frac{1}{2} \times 5.0 \times 10^{-9} v^2$

$1800\left[\frac{3}{2} + \frac{3}{2}\right] = \frac{1}{2} \times 5.0 \times v^2$

$\frac{1800 \times 6}{5} = v^2$

$360 \times 6 = v^2$

$6 \times 6 \times 10 \times 6 = v^2$

$12\sqrt{15} = v$

38. (3)

$$\frac{6}{u} = 1 + \frac{(v+u)\times 1 - 6}{(v-u)}$$

$$\frac{6}{u} = \frac{(v-u)+(v+u)-6}{(v-u)}$$

$$\frac{6}{u} = \frac{2(v-6)}{(v-u)}$$

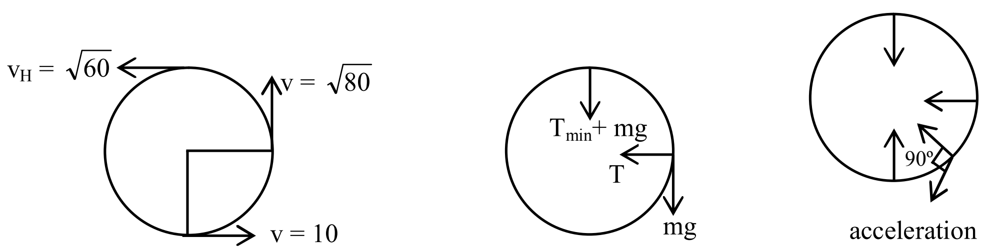
$$\Rightarrow \quad 6(v-u) = 2vu - 6u$$

$$\Rightarrow \quad 6v - 6u = 2vu - 64$$

u = 3 km/h

39. A → p ; B → r ; C → q ; D → s

40. A → p ; B → s ; C → q ; D → r



(A) T_{min} + mg = $\frac{mv_H^2}{R}$ at highest point

(B) a = $\sqrt{\left(\frac{v^2}{R}\right)^2 + g^2}$ = 10 $\sqrt{65}$

(C) a_{min} = $\frac{v_H^2}{R}$ = 60

(D) a_t = 0 at highest point

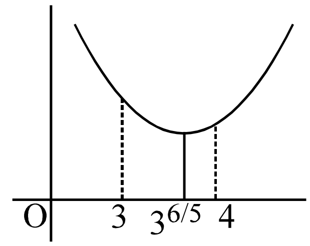
MATHEMATICS

41. (D)

Let $\log_3 n = x$

$y = 5x^2 - 12x + 9$

y is minimum at $x = -\frac{b}{2a} = \frac{12}{10} = \frac{6}{5}$



Here $\log_3 n = \frac{6}{5} \Rightarrow n = 3^{6/5} \cong 3.70$

which is not natural hence minimum occurs at the closest integer

now $4 > 3^{6/5}$

$4^5 > 3^6$

$1024 > 729$ which is true

42. (B)

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{f(h)+|x|h+x\cdot h^2}{h}$$

Also x = y = 0 $\Rightarrow f(0) = 0$

$$\therefore f'(x) = \lim_{h\to 0}\left(\frac{f(h)-f(0)}{h}+|x|+xh\right)$$

$$\Rightarrow f'(x) = f'(0)+|x|$$

43. (A)

$$f(x) = \sqrt{1+x\sqrt{1+(x+1)(x+3)}} = \sqrt{1+x(x+2)} = (x+1) \quad \therefore f'(x) = 1$$

44. (A)

$$\sin^2 x\cos^2 x - \cos^2 x\sin^4 x = 1$$

$$\Rightarrow \sin^2 x\cos^2 x\left(1-\sin^2 x\right) = 1$$

$\Rightarrow \sin^2 x\cos^4 x = 1$, No value of 'x'

45. (C)

For $x = \alpha \in Q^c$, $f(\alpha) = 0$ and $\lim_{x\to\alpha} f(x)$ is also zero because if x moves towards α and attaining irrational value then it is zero, or if attaining rational value then the denominator can be made as large as possible.

