SOLUTIONS

PROGRESS TEST-3

GZRS-1902, GZR-1913 To 1917

GZRK-1905-1906

JEE ADVANCED PATTERN

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CHEMISTRY

1. **(D)**

% of H = 7.7

Assume % of C, O, S be 3x, 2x & 4x respectively

Hence, 3x + 2x + 4x + 7.7 = 100

x = 10.256

Element	%	% At.wt	Simplest Ratio
H	7.7	7.7	6
C	30.767	2.56	2
O	20.511	1.28	1
S	41.022	1.28	1

CF = C_2H_6SO

Mix M.w. = 78

2. **(A)**

Orbital angular momentum = $\frac{h}{2\pi}\sqrt{\ell(\ell+1)}$

For d – orbital $\ell = 2$

$\therefore$ Orbital angular momentum = $\sqrt{6}\times\frac{h}{2\pi}$

3. **(B)**

$$\lambda = \frac{2\pi r}{3} = \frac{2\pi x(3)^2}{3} = 6\pi x$$

4. **(C)**

Energy of 1 quantum = hv

$= 6.626\times10^{-34}\times7.5\times10^{14}$ J

Energy absorbed by 1.5 mole

$1.5\times6.023\times10^{23}\times hv$

$= 4.48\times10^{5}$ J

5. **(B)**

$$E = \frac{hc}{\lambda}$$

$= hc \times (\text{wave no.})$

$= 6.626 \times 10^{-34} \times 3 \times 10^{8} \times 5 \times 10^{5}\ J$

$= 9.93 \times 10^{-20}\ J$

$= 9.93 \times 10^{-23}\ kJ$

6. **(A)**

Chales law

$V \propto T$ at constant P for fixed amount of gas

$\therefore \quad V = kT$ at constant P

$\left(\frac{dV}{dT}\right)_P = K$

7. **(D)**

P = $\frac{C}{V}$ Where C is a constant.

P vs $\frac{1}{V}$ would be a straight line, p vs v will be rectangular hyperbola not straight line

(B) is true because log P = log C – log V.

8. **(A)**

Oleum is SO_3 + H_2SO_4

$SO_3 + \underset{9g}{H_2O} \rightarrow H_2SO_4$

$n_{SO_3} = n_{H_2O}$

$\frac{wt}{80} = \frac{9}{80} \Rightarrow wt_{SO_3} = 40\,g$ in 100g oleum

9. **(B, C)**

3 mole C_2H_6

$$\frac{+\frac{60}{30} = 2 \text{ mole } C_2H_6}{5 \text{ mole } C_2H_6} - \frac{2.4 \times 10^{24}}{6 \times 10^{23}}$$

(5–4) mole C_2H_6 = 1 mole C_2H_6

$$C_2H_6 + \frac{7}{2}O_2 \rightarrow 2CO_2 + 3H_2O$$

$$V_{CO_2} = 2 \times 22.4 = 44.8 \text{ lit}$$

$$M_{H_2O} = 3 \times 18 = 54\,g$$

10. (B,C,D)

$$V = 2.18 \times 10^6 \times \left(\frac{z}{h}\right)$$

$$x = 0.529 \times \frac{h^2}{z}$$

$$KE = 13.6 \times \frac{z^2}{n^2}$$

$$PE = -27.2 \times \frac{z^2}{n^2}$$

11. (A, C)

$$[\text{Cation}] = \frac{1 \times 100 + 3 \times 100 + 1 \times 200}{400} = 6/4 = 3/2\,M$$

$$[\text{Anion}] = \frac{1 \times 100 + 3 \times 100 + 2 \times 1 \times 200}{400} = 2\,M$$

12. (A, C)

At constant T (for fixed amount of gas)

PV = constant.

Also, $d = \frac{PM}{RT}$

$$\frac{d}{P} = \frac{M}{RT} = \text{constant}$$

13. (8)

BE = 246.5 kJ/mole = 2.56 eV/molecule

$$\text{Ephoton} = \frac{12400}{4450} = 2.78\,eV$$

KE of iodine atom = 2.78–2.56 = 0.22.

