SOLUTIONS

MEAITS 2018

PART TEST-1

(MAIN PATTERN)

Test Date: 18-11-2017

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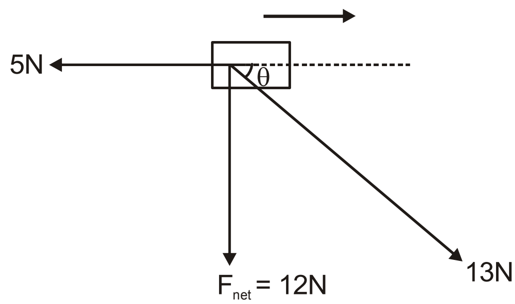
Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd., Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

PHYSICS

1. (C)

13N will act in such a direction that resultant force along with pseudo force will be perpendicular to direction of motion of train.



Hence 13 cos θ = 5 $\Rightarrow \cos\theta = \frac{5}{12}$. $\therefore \sin\theta = \frac{12}{13}$

$\therefore f_{net} = 13\sin\theta = 12N$

$\therefore f_r = \mu mg = 10N$

$\therefore a = \frac{12-10}{2} = 1\,m/s^2$

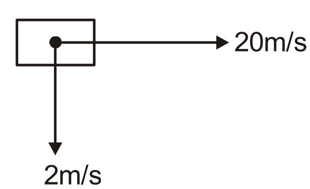
Now velocity of train after 8 sec

= u + at

= 0 + 2.5 × 8 = 20m/s

and velocity due to applied force

= 0 + 1 × 2 = 2 m/s



$\therefore u_{net} = \sqrt{20^2 + 2^2} = \sqrt{404}$

$\therefore K.E. = \frac{1}{2}mv^2$

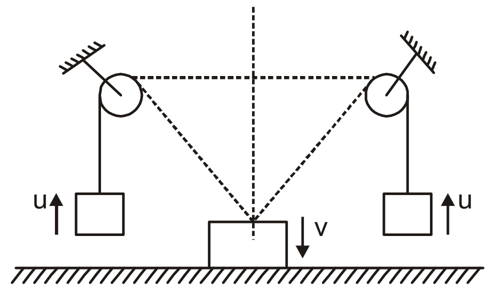
$= \frac{1}{2} \times 2 \times 404$

$= 404J$ Ans.

2. **(B)**

When 2m fall by 3 meter, m will rise through 1 meter each.

From energy conservation



$$2mg \times 3 - 2mg \times 1 = \frac{1}{2}mu^2 \times 2 + \frac{1}{2}(2m)v^2 \quad ....(1)$$

Also from constrain relation, $\frac{3}{5}v = u$ ...(2)

from (1) & (2)

$$u = 10 \times \sqrt{\frac{5}{17}} \text{ m/s}$$

3. **(B)**

$$v_x = 18\cos 60° = 9\text{m/s} \ \& \ u_y = 18\sin 60° = 9\sqrt{3} \text{ m/s}$$

$$\sqrt{9^2 + v_y^2} = 15 \Rightarrow v_y = 12 \text{ m/s}$$

$$\therefore \ 12 = 9\sqrt{3} - 10t \ ....$$

$$\therefore \ t = \frac{9\sqrt{3} - 12}{10} \text{ sec.}$$

$$t_f = \frac{2v\sin\theta}{g} = \frac{9\sqrt{3}}{5} \text{ sec.}$$

$$\therefore \ \Delta t = \frac{9\sqrt{3}}{5} - 2 \times \frac{9\sqrt{3} - 12}{10} = \frac{12}{5} \text{ sec.}$$

4. **(D)**

$$\frac{dv}{dt} = \frac{v^2}{R} \quad \Rightarrow \int_{v_0}^{v} \frac{dv}{v^2} = \frac{1}{2}\int_0^t dt$$

$$\Rightarrow v = \frac{v_0 R}{R - v_0 t} \quad \Rightarrow \frac{dx}{dt} = \frac{v_0 R}{R - v_0 t}$$

$$\Rightarrow \int_0^{2\pi R} dx = v_0 R \int_0^t \frac{dt}{R - v_0 t}$$

$$\Rightarrow 2\pi R = \frac{v_0 R}{-v_0}\left[\ell n(R - v_0 t)\right]_0^t \quad \Rightarrow t = \frac{R}{v_0}\left(1 - e^{-2\pi}\right)$$

5. (B)

$\mu = \mu_0 (1 + \alpha\Delta\theta)$

$\therefore \quad \Delta\mu = \mu_0 \alpha \Delta\theta$

shift $y = \dfrac{(\mu_2 - \mu_1) t \times D}{d}$

one fringe width = $\dfrac{D}{d}.\lambda$

Hence number of fringes crossed is

$n = \dfrac{y}{w} = \dfrac{(\mu_2 - \mu_1) \times t}{\lambda}$

$= \dfrac{\mu_0 \alpha \Delta\theta \times t}{\lambda}$

$= 10^{-4}$

6. (D)

Conceptual

7. (B)

$$\mu_1 = \frac{\sin\frac{A+\delta_1}{2}}{\sin\frac{A}{2}}, \mu_2 = \frac{\sin\frac{A+\delta_2}{2}}{\sin\frac{A}{2}}, \mu_3 = \frac{\sin\frac{A+\delta_3}{2}}{\sin\frac{A}{2}}$$

Using the relation $2\delta_2 = \delta_1 + \delta_3$ (for A.P.)

we can get $\dfrac{\mu_1 + \mu_3}{\mu_2} = \dfrac{\sin\frac{A+\delta_1}{2} + \sin\frac{A+\delta_3}{2}}{\sin\frac{A+\delta_2}{2}} = \dfrac{2\sin\frac{2A+\delta_1+\delta_3}{4} \cdot \cos\frac{\delta_1-\delta_3}{4}}{\sin\frac{A+\delta_2}{2}}$

$$= \frac{2\sin\frac{A+\delta_1}{2} \cdot \cos\frac{\delta_1-\delta_3}{2}}{\sin\frac{A+\delta_2}{2}}$$

$$= 2\cos\frac{\delta_1-\delta_3}{4}$$

8. (D)

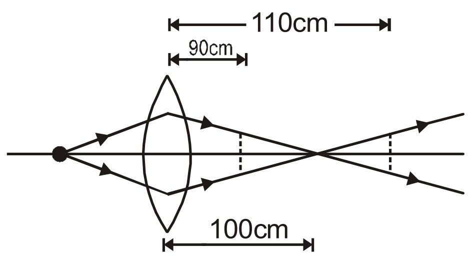
Since object at focal length of 1st lens, hence d_1 can have any value, but image fromed by 2nd lense must lies at 2nd focal length of 3rd lens.