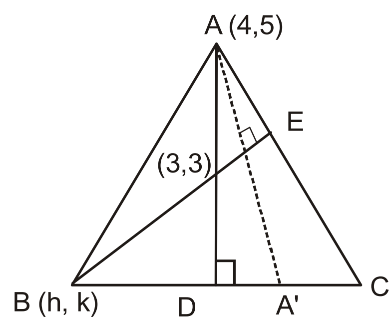
46. **(C)**

$$f(x)=\begin{cases}-g(x) & x<-3\\ 0 & x=-3\\ g(x) & -3<x<-2\\ 0 & x=-2\\ -g(x) & -2<x<-1\\ 0 & x=-1\\ g(x) & x>-1\end{cases}$$

47. **(D)**

The image of A in y = x will lie on BC

A' = (5, 4)



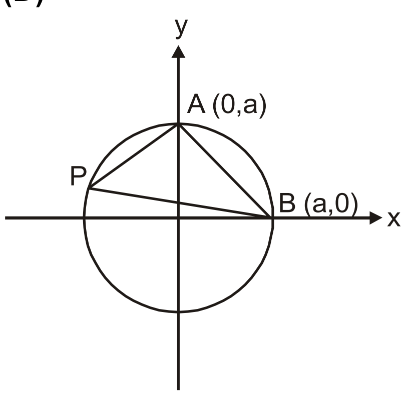
$AD \perp BC$

$$2\left(\frac{4-k}{5-h}\right)=-1 \;\; \Rightarrow 8-2k=-5+h$$

$\because h=k$

$$\therefore h=k=\frac{13}{3}$$

48. **(D)**



Let $P\equiv(a\cos\theta, a\sin\theta)$

and centroid of ΔAPB be (h, k).

Then $h=\dfrac{a\cos\theta+0+a}{3}, k=\dfrac{a\sin\theta+a+0}{3}$

$\Rightarrow \cos\theta = \frac{3h}{a} - 1, \sin\theta = \frac{3k}{a} - 1$

$\because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{3h}{a} - 1\right)^2 + \left(\frac{3k}{a} - 1\right)^2 = 1$

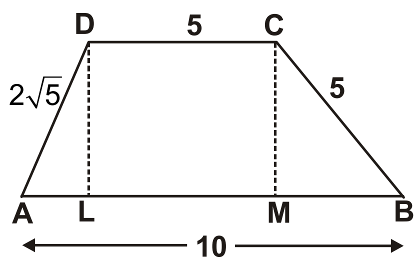
$\Rightarrow 9h^2 + 9k^2 - 6ah - 6ak + a^2 = 0$

so locus of centroid is

$9x^2 + 9y^2 - 6ax - 6ay + a^2 = 0$

49. (A, D)

Let us take distance between paralle sides ℓ then BM + AL = 5



$\Rightarrow \sqrt{25 - \ell^2} + \sqrt{20 - \ell^2} = 5 \quad \Rightarrow \ell = 4 \qquad \Rightarrow AL = 2, BM = 3$

hence area of trapezium is

$= 5 \times 4 + \frac{1}{2} \times 3 \times 4 + \frac{1}{2} \times 2 \times 4 = 30$ sq. units

Also, $\tan A = \frac{4}{2} = 2 \Rightarrow A = \tan^{-1} 2$

Now, area of region enclosed by locus of

P is = $2 \times (10 + 5 + 5 + 2\sqrt{5}) + \pi(2)^2 + 30$ = $70 + 4\left(\sqrt{5} + \pi\right)$

50. (B,C,D)

Let $(x_1, y_1) \equiv (at^2, 2at)$.

Tangent at this point is $yt = x + at^2$.

Any point on this tangent is $\left(h, \frac{h + at^2}{t}\right)$.

Chord of contact with respect to the circle is $hx+\left(\frac{h+at^2}{t}\right)y=a^2$

i.e. $(aty-a^2)+h\left(x+\frac{y}{t}\right)=0$

Which is the family of the straight lines passing through the point of intersection of

$ty-a=0$, and $x+\frac{y}{t}=0\cdot$

So the fixed point is $\left(-\frac{a}{t^2},\frac{a}{t}\right)$

$\therefore\ x_2=-\frac{a}{t^2}$ and $y_2=\frac{a}{t}$

51. (A,B,C,D)