$\% = \dfrac{0.22}{2.78} \times 100 = 8$

14. (8)

$\text{Molality} = \dfrac{3.2}{1 \times 0.4} = 8\text{ m}$

15. (5)

$M_f = 2.6 = \dfrac{x \times 5 + 20 \times 2}{(20 + x)} = \dfrac{40 + 5x}{20 + x} \Rightarrow x = 5$

16. (9)

No. of orbital $= n^2 = 3^2 = 9$

with $m_s = -1/2$, maximum no. of e^-s = 9

17. (3)

$\Delta E = E_3 - E_2 = 1.9\,\text{eV}.z^2 = 17$

$(z^2 = 3)$

18. (3)

No. of 9–atom of C = no. of mole of CO_2 = $\dfrac{132}{44} = 3$

19. (A) - S; (B) - R; (C) - Q; (D) - P

$K \cdot E \cdot \Big|_{n=2}^{He^+} = 13.6\ \text{eV}$

$\lambda = \dfrac{h}{\sqrt{2mK \cdot E}} = \sqrt{\dfrac{h^2}{2mK \cdot E}} = \sqrt{\dfrac{150}{13.6}}\ A^\circ$

20. A $\rightarrow$ P; B $\rightarrow$ P, Q, S; C $\rightarrow$ Q, S; D $\rightarrow$ P, R

A, B has 3 radial nodes :

In 'A' φ is non–zero at r = 0, so it is 4s–orbital.

$\Rightarrow$ A : P

B : P, Q,S because both 4s, $5p_x$ and $6d_{xy}$ has 3 radial node

C : p & d – orbitals have angular dependence

$\Rightarrow$ C : Q, S

D : s–orbital has no. angular dependence

$\Rightarrow$ D : P, R.

PHYSICS

21. **(B)**

22. **[C]**

$a_1 = a_2 = 1$ and

$a_1^2 + a_2^2 + 2a_1a_2 \cos\theta = (\sqrt{3})^2 = 3$

Or $1 + 1 + 2\cos\theta = 3$ or $\cos\theta = \frac{1}{2}$

Now $(\vec{a}_1 - \vec{a}_2)\ (2\vec{a}_1 + \vec{a}_2) = 2a_1^2 - a_2^2 - a_1a_2 \cos\theta$

$= 2 - 1 - \frac{1}{2} = \frac{1}{2}$

23. **(C)**

24. **(b)**

$$\frac{\Delta T}{T} \times 100 = \frac{\frac{1}{5}}{25} \times 100 = 0.8\%$$

25. **(a)**

Dimension of $\frac{a}{V^2}$ must be that of pressure

$$\frac{MLT^{-2}}{L^2} = \frac{a}{L^6}$$

$$a = ML^5T^{-2}$$

26. **(C)**

$mg - B = mf$

$\therefore \quad B - (m - m')g = (m - m')f$

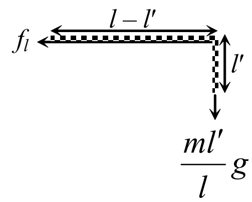
$\Rightarrow \quad m'g = (2m - m')f \quad \Rightarrow m' = \frac{2mf}{g + f}$

$\therefore \quad \Rightarrow \quad w' = \frac{2wf}{g + f}$

27. **(C)**

Frictional force will balance the weight of hanging portion of rope. $\frac{ml'}{l}g = \mu \frac{m}{l}(l - l')g$

$l' = \mu l - \mu l'$

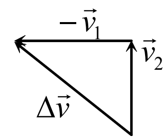


$l' = \frac{\mu l}{1 + \mu}$

28. **(b)**

$|\Delta \vec{v}| = 5\,m/s$

$\vec{a}$ **= 1** m/s^2 (towards north–west)