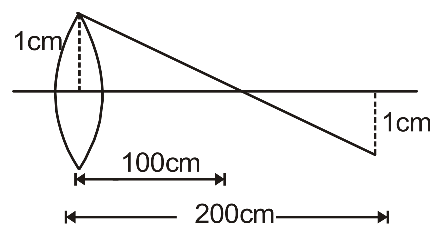
$$\therefore \quad d_2 = 20 - 5 = 15\text{cm}$$

9. (C)

Clearly $\frac{1}{v} - \frac{1}{-25} = \frac{1}{20} \quad \Rightarrow \quad v = 100 \text{ cm}$

Intensity at x = 100 cm will be maximum and it will be same at equal distances from 100cm i.e. same at 90cm & 110cm.





Hence (ii), (iii) & (iv) are correct

10. (C)

Magnefying power,

$$MP = \frac{f_0}{f_e}\left[1 + \frac{f_e}{D}\right] = \frac{60}{3}\left[1 + \frac{3}{25}\right]$$

$$= 22.4$$

But $MP = \frac{\theta}{\theta_0} \qquad \therefore \theta = 22.4 \times \frac{1^\circ}{2} = 11.2^\circ$

11. (A)

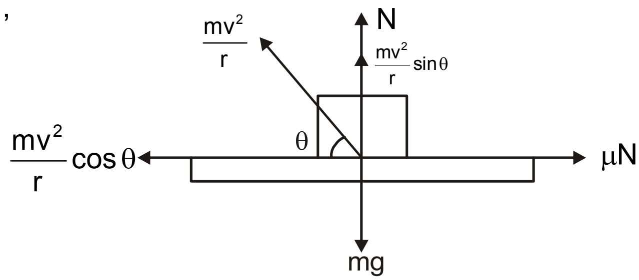
Applying snell's law between O & P,

$$2 \times \sin 60^\circ = \frac{2}{1 + H^2} \times \sin 90^\circ$$

$$\frac{\sqrt{3}}{2} = \frac{1}{H^2 + 1} \qquad \Rightarrow \qquad H = \sqrt{\frac{2}{\sqrt{3}} - 1}$$

12. (D)

f.b.d. of block'A' w.r.t. platform is shown in fig. along with centrifugal force at angular position θ,



which gives $\mu N = \frac{mv^2}{r}$ ...(1)

& $N + \frac{mv^2}{r}\sin\theta = mg$ ...(2)

From (1) & (2)

$\mu = \frac{\frac{v^2}{r}\cos\theta}{g - \frac{v^2}{r}\sin\theta}$ Now for μ to be mnimum, $\frac{d\mu}{d\theta} = 0$

$$\Rightarrow \frac{\left(g - \frac{v^2}{r}\sin\theta\right)\left(-\frac{v^2}{r}\sin\theta\right) - \left(\frac{v^2}{r}\cos\theta\right)\left(0 - \frac{v^2}{r}\cos\theta\right)}{\left(g - \frac{v^2}{r}\sin\theta\right)^2} = 0$$

$$\Rightarrow -\frac{v^2}{r}g\sin\theta + \left(\frac{v^2}{r}\sin\theta\right)^2 + \left(\frac{v^2}{r}\cos\theta\right)^2 = 0$$

$$\Rightarrow \frac{v^2}{r}g\sin\theta = \left(\frac{v^2}{r}\right)^2 \quad \Rightarrow \sin\theta = \frac{v^2}{rg}$$

or $\sin\theta = \frac{0.7 \times 0.7}{0.2 \times 9.8} = 0.25 = \frac{1}{4}$

$\therefore \cos\theta = \sqrt{1 - \frac{1}{16}} = \sqrt{\frac{15}{16}}$

$$\therefore \mu_{min^m} = \frac{\frac{0.7 \times 0.7}{0.2} \times \sqrt{\frac{15}{16}}}{9.8 - \frac{0.7 \times 0.7}{0.2} \times \frac{1}{4}} = 0.258$$

13. (C)

Distance between successive wave fringes is equal to wavelength. THe plane wave X - Y will reach at P after three wave lengths

$\therefore$ D = 3λ

$\therefore \quad t = \frac{3\lambda}{C}$

14. (B)

Apperent depth = $\frac{\text{Real depth}}{\mu}$

Hence for the 1st medium apperent distance is

$$\ell = \int \frac{dx}{\mu} = \int_0^{10} \frac{dx}{\frac{22}{(1+2x)}}$$

$$= \frac{1}{22}\int_0^{10}(1+2x)\,dx = \frac{1}{22}[10+100] = 5cm$$

Hence $u = -(5+10) = -15cm \quad f = +10cm$

$\therefore \quad \frac{1}{v} - \frac{1}{-15} = \frac{1}{10} \qquad \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{15}$

$\Rightarrow \quad v = 30cm$

15. (C)

For AB, acceleration's

a = g sinθ – μg cosθ

$$= g\sin\theta - \frac{\tan\theta}{2}.g\cos\theta = \frac{g\sin\theta}{2} \text{ downward}$$

for BC

$$a = g\sin\theta - \frac{3g\sin\theta}{2} = -\frac{g\sin\theta}{2}$$

i.e. upward acceleration $\frac{g\sin\theta}{2}$

Hence block will again come to rest.

16. (D)

Intensity of polarised light from first from first polarizer is

$$\frac{100}{2} = 50$$

Hence new intensity is

$$I = 50\cos^2 60° = \frac{50}{4} = 12.5\%$$

17. (D)

Height of fall of ring is h = L tan 37° = 0.7 × $\frac{3}{4}$.