(A) $\sin\left(\frac{11\pi}{12}\right)\cdot\sin\left(\frac{5\pi}{12}\right)=\sin\left(\frac{\pi}{12}\right)\cdot\cos\left(\frac{\pi}{12}\right)=\frac{1}{2}\sin\left(\frac{\pi}{6}\right)=\frac{1}{4}\in Q$

(B)

$\operatorname{cosec}\left(\frac{9\pi}{10}\right)\cdot\sec\left(\frac{4\pi}{5}\right)=-\operatorname{cosec}\left(\frac{\pi}{10}\right)\cdot\sec\left(\frac{\pi}{5}\right)=\frac{1}{\sin 18^\circ\cdot\cos 36^\circ}=\frac{-16}{(\sqrt{5}-1)(\sqrt{5}+1)}=-4\in Q$

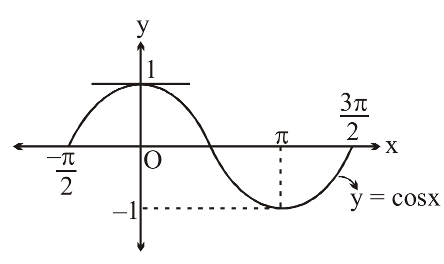
(C) $\sin^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{\pi}{8}\right)=1-\frac{1}{2}\sin^2\left(\frac{\pi}{4}\right)=1-\frac{1}{4}=\frac{3}{4}\in Q$

(D) $2\cos^2\frac{\pi}{9}\cdot 2\cos^2\frac{2\pi}{9}\cdot 2\cos^2\frac{4\pi}{9}=8(\cos 20^\circ\cdot\cos 40^\circ\cdot\cos 80^\circ)^2=\frac{1}{8}\in Q$

52. (A, C, D)

$[f(x)=\lim_{n\to\infty} x\left\{\frac{3}{2}+[\cos x]\left(\sqrt{n^2+1}-\sqrt{n^2-3n+1}\right)\right\}$

$=\frac{3x}{2}+x\cdot[\cos x]\cdot\lim_{n\to\infty}\left(\sqrt{n^2+1}-\sqrt{n^2-3n+1}\right)$

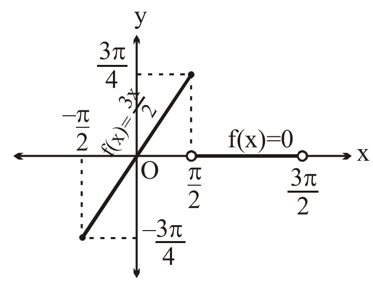


$=\frac{3x}{2}+x[\cos x]\cdot\lim_{n\to\infty}\frac{n^2+1-(n^2-3n+1)}{\sqrt{n^2+1}+\sqrt{n^2-3n+1}}$

$=\frac{3x}{2}+x[\cos x]\cdot\frac{3}{2}=\frac{3x}{2}\left(1+[\cos x]\right)$ [cos x]

$$= \begin{cases} 0 & x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\} \\ 1 & x = 0 \\ -1 & x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \end{cases}$$

$$f(x) = \begin{cases} \frac{3x}{2} & -\frac{\pi}{2} \le x \le \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$$



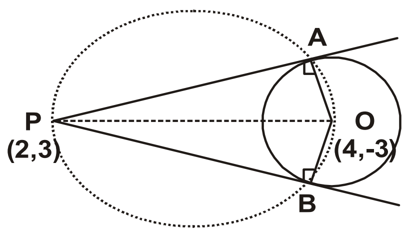
$\therefore$ Graph of f (x) in $\left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$

From the graph it is clear that options (A), (C) and (D) are correct]

53. (7)

Centre of the given circle O(4,–3)

The circumcircle of ΔPAB is $(x-2)(x-4)+(y-3)(y+3)=0$



$\Rightarrow x^2 + y^2 - 6x - 1 = 0$ ....(i)

Director circle of ellipse is

$(x+5)^2 + (y-3)^2 = 9 + b^2$ ....(ii)

From (i) and (ii) applying condition of orthogonality, we get

$2[(-3)(5) + 0(-3)] = -1 + 25 - b^2$

$\Rightarrow b^2 = 54 \Rightarrow [b] = 7$

54 (4)