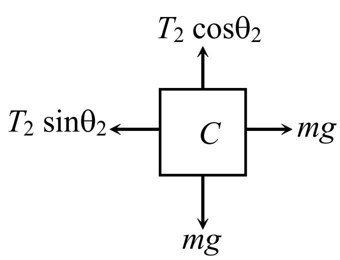
29. **(A, C, D)**

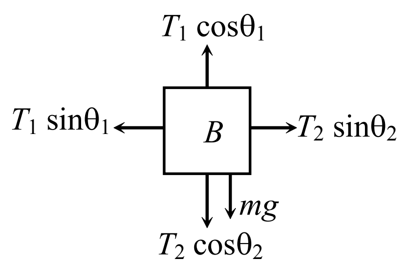
$T_2 \sin\theta_2 = mg$ … (i)

$T_2 \cos\theta_2 = mg$ …(ii)

$T_1 \cos\theta_1 = mg + T_2 \cos\theta_2$

$T_1 \sin\theta_1 = T_2 \sin\theta_2$





30. **(c, d)**

Resultant force may not be zero for coplanar forces. Hence (a) is not true

Since magnitudes are not equal (b) can not be true. In (c) and (d), net force is zero

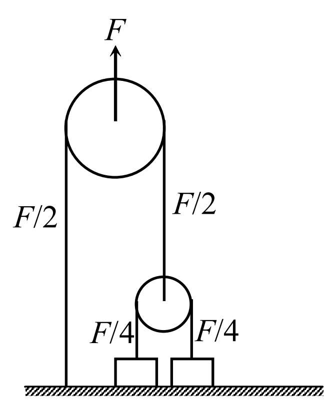
31. **(A) (B) (C)**

$a_1 > 0$ when $\frac{F}{4} > 50$, $F > 200$

$a_2 > 0$ when $\frac{F}{4} > 100$, $F > 400$

F = 300 N

$a_1 = \frac{F/4 - 50}{5} = \frac{300/4 - 50}{5} =$ 5 m/s^2



$a_2 = 0$

If $F = 500$ N

$a_1 = 15$ m/s^2 , $a_2 = 2.5$ m/s^2

32. (A), (B)

$$\Delta u = \left(u^2 + u^2 + 2u^2 \cos 120^0\right)^{\frac{1}{2}}$$

$$\Delta u = u = 60 \text{ ms}^{-1}$$

$$\text{Average acceleration} = \frac{\Delta u}{t} = \frac{u}{\frac{r\theta}{4}} = 11.5 \text{ ms}^{-2}$$

$$\text{Instantaneous acceleration} = \frac{u^2}{r} = 12 \text{ ms}^{-2}$$

33. (4)

$\because \quad \sin x + \cos x = \sqrt{2}$

on squaring $\Rightarrow \sin^2 x + \cos^2 x + 2\sin x \cos x = 2$

$\sin 2x = 1$ ....(1)

and $\dfrac{1}{\sin^6 x + \cos^6 x} = \dfrac{1}{1 - 3\sin^2 x \cos^2 x}$

$= \dfrac{1}{1 - \frac{3}{4}\sin^2 2x} = \dfrac{1}{1 - \frac{3}{4}(1)^2}$ using eq^n .... (1)

$= \dfrac{4}{4-3} = 4$

34. (9)

$y = \sqrt{3 + x^2}$

$$\frac{dy}{dx} = \frac{x}{\sqrt{3 + x^2}}$$

$$\frac{d^2y}{dx^2} = \frac{\sqrt{(3 + x^2)} - \frac{x^2}{\sqrt{3 + x^2}}}{(3 + x^2)}$$

$$= \frac{3 + x^2 - x^2}{(3 + x^2)^{3/2}} = \frac{3}{(3 + x^2)^{3/2}}$$

$$\left(\frac{d^2y}{dx^2}\right) = \frac{3}{(3+6)^{3/2}} = \frac{3}{27}$$

$$\frac{1}{\frac{d^2y}{dx^2}} = 9$$

35. (2)

$$y = \frac{x^3}{3} - \frac{5}{2}x^2 + 6x + 4$$

$$\frac{dy}{dx} = x^2 - 5x + 6 \text{ ....(1)}$$

if $x^2 - 5x + 6 = 0$

x = 2, 3

Again diff. (1)