Distance moved by M up the plane is $\Delta\ell = L\sec 37° - L = \frac{L}{4}$

From constrain relation $v_M = \frac{4}{5}v_m$ ...(1)

Now work energy theorem gives

$$-mgh + Mg\Delta\ell\sin 37° + \frac{1}{2}mv_m^2 + \frac{1}{2}Mv_M^2 = 0 \qquad ...(2)$$

From (1) & (2)

$v_m = 0,$ i.e. ring will instantaneously at rest

18. (A)

The virtual image formed by the lens at the naked eyes true near point

$\therefore$ P = 25 cm

Now $f = \frac{1}{P} = \frac{1}{3}m = 33.3\,cm$

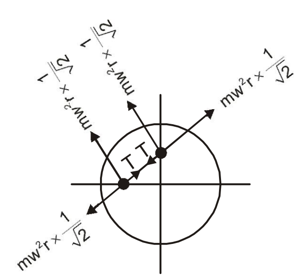
$$\therefore \frac{1}{25} + \frac{1}{q} = \frac{1}{33.3}$$

$\Rightarrow$ q = – 100cm

i.e. 1m away for clear vision

19. (A)

Considering centrifugal forces an each particles



$mw^2 r\cos 45° = \mu mg$

$$\omega_{max} = \sqrt{\frac{\sqrt{2}\,\mu g}{r}}$$

20. (A)

The detector will receive the max^m. light when the image of point source of light coincide with the position of detector.

Velocity of object w.r.t lens = +10m/s

Velocity of detector w.r.t. lens = 20 m/s

Hence after time t

u = –(100 – 10t) & v = 20t, f = 10cm.

$$\therefore \quad \frac{1}{20t} - \frac{1}{-(100-10t)} = \frac{1}{10}$$

$$\Rightarrow \quad 2t^2 - 19t + 10 = 0$$

$$\Rightarrow \quad t = 0.56 \text{ sec and } 8.94 \text{ sec.}$$

21. (B)

Snell's law at 2nd face gives

$\mu \sin 30° = \sin\beta \qquad \Rightarrow \beta = 60°$

$\therefore$ 1st deviation $\delta = 60° - 30° = 30°$ (by prism)

Incidence angle θ that ray makes with principle axis is

$$\tan\theta = \frac{R/\sqrt{3}}{R} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 30° = \delta$$

It means that ray incidence on sphere normally and hence retrace its path from silvered face. Hence total angle of deviation is 180°

22. (B)

For first minimum, $\theta = \dfrac{\lambda}{a}$

and $\Delta\phi = \dfrac{2\pi}{\lambda}.\Delta x = \dfrac{2\pi}{\lambda}.\, a \sin\theta$

$= \dfrac{2\pi}{\lambda}.a\theta$ (for small angle)

$= \dfrac{2\pi}{\lambda}.a \times \dfrac{\lambda}{a}$

= 2λ

23. (B)

Clerly $V_A \cos\beta = V_B \cos\alpha$

(from constrain relation)

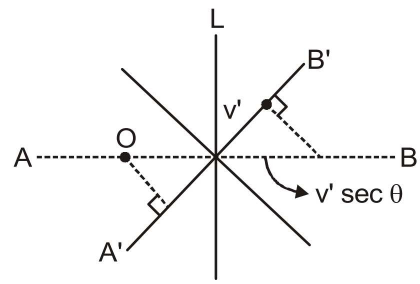
24. (A)

The ray will always be normal, its x and y coordinate remains same. Hence optical length along z-axis will be $s = \int_0^c \mu dz = \int_0^c Az^3 dz = \frac{AC^4}{4}$

25. (A)

After rotation

$$u' = -2f\cos\theta$$



$$\therefore \quad \frac{1}{v'} - \frac{1}{-2f\cos\theta} = \frac{1}{f}$$

$$\therefore \quad v' = \frac{2f\cos\theta}{2\cos\theta - 1}.$$ (for small θ, v' will be + ve)

so the image distane along initial OA is

$$v = v'\sec\theta = \frac{2f}{2\cos\theta - 1}$$

Hence velocity of image is

$$\frac{dv}{dt} = \frac{-2f}{(2\cos\theta - 1)^2}(-2\omega\sin\theta)$$

$$= \frac{4f\omega\sin\theta}{(2\cos\theta - 1)^2}$$

for very small angle, i.e, $\theta = 2°$

$$\sin\theta \simeq \theta \simeq 2° = \frac{\pi}{90}\text{rod.}$$

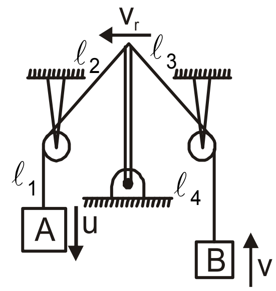
and $\cos\theta \simeq 1$

$\therefore \quad v_{image} = \frac{4f\omega\pi}{40}$ m/s along A to B i.e., away from pole.

26. (A)

Let magnitude of velocity of top of rod is V_R towards left and speed of block B is v.

$\ell_1 + \ell_2 = \text{constant}$



$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} = 0$$

$\Rightarrow\ u - v_R \cos 60° = 0 \quad ...(1)$

Also $\ell_3 + \ell_4 = \text{constant}$

$$\Rightarrow\ \frac{d\ell_3}{dt} + \frac{d\ell_4}{dt} = 0$$

$\Rightarrow\ v_R \cos 30° - v = 0$

from (1) & (2)

$v = \sqrt{3}\,u$

27. (D)

Work-energy theorem gives

increase in K.E. = work-done

or $\quad \frac{1}{2}mv_2^2 - \frac{1}{2}m.\frac{2F_0x_0}{m} = \frac{1}{2}\times 2F_0 \times 3x_0 + 3F_0x_0$

or $\quad v_2 = \sqrt{\frac{14F_0x_0}{m}}$

28. (D)

Line OP is also incline at 53° and initial velocity is zero. Hence time of callision is

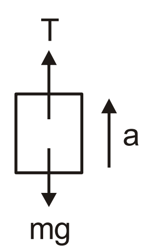
$$t = \frac{120}{40\cos 53°} = 5\,\text{sec}.$$