Suppose px + qy =1 intersect the parabola $y^2 = 4ax$ at $(at^2, 2at)$ then

$pat^2 + q \times 2at = 1 \qquad \Rightarrow t_1 + t_2 = \frac{-2aq}{pa} = \frac{-2q}{p}$

but we know that for co-normal points $(at_1^2, 2at_1), (at_2^2, 2at_2), (at_3^2, 2at_3)$

$t_1 + t_2 + t_3 = 0 \Rightarrow t_3 = \frac{2q}{p}$

Hence 3rd point is $\left(\frac{4aq^2}{p^2}, \frac{4aq}{p}\right) \Rightarrow m = 4$

55. (2)

$a = \min\{(x_1 - x_2)^2 + (y_1 - y_2)^2\}$

where $P(x_1, y_1)$ lies on $y = \sqrt{x-1} \Rightarrow y^2 = x - 1$, y being positive

and $Q(x_2, y_2)$ lies on $y = x^2 + 1$

Here value of 'a' can be obtained if PQ is common normal to $y^2 = x - 1$ and $y = x^2 + 1$.
Two parabolas are image of each other in the line y = x
therefore slope of common normal is – 1

Here, $x_1 - 1 = \frac{1}{4}(-1)^2 \Rightarrow x_1 = 5/4$

and $y_1 = -\frac{1}{2}(-1) = \frac{1}{2}$

Hence P is $\left(\frac{5}{4}, \frac{1}{2}\right) \Rightarrow$ Q is $\left(\frac{1}{2}, \frac{5}{4}\right)$

$\Rightarrow PQ = \sqrt{\left(\frac{5}{4} - \frac{1}{2}\right)^2 + \left(\frac{1}{2} - \frac{5}{4}\right)^2} = \sqrt{2 \times \frac{9}{16}} = \frac{3}{2\sqrt{2}}$

$\Rightarrow a = PQ^2 = \frac{9}{8}$ hence, $\sqrt{2\sqrt{2a} + 1} = 2$

56. (2)

In the triangle, $\tan A . \tan B . \tan C = \tan A + \tan B + \tan C$

or $\frac{1}{2} . \left(\frac{2k+1}{2}\right)\left(\frac{4k+1}{2}\right) = \frac{3}{2} + 3k$

or $\frac{8k^2 + 6k + 1}{8} = \frac{3 + 6k}{2} \Rightarrow 8k^2 + 6k + 1 = 12 + 24K \Rightarrow 8k^2 - 18k - 11 = 0$

$(2k+1)(4k-11) = 0 \Rightarrow k = -\frac{1}{2}$ or $k = \frac{11}{4}$ $\therefore [k] = 2$

57. (4)

Three vertices lies on the circle $x^2 + y^2 = 25$

Let orthocentre is H(h,k) then

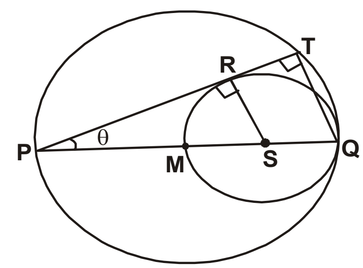
$h = 5\sin\theta + 5\cos\theta + 3 \qquad k = 5\sin\theta - 5\cos\theta + 4$

Hence $\sin\theta = \dfrac{h+k-7}{10}, \cos\theta = \dfrac{h-k+1}{10} \Rightarrow (h+k-7)^2 + (h-k+1)^2 = 100$

$\Rightarrow (x+y-7)^2 + (x-y+1)^2 = 100 \qquad \therefore \alpha = -7, \beta = 1, \gamma = \pm 10$ and $|\gamma| + \alpha + \beta = 4$

58. (6)

Take $\angle QPT = \theta$



than $\sin\theta = \dfrac{RS}{PS} = \dfrac{r}{3r} = \dfrac{1}{3} \Rightarrow \cos\theta = \dfrac{2\sqrt{2}}{3}$

$\Rightarrow \dfrac{PT}{PQ} = \dfrac{2\sqrt{2}}{3} \qquad \Rightarrow \dfrac{PT}{12} = \dfrac{2\sqrt{2}}{3}$

$\Rightarrow PT = 8\sqrt{2} \qquad \Rightarrow m = 8, n = 2$

59. (A - r); (B - s) ; (C - p) ; (D - q)