$$\frac{d^2y}{dx^2} = 2x - 5$$

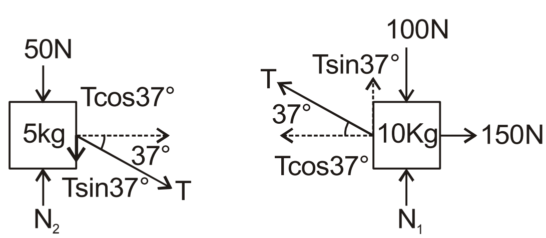
at x= 2 $\quad \frac{d^2y}{dx^2} = -1 < 0$

so maximum at x = 2.

36. (2)

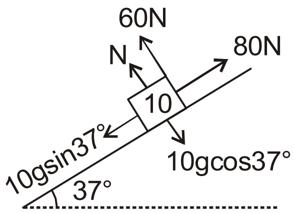
$a = 10ms^{-2}$

150 – Tcos37° = 10a



$\therefore \quad T = \frac{125}{2} N$

37. (2)



$$a = \frac{80N - 100\sin 37°}{10} = \frac{80-60}{10} = 2ms^{-2}$$

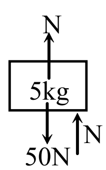
38. (1)

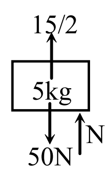
$\int_1^2 ydx =$ area under the graph = $\frac{1}{2} \times 2 \times 1 = 1$ **a**

39. (A – q) ; (B – s) ; (C – p) ; (D – r)

No force is acting on 3 kg block.

$\frac{15}{3}$ **=** 5 m/s^2





N = 50 N

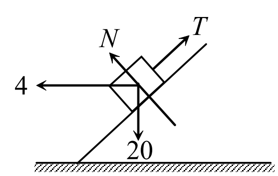
N = 50 –15/2 = 42.5 N

40. (A – q) ; (B – p) ; (C – r) ; (D – s)

Acceleration of 2kg relative to wedge = 2 m/s^2

Acceleration of 2 kg relative to ground

= $\sqrt{2^2 + 2^2 + 2\times 2\times \cos 120°}$ = 2 m/s^2



$$20\sin 60° + 4\cos 60° - T = 4$$

$N = 20\cos 60° - 4\sin 60° = 10 - 2\sqrt{3}$ N

Net force = 2 × 2 = 4 N

MATHEMATICS

41. (C)

a, ar, ar^2, ar^3 (G.P.)

$a-2, ar-7, ar^2-9, ar^3-5$ (A.P.)

$\therefore\ 2(ar-7) = (a-2) + (ar^2-9)$

$\Rightarrow\ 2ar - 14 = a(1+r^2) - 11$

$\Rightarrow\ a(1-r)(r-1) = 3$ ....(i)

Also $2(ar^2-9) = (ar-7) + (ar^3-5)$

$\Rightarrow\ 2ar^2 - 18 = ar(1+r^2) - 12$

$\Rightarrow\ a.r(r-1)(1-r) = 6$ ....(ii)

From (i) & (ii), $r = 2$ and $a = -3$

$\therefore$ third term of A. P. $= ar^2 - 9 = (-3).(2)^2 - 9 = -12 - 9 = -21$

42. (B)

$$x = \frac{z^{1/3}}{2}, y = \frac{z^{1/6}}{5}; \text{if } xy = z^{3/2}; \frac{z^{1/3}}{2}.\frac{z^{1/6}}{5} = z^{3/2} \Rightarrow z = \frac{1}{10}$$

43. (C)

Let $s = \cos 0 + \cos 2\theta + \cos 4\theta + ... + \cos 10\theta$, then

$$2\sin\theta.s = 2\sin\theta[\cos 0 + \cos 2\theta + \cos 4\theta + ... + \cos 10\theta]$$

$$= (\sin\theta + \sin\theta) + (\sin 3\theta - \sin\theta) + ... + (\sin 11\theta - \sin 9\theta)$$

$$\Rightarrow 2\sin\theta \cdot s = (\sin\theta + \sin 11\theta)$$

or, $2\sin\theta \cdot s = 2\sin 6\theta \cdot \sin 5\theta$

$$\therefore s = \frac{2\sin 6\theta \cdot \cos 5\theta}{2\sin\theta} = \frac{\sin n\theta \cdot \cos m\theta}{\sin\theta}$$