29. (A)

Consider an instant when block is at height y. Then pulling force is

$T = 600 - 20y$

Hence f.b.d. of block gives,



T – mg = ma

$\Rightarrow (600 - 20y) - 400 = 40a$

$200 - 20y = 40a$

$$\Rightarrow\ 5 - 0.5y = v\frac{dv}{dy}$$

$$\Rightarrow \quad 5\left[y\right]_0^5 - \frac{1}{2}\left[\frac{y^2}{2}\right]_0^5 = \frac{v^2}{2}$$

$$\Rightarrow \quad v = \sqrt{\frac{75}{2}} = 6.12\,\text{m/s}$$

30. (C)

If H is maximum height, then

$u_y^2 = 2gH$ & $v_y^2 = 2 \times 2gH$

$$\therefore \quad \tan\phi = \frac{v_y}{u_x} = \frac{\sqrt{2}\,u_y}{u_x} = \sqrt{2}\tan\theta$$

CHEMISTRY

31. (B)

Since P and V of vessel does not change during heating. Hence, total kinetic energy of vessel does not change.

32. (D)

$4.9\%\ w/v \equiv 0.5MH_2SO_4$

Moles of H_2SO_4 = 3×.5= 1.5 mole

Moles of SO_3 = 1.5 mole

Let mole of O_2 in mixture = x mole

$$\frac{1.5 \times 80 + x \times 32}{1.5 + x} = 60.8 \quad \therefore x = 1$$

Moles in mixture = 1.5 + 1 = 2.5 mole

Vol (NTP) = 22.4 × 2.5 = 56 L

33. (C)

C = 120 gm = 10 mole

O_2 = 179.2L =8 mole

$$\underset{c}{C} + O_2 \longrightarrow \underset{a}{CO} + \underset{b}{CO_2}$$

a+b = 10 & a+2b = 8 × 2

b = 6 mole a = 4 mole

	$2NaOH$	$+ CO_2 \longrightarrow$	$Na_2CO_3 + H_2O$
initial mole	10	6	0
final mole	0	1	5

	$Na_2CO_3 + CO_2 + H_2O \longrightarrow 2NaHCO_3$			
	Na_2CO_3	CO_2	H_2O	$2NaHCO_3$
initial mole	5	1		0
final mole	0	0		2

Molarity $= \frac{2}{5} = 0.4M$

34. **(B)**

$Cu_2S \longrightarrow Cu^{++} + SO_2$ nf = 1×2+6 = 8

$CuS \longrightarrow Cu^{++} + SO_2$ nf = 6

Let x mole of each is present :

Total equivalent of mixture = 8x + 6x = 14 x

Total equivalent of MnO_4^- = 200 × 5 × 0.75 × 10^{-3} = 0.750

Equivalent of Excess MnO_4^- = 190 × 1 × 10^{-3} = 0.190

Equivalent used for sample = 0.750 – 0.190 = 0.56

$\Rightarrow$ 14 x = 0.56 $\Rightarrow$ x = 0.04

Maximum mole obtain = 0.04 × 3 = 0.12

35. **(D)**

$$C_xH_{3x}O_y(g) + \left[\frac{7x}{4} - \frac{y}{2}\right]O_2(g) \longrightarrow XCO_2(g) + \frac{3x}{2}H_2O(\ell)$$

16 ml $\quad 16\left(\frac{7x}{4} - \frac{y}{2}\right) \quad$ 16 x

Vol of CO_2 = 16x = 54–22

x = 2

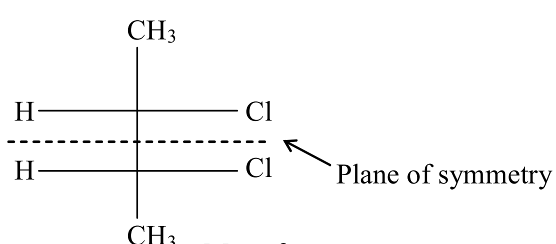
Now V_{O_2} left $= 22ml$

$$70 - 16\left[\frac{7x}{4} - \frac{y}{2}\right] = 22$$

y = 1

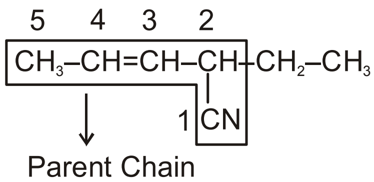
$C_2H_6O_1$

36. **(C)**

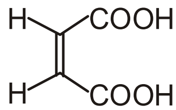


Meso form

37. (C)



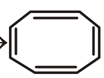
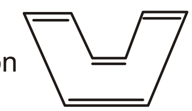
38. (A)



39. (C)

H_2CO_3 is more Acidic than phenol.

40. (D)

B →  This compound exist in boat conformation  so resonance is not possible.

C → These two are Tautomers.

41. (A)

42. (A)

Forth ionisation energy abruptly increases. This shows that fourth electron is removed from inert gas core.

43. (D)

Cr. has maximum oxidation number (+6) in K_2CrO_4 and thus, has minimum ionic radius.

44. (D)

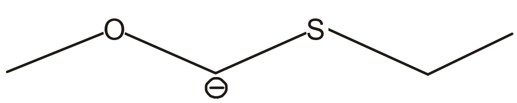
BeO, SnO & ZnO are amphoteric in nature

45. (A)

r : $Co \simeq Ni$

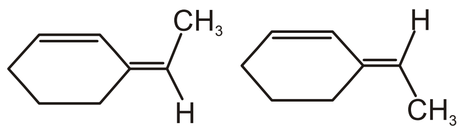
r : $Rh \simeq Ir$ → due to lanthanide contraction

46. (B)



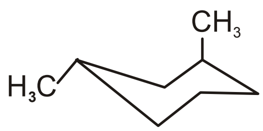
–ve charge is stablished by both oxygen, sulfur-I effect and (sulfur has empty d-orbital, it can stabilise adjacent negative charge).

47. (D)



Z E

48. (B)



In 1,3- di substituted cyclohexanes (a,e) is more stable.

49. (B)

For racemisation enol intermediate must be achiral.