A. Eqn. of tangent at $(2\cos\theta, \sqrt{3}\sin\theta)$ is $\dfrac{x}{2}\cos\theta + \dfrac{y}{\sqrt{3}}\sin\theta = 1$. Mid point (h,k) of portion of tangent between coordinate axes is $(\sec\theta, \dfrac{\sqrt{3}}{2}\operatorname{cosec}\theta)$

$\therefore h = \sec\theta, k = \dfrac{\sqrt{3}}{2}\operatorname{cosec}\theta \qquad \Rightarrow \dfrac{1}{h^2} + \left(\dfrac{\sqrt{3}}{2k}\right)^2 = 1 \Rightarrow \dfrac{4}{h^2} + \dfrac{3}{k^2} = 4$

B. Eqn. of chord having mid point (x_1, y_1) is $\dfrac{x\,x_1}{4} + \dfrac{y\,y_1}{3} = \dfrac{x_1^2}{4} + \dfrac{y_1^2}{3}$

Eqn of pair of lines joining origin to extremities of the chord is

$$\frac{x^2}{4} + \frac{y^2}{3} = 1\left(\frac{\frac{x\,x_1}{4} + \frac{y\,y_1}{3}}{\frac{x_1^2}{4} + \frac{y_1^2}{3}}\right)^2$$

The lines are mutually perpendicular

$\therefore$ coeff. of x^2 + coeff of $y^2 = 0$

$$\Rightarrow \quad \frac{1}{4}\left(\frac{x_1^2}{4}+\frac{y_1^2}{3}\right)^2-\frac{x_1^2}{16}+\frac{1}{3}\left(\frac{x_1^2}{4}+\frac{y_1^2}{3}\right)^2-\frac{y_1^2}{9}=0 \;\Rightarrow\; 7\left(\frac{x_1^2}{4}+\frac{y_1^2}{3}\right)^2=12\left(\frac{x_1^2}{16}+\frac{y_1^2}{9}\right)$$

C. Eqn. of chord of contact from (h,k) is $\frac{hx}{4}+\frac{ky}{3}=1$ which passes through the focus (1, 0) or (–1, 0)

$$\therefore \quad \pm\frac{h}{4}=1 \Rightarrow h^2=16$$

D. Eqn. of normal at $(2\cos\theta,\sqrt{3}\sin\theta)$ is $2x\sec\theta-\sqrt{3}y\,\text{cosec}\theta=2^2-\sqrt{3}^2=1$ ..... (1)

Eqn. of chord with mid point (x_1, y_1) is $\frac{x\,x_1}{4}+\frac{y\,y_1}{3}=\frac{x_1^2}{4}+\frac{y_1^2}{3}$ ..... (2)

Comparing (1) and (2), $\frac{2\sec\theta}{x_1/4}=-\frac{\sqrt{3}\,\text{cosec}\theta}{y_1/3}=\frac{1}{\frac{x_1^2}{4}+\frac{y_1^2}{3}}$

$$\Rightarrow \cos\theta=\frac{8}{x_1}\left(\frac{x_1^2}{4}+\frac{y_1^2}{3}\right),\ \sin\theta=\frac{3\sqrt{3}}{y_1}\left(\frac{x_1^2}{4}+\frac{y_1^2}{3}\right)$$

$$\Rightarrow \left(\frac{64}{x_1^2}+\frac{27}{y_1^2}\right)\left(\frac{x_1^2}{4}+\frac{y_1^2}{3}\right)^2=1$$

60. (A - q); (B - r); (C - p); (D - s)

Equation of tangent to parabola = $y=mx+\frac{a}{m}$ which passes through P(6,5) $\Rightarrow m=\frac{1}{2}$ or $\frac{1}{3}$

points of contact Q and R = (4,4) and (9,6) and area of $\Delta PQR=\frac{1}{2}$

$C_2:(x-9)^2+(y-6)^2+\lambda(x-3y+9)=0$; $C_1:(x-4)^2+(y-4)^2+\mu(x-2y+4)=0$

now C_2 & C_1 pass through (1,0)

$C_2: x^2+y^2-28x+18y+27=0$; $C_1: x^2+y^2+13x+2y+12=0$

Centroid of ΔPQR is $\left(\frac{19}{3},5\right)$