$\therefore\ n = 6, m = 5$

44. (C)

$$\cos\frac{\pi}{7}.\cos\frac{2\pi}{7} \cdot \cos\frac{4\pi}{7} - \cos\left(\frac{\pi}{2} - \frac{\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{5\pi}{14}\right)$$

$$= \cos\frac{\pi}{7} \cdot \cos\frac{2\pi}{7} \cdot \cos\frac{4\pi}{7} - \cos\frac{3\pi}{7}\cos\frac{2\pi}{7} \cdot \cos\frac{\pi}{7}$$

$$= 2\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$$

$$= 2\frac{\sin\frac{8\pi}{7}}{2^3\sin\frac{\pi}{7}} = -\frac{1}{4}$$

45. (B)

$A.M \geq G.M$

$\frac{a+b+c}{3} \geq (abc)^{\frac{1}{3}}$; for (a, b, c > 0)

$\Rightarrow\ a+b+c \geq 3(abc)^{\frac{1}{3}}$

but given ab^2c^3, $a^2b^3c^4$, $a^3b^4c^5$ are in A.P

Hence $2abc = 1 + a^2b^2c^2 \Rightarrow (abc-1)^2 = 0 \Rightarrow abc = 1$

hence minimum value of

$a+b+c = 3(abc)^{\frac{1}{3}} = 3.(1)^{\frac{1}{3}} = 3$

46. (A)

The given expression is

$$1+2\sin 3x \sin 2x + \frac{1-\cos 4x}{2} + \frac{1-\cos 6x}{2}$$

$$\Rightarrow 1+2\sin 3x \sin 2x + \sin^2 2x + \sin^2 3x$$

$$\Rightarrow 1+\left(\sin 2x + \sin 3x\right)^2$$

Thus least value is 1

47. (A)

$$S_n = \sum_{r=1}^{n}(r^2-r+1)(r!)$$

$$T_r = (r^2-r+1)(r!) = \left[(r^2-1)-(r-2)\right]r!$$

$$= (r-1).(r+1)! - (r-2).r!$$

$$\therefore\ T_1 = (1-1).(1+1)! - (1-2)1!$$

$$T_2 = (2-1).(2+1)! - (2-2)2!$$

$$T_3 = (3-1).(3+1)! - (3-2).3!$$

. . . .

$$T_{n-1} = (n-2).n! - (n-3).(n-1)!$$

$T_n = (n-1).(n+1)! \; - (n-2).n!$

$\therefore \; S_n = (n-1).(n+1)! + 1 = \; 1 + (n-1)!(n-1)(n+1)n$

$= 1 + (n-1)!(n^3 - n)$

48. (B)

$B \cap (A \cup B) = B$

49 (A, C, D)

$$T_r = \frac{r\sqrt{r+1} - (r+1)\sqrt{r}}{r^2(r+1) - (r+1)^2 r} \; = \frac{(r+1)\sqrt{r} - r\sqrt{r+1}}{r(r+1)} = \frac{1}{\sqrt{r}} - \frac{1}{\sqrt{r+1}}$$

$$\frac{T_r}{T_{r+1}} = \frac{\sqrt{r+1} - \sqrt{r}}{\sqrt{r+2} - \sqrt{r+1}} \cdot \frac{\sqrt{r+2}}{\sqrt{r}} \; = \frac{\sqrt{r+2} + \sqrt{r+1}}{\sqrt{r+1} + \sqrt{r}} \cdot \frac{\sqrt{r+2}}{\sqrt{r}} > 1 \Rightarrow T_r > T_{r+1}$$

$$\sum_{r=1}^{n} T_r = \left(\frac{1}{1} - \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right) + ... + \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right) = 1 - \frac{1}{\sqrt{n+1}} < 1$$