50. (C)

By fractional distillation :

(1) Enantiomers can't be seperated

(2) Distereomers can be seperated

51. (C)

$$2FeS + \frac{7}{2}O_2 \longrightarrow Fe_2O_3 + 2SO_2$$

2mole $\quad$ 1mole

$88 \times 2 = 176$ $\quad$ $160g$

16gm loss is observed when 176 g FeS is present.

If mass of sample is W gm.

$$\Rightarrow W \times \frac{4}{100} = 16$$

$$\Rightarrow W = 400 \text{ gm}$$

$$\therefore \text{ \% of FeS} = \frac{176}{400} \times 100 = 44\%$$

52. (B)

$$\frac{r_{SO_3}}{r_{O_2}} = \frac{w/80}{w/32}\sqrt{\frac{32}{80}} = \frac{2\sqrt{2}}{5\sqrt{5}}$$

53. (A)

$$\frac{P_C}{T_C} = \frac{R}{8b};\ \left(\frac{P_C}{T_C}\right)_{CH_4} = \left(\frac{P_C}{T_C}\right)_{CO_2}$$

$$\frac{P}{200} = \frac{75}{300}$$

$$P = \frac{2 \times 75}{3} = \frac{150}{3} = 50\,\text{atm}$$

54. (A)

$$6Cu_3P + 124H^+ + 11Cr_2O_7 \longrightarrow 18Cu^{++} + 6H_3PO_4 + 22Cr^{3+} + 53H_2O$$

55. (B)

$$\frac{1}{2}MU^2 = \frac{3}{2}\frac{RT}{N}$$

$$\frac{1}{2} \times 8.314 \times 10^{-23} \times U_{rms}^2 = \frac{3}{2} \times \frac{8.314 \times 600}{6 \times 10^{23}}$$

$$U_{rms}^2 = 300$$

U_{rms} = 17.32 m/s

56. (D)

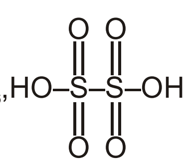
Xenon has highest boiling point because it has maximum vander waals forces due to the possession of more electrons

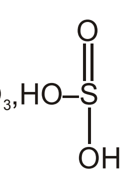
Select the correct answer using the codes given below :

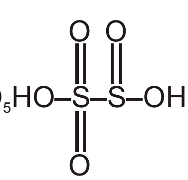
57. (C)

XeF_6 has distorted octahedral structure Xenon in XeF_6 undergoes sp^3d^3 hybridization giving pentagonal bipyramid with one axis is occupied by a lone pair.

58. (B)

$H_2S_2O_6$,  4π – bonds;

H_2SO_3,  1π – bonds;

$H_2S_2O_5$  3π – bonds

59. (C)

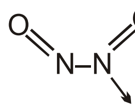
$I \longrightarrow PCl_5(sp^3d)\ \ \ddot{I}Cl_3(sp^3d)$

$PCl_6^-\,(sp^3d^2)\ \ \ddot{I}Cl_2^+(sp^3)$

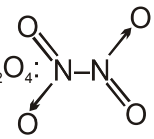
60. (C)

N_2O: $N{=}N{=}O \longleftrightarrow N{\equiv}N{-}O$

N_2O_3:



N_2O_4:



MATHEMATICS

61. (B)

$f(x) + f(x + 2) = f(x + 1)$ ...(i)

Replace x by x + 1

$f(x+1) + f(x+3) = f(x + 2)$ ...(ii)

(i) + (ii) $\Rightarrow$ $f(x)+f(x+3)=0$

$\Rightarrow$ f(x) is periodic with period 6

$\therefore$ $f(1) = f(1 + 2.6) = f(13) = f(-13)$ $\{\because$ f is even$\}$

$f(-5)=f(-5+6)=f(1)=4$

$\therefore$ $f(-5)+f(1)=8$

62. (B)

$f(x+y)=f(x)+f(y)+2xy-2$

Differentiating both sides w.r. to x, treating y as constant

$f'(x+y)=f'(x)+2y$

Put x = 0

$f'(y)=f'(0)+2y$

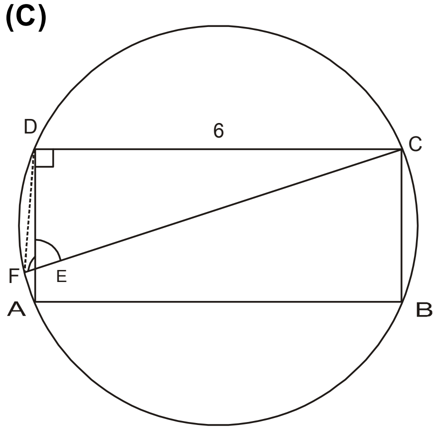
$\Rightarrow$ $f(y)=-2y+y^2+c$

$\because$ $f(0)=2$ $\Rightarrow c=2$

$\therefore$ $f(y)=y^2-2y+2$

$f(2) = 2$

63. (C)



DE = 6 DA = 8 EA = 2

CD = 6 $\angle CED = 45°$ $EC = 6\sqrt{2}$

As AE.ED = FE.EC

$\Rightarrow \; 2.6 = FE = 6\sqrt{2} \qquad \Rightarrow \; FE = \sqrt{2}$

Now, in ΔDEF $\cos E = \dfrac{ED^2 + EF^2 - FD^2}{2.ED.EF}$

$$-\frac{1}{\sqrt{2}} = \frac{36 + 2 - FD^2}{2.6.\sqrt{2}}$$

$\therefore \; FD^2 = 50$

$FD = 5\sqrt{2}$

64. (A)

Vertices of quadrilateral will be $(\pm 3, \pm 2)$.

65. (C)

$$f(x) = \left| \sqrt{(x-3)^2 + (x^2-2)^2} - \sqrt{x^2 + (x^2-6)^2} \right|$$

Let P (x, x^2)

$A \equiv (3,2)$

$B \equiv (0,6)$

$|PA - PB| \le AB$

$\therefore \; \max^m \text{ value} = AB$

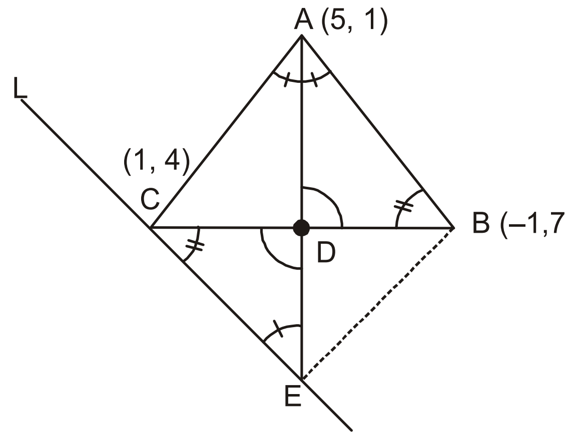
$= \sqrt{9 + 16} = 5$

66. (A)

$$\frac{CD}{DB} = \frac{AC}{AB}$$

$$= \frac{5}{10} = \frac{1}{2}$$

$$\therefore \; D \equiv \left(\frac{1}{3}, \frac{1}{3}\right)$$



Δ CDE ~ ΔBDA

$$\therefore \quad \frac{DE}{DA} = \underbrace{\frac{CD}{BD} = \frac{AC}{AB}} = \frac{1}{2}$$

$$\therefore \frac{ED}{DA} = \frac{1}{2}$$

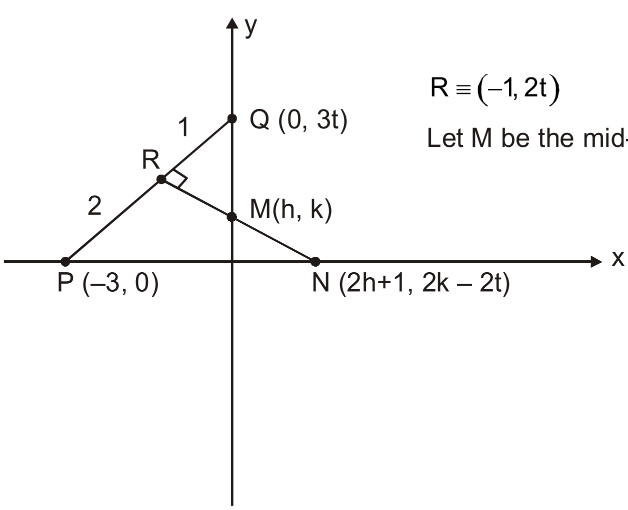
$$\therefore \quad E \equiv (-2, 0)$$

$$ar(\Delta\, ADC) = \frac{25}{3}\text{sq units}$$

and

$$ar\,(\Delta\, BDE) = \frac{25}{3}\text{sq units}$$

67. (D)



$R \equiv (-1, 2t)$

Let M be the mid-point of RN

$\because$ RN $\perp_r$ PQ $\Rightarrow m_{RN}.m_{PQ} = -1$

$$\left(\frac{2k-4t}{2h+2}\right)\left(\frac{3t}{3}\right) = -1$$

$$(k-2t)\,t = -(h+1)$$

$\Rightarrow 2t^2 - kt = h+1$

$\Rightarrow 2k^2 - k^2 = h + 1$ $\left[\because \text{ N lies on the x-axis } \therefore 2k - 2t = 0 \quad \boxed{t = k}\right]$

$\Rightarrow k^2 = h + 1$

$\therefore$ Locus $y^2 = x + 1$

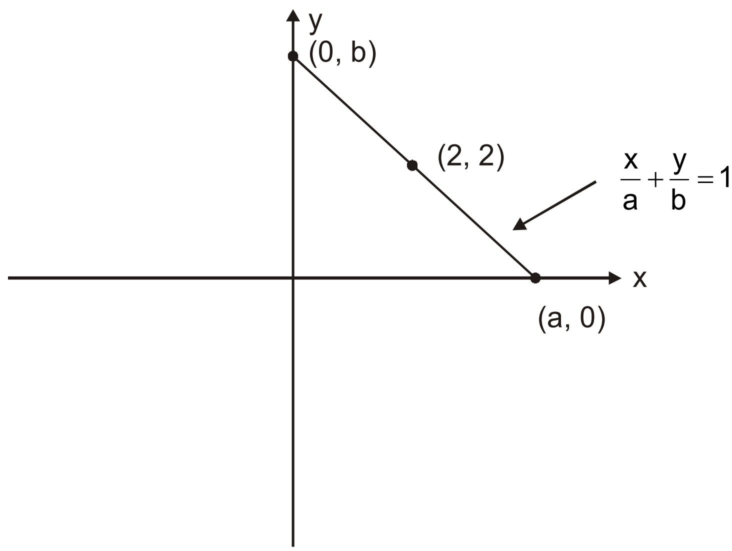
68. (D)

$$\alpha = \frac{a+b}{2}$$

$$\boxed{a + b = 2\alpha}$$

$$\beta = \sqrt{ab}$$

$$\beta^2 = ab$$



$$\frac{2}{a} + \frac{2}{b} = 1$$

$\Rightarrow$ 2 (a + b) = ab

$\Rightarrow 2.2\alpha = \beta^2$

$$\boxed{\beta^2 = 4\alpha}$$

$\therefore$ $p(\alpha, \beta)$ Lies on the parabola is $y^2 = 4x$

69. (A)

Let $P \equiv (3\cos\theta, 3\sin\theta)$

$Q \equiv (-3\cos\theta, -3\sin\theta)$

$$\alpha\beta = \left|\frac{3\cos\theta + 3\sin\theta - 2}{\sqrt{2}}\right| \left|\frac{-3\cos\theta - 3\sin\theta - 2}{\sqrt{2}}\right|$$

$$= \left|\frac{(3\cos\theta + 3\sin\theta)^2 - 4}{2}\right|$$

$$= \left|\frac{9+9\sin 2\theta - 4}{2}\right|$$

$$= \left|\frac{5+9\sin 2\theta}{2}\right|$$

$$\alpha\beta_{max^m} = 7$$

70. (D)

EOT at (1, 2) on circle C_1 : x + 2y = 5 ....(i)