50. (B, D)

51. (B, D)

Dividing by $\cos(2012^\circ)$, we get

$$\tan\theta = \frac{1 + \tan 2012^\circ}{1 - \tan 2012^\circ}$$

$\Rightarrow \tan\theta = \tan(2012^\circ + 45^\circ) = \tan 2057^\circ$

$\Rightarrow$ Hence θ = k (180^0) + 2057^0

Put k = –10

θ = 2057º – 1800º = 257º

If k = –11 $\Rightarrow$ θ = 77º

52. (A, B, D)

$$P(x) = \left(1 + \cos\frac{\pi}{6x}\right)\left(1 + \sin\frac{\pi}{6x}\right)\left(1 - \sin\frac{\pi}{6x}\right)\left(1 - \cos\frac{\pi}{6x}\right)$$

$$P(x) = \frac{1}{4}\sin^2\left(\frac{\pi}{3x}\right)$$

53. (1)

$$x=\frac{1}{9}(999....9)=\frac{1}{9}\left(10^{20}-1\right);\ y=\frac{1}{3}(999....9)=\frac{1}{3}\left(10^{10}-1\right);\ z=\frac{2}{9}(999....9)=\frac{2}{9}\left(10^{10}-1\right)$$

54. (4)

If A + B = 45^0 then (1+ tanA)(1+ tanB) = 2

55. (3)

56. (4)

57. (1)

3x, 4y, 5z are in G.P.

$\therefore$ $16y^2 = 15xz$ ....(1)

and $y = \dfrac{2xz}{x+z}$ ....(2)

using (2) in (1)

$16 \times 4x^2z^2 = 15(x + z)^2xz$

$$\frac{(x+z)^2}{xz} = \frac{64}{15};\ \frac{x}{z}+\frac{z}{x}+2=\frac{64}{15};\qquad \therefore\ \frac{x}{z}+\frac{z}{x}=\frac{34}{15}$$

$\therefore$ m = 34 and n = 15

$\therefore$ m + n = 34 + 15 = 49

58. (4)

$2\sin^2 x + 5\sin x - 3 = 0$

$$\Rightarrow \sin x = -3, \frac{1}{2}$$

$$x=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6},\frac{17\pi}{6}$$

59. (A$\rightarrow$p; B$\rightarrow$p; C$\rightarrow$p; D$\rightarrow$p)

$$f_n(\theta)=\tan\frac{\theta}{2}\cdot\frac{2\cos^2\frac{\theta}{2}}{\cos\theta}\cdot\frac{2\cos^2\theta}{\cos 2\theta}\cdot....\frac{2\cos^2 2^{n-1}\theta}{\cos 2^n\theta}$$

$$=\frac{\sin\theta}{\cos 2^n\theta}\cdot 2^n\cdot\left\{\frac{2\sin\theta.\cos\theta.\cos 2\theta\ldots\cdot\cos 2^{n-1}\theta}{2\sin\theta}\right\}$$

$$= \frac{\sin\theta}{\cos 2^n\theta} \cdot 2^n \cdot \frac{\sin 2^n\theta}{2^n \sin\theta} = \tan 2^n\theta$$

60. ($A \to s$; $B \to r$; $C \to q$; $D \to p$)

(B) $\left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x \geq 1$

$\therefore \cos^x \alpha + \sin^x \alpha \geq 1$, where $\cos\alpha = \frac{5}{13}$

Equally holds for x = 2. If x < 2, both cos α sin α increase (being positive fractions). So, $\cos^x \alpha + \sin^x \alpha > 1$ if x < 2. Thus $x \leq 2$.

(C) $\frac{x^2 - 4x + 3}{x + 1} > 0 \Rightarrow (x-1)(x-3)(x+1) > 0, x \neq -1.$

(D) Solve the inequation $x^2 - 3x + 2 \leq 0$ and $2x^2 - 3x - 5 \geq 0$. and take intersection of both.