Let P be $(\alpha_1\beta)$

Eq^n of chord of contact of tangents from the point P to the circle C_2 : $\alpha x + \beta y = 9$ ....(ii)

Eq^ns (i) and (ii) represent same line

$\therefore$ on comparing : we get

$$\frac{\alpha}{1} = \frac{\beta}{2} = \frac{-9}{-5}$$

$$\Rightarrow \alpha = \frac{9}{5}, \beta = \frac{18}{5}$$

71. (B)

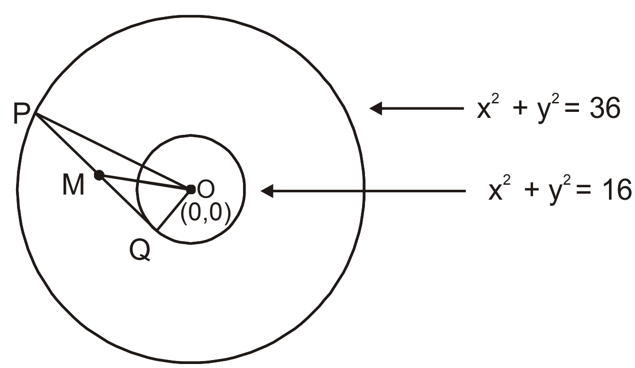
Two circles intersects each other orthogonally.

$\therefore$ Centres of the given circles will be diametric ends of circumcircle of $\square$ ABCD.

$\therefore$ Diameter = $\sqrt{3^2+4^2} = 5$

72. (A)

M is mid point of PQ



In ΔOPQ

$$OP^2 + OQ^2 = 2\left(OM^2 + PM^2\right)$$

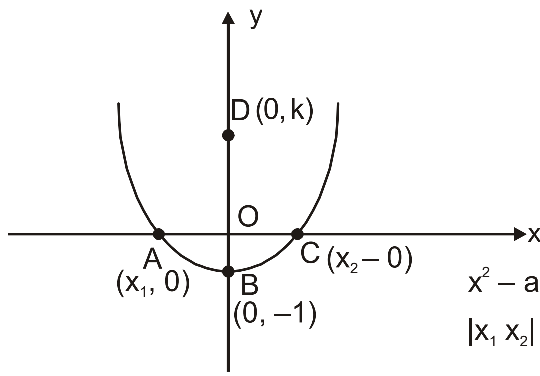
$$\Rightarrow 36 + 16 = 2\left(OM^2 + 4\right)$$

$$\Rightarrow OM = \sqrt{22}$$

$\Rightarrow$ Locus of M is circle

$\because$ Its distance from a fixed point (origin) is constant and this constant distance is radius

73. (D)



$x^2 - ax - 1 = 0$ with roots x_1, x_2

$|x_1 x_2| = 1$

$\because$ ABCD are Concyclic

$\therefore$ OA . OC = OB. OD

$|x_1||x_2| = 1.k$

$\boxed{\Rightarrow\ k = 1}$

74. (B)

Let $h(x) = f(x) - 4g(x)$

Applying LMVT over (0, 1)

$$h'(c) = \frac{h(1) - h(0)}{1 - 0}$$

$$f'(c) - 4g'(c) = \frac{((f(1)) - 4\,g(1)) - (f(0) - 4g(0))}{1}$$

$0 = 8 - 4\,g\,(1) - 0 \qquad \{\because\ \text{Given } f'(c) = 4g'(c)\}$

$\therefore$ g(1) = 2

75. (C)

at $x = 0$

$y = 1$

$y = e^{2x} + x^3$

$$\frac{dy}{dx} = 2e^{2x} + 3x^2$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 2$$

$\therefore$ EOT at x = 0; y – 1 = 2 (x – 0)

$$\boxed{y-2x-1=0}$$

Distance from origin = $\left|\dfrac{0-1}{\sqrt{1+4}}\right| = \dfrac{1}{\sqrt{5}}$

76. (C)

f(g(x)) = x

$$f'(g(x)).g'(x)=1$$

$$\therefore \quad g'(3)=\frac{1}{f'(g(3))}$$

$$=\frac{1}{f'(0)} \qquad \left\{\begin{matrix}\because\ f(0)=3\\ \therefore\ g(3)=0\end{matrix}\right\}$$

$$=\frac{1}{\left(\frac{1}{3}\right)} \quad \begin{cases}\because\ f'(x)=5x^4+\frac{1}{3}e^{\frac{x}{3}}\\ f'(0)=\frac{1}{3}\end{cases}$$

= 3

77. (C)

$$g'(x)=2f\left(5-3f(x)\right).f'\left(5-3f(x)\right).\left(-3f'(x)\right)$$

$$g'(2)=2f\left(2\right).f'\left(2\right).\left(-3f'(2)\right)$$

$$=\ 2.1.\left(-1\right).-3.\left(-1\right)$$

= –6

78. (B)

Clearly $\alpha=e^{\frac{1}{2}}$

and $\beta=\dfrac{1+2+3}{3}=2$

$$\therefore \quad \ln\left(\alpha^{\beta}\right)=\ln\left(\left(e^{\frac{1}{2}}\right)^2\right)$$

= 1

79. (B)

$$\lim_{x\to\infty}\left(\frac{2x^2+3x+1}{x+2}-ax-b\right)=5$$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{2x^2+3x+1-ax^2-bx-2ax-2b}{x+2}\right)=5$$

$$\Rightarrow \lim_{x\to\infty}\frac{(2-a)x^2+(3-b-2a)x+1-2b}{x+2}=5$$

$\therefore$ 2 – a = 0

$\Rightarrow$ a = 2

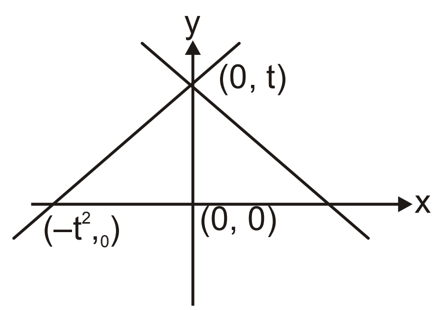
3 – b – 2a = 5

b = – 2 – 2a

= –6

80. (A)

EOT : $yt = x + t^2$



$$\text{Centroid} \equiv (h, k) \equiv \left(\frac{-t^2}{3}, \frac{t}{3}\right)$$

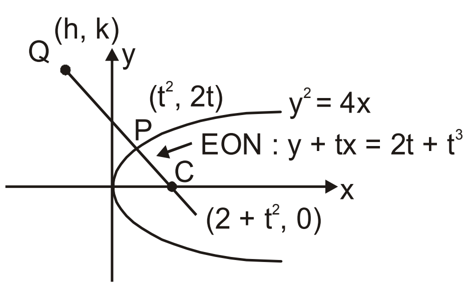
$h = \dfrac{-t^2}{3}$ $\qquad$ $K = \dfrac{t}{3}$

$\Rightarrow t^2 = -3h$ $\qquad$ $t = 3K$

$\therefore 9K^2 = -3h$

$\therefore$ Locus : $3y^2 = -x$

81. (B)



$$h = 2t^2 - 2 - t^2 = t^2 - 2$$

$K = 4t \;\Rightarrow\; t = \frac{K}{4}$

$\therefore \quad h = \frac{K^2}{16} - 2$

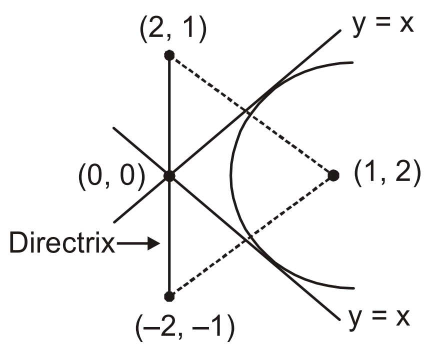
$\Rightarrow K^2 = 16\,(h + 2)$

$\therefore$ Locus $y^2 = 16\,(x + 2)$

focus = (2, 0)

82. (A)

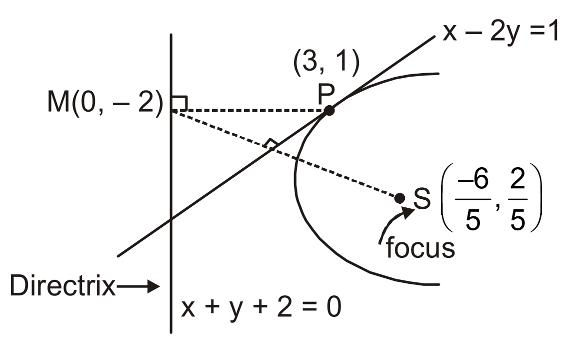
Image of focus w.r. to any tangent of the parabola, lies on it directrix.



Eqn of directrix : $y = \frac{1}{2}x$

$= 2y - x = 0$

83. (A)



$\therefore$ Semi latus Rectum = $\left|\frac{-\frac{6}{5} + \frac{2}{5} + 2}{\sqrt{2}}\right| = \frac{6}{5\sqrt{2}}$

84. (C)

$$f(0)=\underset{x\to 0}{\mathrm{Lt}}\, f(x)=\underset{x\to 0}{\mathrm{Lt}}\,\frac{e^{3x}-1-3x}{3x\left(e^{3x}-1\right)}$$

$$=\underset{x\to 0}{\mathrm{Lt}}\,\frac{\left(1+3x+\frac{(3x)^2}{2!}\frac{(3x)^3}{3!}+....\right)-1-3x}{3x\left(1+3x+\frac{(3x)^2}{2!}+....-1\right)}$$

$$=\frac{1}{2}$$

85. (B)

$$\sin^{-1}(\sin x)+\frac{x}{16}=\frac{1}{2}$$

$$\sin^{-1}(\sin x)=\frac{1}{2}-\frac{x}{16}$$

$$f(x)=\sin^{-1}(\sin x)$$

$$g(x)=\frac{1}{2}-\frac{x}{16}$$

Draw the graph of both functions, and count the point of intersection to get the number of sol^{n}.

86. (B)

$$f(x)=\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)\quad \begin{cases} x\in(\pi,2\pi) \\ \Rightarrow \frac{x}{2}\in\left(\frac{\pi}{2},\pi\right)\end{cases}$$

$$=\tan^{-1}\left(\left|\tan\frac{x}{2}\right|\right)$$

$$=\tan^{-1}\left(-\tan\frac{x}{2}\right)$$

$$=-\tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$=-\left(\frac{x}{2}-\pi\right)=\pi-\frac{x}{2}$$

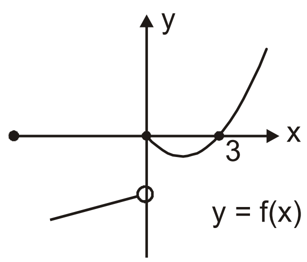
87. (C)

Area of ΔOAB will be least only when P (2, 4) is mid-point of AB.

88. (C)

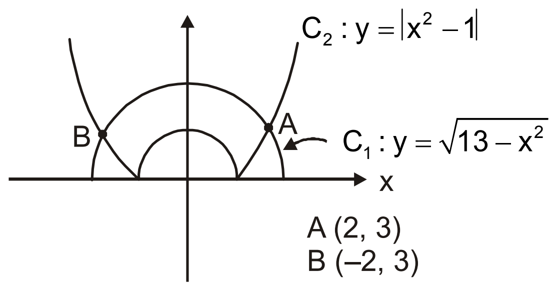
Draw the graph

89. (C)



Use transformation of Graph and check the correct option.

90. (A)



$$m_1 = \left.\frac{dy}{dx}\right|_A^{C_1} = \frac{1}{2\sqrt{13-4}}\,.\,(-2).2 = \frac{-2}{3}$$

$$m_2 = \left.\frac{dy}{dx}\right|_A^{C_2} = 2.2 = 4$$

Let θ be the acute angle bet^n tangent at

$$\tan\theta = \left|\frac{m_1 - m_1}{1 + m_1 m_2}\right| = \left|\frac{-\frac{2}{3} - 4}{1 + \left(\frac{-2}{3}\right)4}\right| = \frac{14}{5}$$