SOLUTIONS

PROGRESS TEST-2

GZ-1926 & GZK-1909

JEE ADVANCED PATTERN

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CHEMISTRY

1. (B)

$$\tan\theta = 1.5\times10^{22} = \frac{\text{No. of atom}}{\text{wt}}$$

$$\frac{N_A}{\text{at.wt.}} = 1.5\times10^{22}$$

$$\text{At.wt.} = \frac{6.023\times10^{23}}{1.5\times10^{22}}$$

$$= 4\times10 = 40$$

2. (B)

$$w_{A\ell F_3} = 4.2\text{g}$$

$$\text{mole of } A\ell F_3 = \frac{4.2}{84} = 0.05 \text{ mole}$$

$$= 1.5 \text{ mole } F^-$$

3. (A)

$$22.4\text{L of gas at STP} \equiv 6.02\times10^{23}$$

$$22400 \text{ mL of gas at STP} \equiv 6.023\times10^{23}$$

$$1\text{mL of gas at STP} \equiv \frac{6.023\times10^{23}}{22400} = 2.7\times10^{19}$$

4. (B)

$$\text{Mole} = \frac{448}{22400} = \frac{2}{3a}$$

$$a = \frac{2\times22400}{3\times448}\text{amu}$$

= 33.33 amu

= $33.33 \times 1.66 \times 10^{-24}$ g

5. (D)

$$\frac{N_{(\alpha,60^\circ)}}{N_{(\alpha,90^\circ)}} = \frac{\text{Sin}^4\left(\frac{90^\circ}{2}\right)}{\text{Sin}^4\left(\frac{60^\circ}{2}\right)} \Rightarrow \frac{12}{N_{(\alpha,90^\circ)}} = \frac{\left(\frac{1}{\sqrt{2}}\right)^4}{\left(\frac{1}{2}\right)^4} = 4$$

$$\therefore N_{(\alpha,90^\circ)} = 3$$

6. (B)

$$E_P = E \times 3^2 + \frac{1}{2}mv^2$$

$$\therefore V = \sqrt{\frac{2(E_P - 9E)}{m}}$$

7. (B)

Stopping potential = 0.5 ev $\Rightarrow$ K.E.=0.5 ev

$$E_{Photon} = \frac{hc}{\lambda} = \frac{1240}{265} = 4.68\,ev$$

$\therefore$ Work function = 4.68–0.5 = 4.18 ev

8. (D)

$$E = \frac{nhc}{\lambda}$$

or, $$40 \times 60 \times \frac{50}{100} \times \frac{1}{1.6 \times 10^{-19}} = \frac{n \times 12400}{6200}$$

$\therefore\; n = 3.75 \times 10^{21}$

9. (A), (B), (C), (D)

Molarity = 2M →
- n_{solute} = 2 mole
- $V_{solution}$ = 1 L = 1000 mL
- $d_{solution}$ = 1.20 g/mL
- $w_{solution}$ = 1200 g
- w_{solute} = 2 × 60 g
- $w_{solvent}$ = 1080 g

Molality = $\frac{2}{1080} \times 1000m$

Xsolute = $\frac{2}{2 + \frac{1080}{18}}$

$$\%\frac{w}{w} = \frac{120}{1200} \times 100 = 10\%$$

$$\%\frac{w}{v} = \frac{120}{1000} \times 100 = 12\%$$

10. (B), (C), (D)

$$M_R = \frac{\frac{11.2}{11.2}\times 100 + \frac{22.4}{11.2}\times 200 + \frac{33.6}{11.2}\times 200}{1000} = \frac{1100}{1000}$$

= 1.1 M = 1.1 mol/L = 1.1 × 34 g/L

V.S = 1.1 × 11.2 = 12.32 V

11. (B,D)

$$\nu = \frac{c}{\lambda} = \frac{3\times 10^{8}}{600\times 10^{-9}} = 5\times 10^{14}\ \text{sec}^{-1}$$

$$\text{Energy of photon} = \frac{12400}{6000} = 2.07\,\text{ev}$$

$$\text{Wave number} = \frac{1}{\lambda} = \frac{1}{600\times 10^{-9}} = 1.67\times 10^{6}\ \text{m}^{-1}$$

12. (A,B,C,D)

$$\frac{hc}{\lambda} = \phi + kE_{max}$$

$$\text{or, } \frac{1240}{280} = 2.5 + kE_{max}$$

$$\therefore kE_{max} = 1.93\,\text{ev}$$

and stopping potential = 1.93 volts

13. (1)

$$x_{NaOH} = .01; n_{NaOH} = 1\text{mole} : w_{NaOH} = 40g$$

$$d_{solution} = 1.4\,g/mL;\ w_{solution} = 202g$$

$$v_{solution} = \frac{202}{1.4}mL$$

$$\text{Molarity} = \frac{1}{\left(\frac{202}{1.4}\right)}\times 1000 = 6.93M \qquad \text{or} \qquad \frac{6.43}{6.93} = 1$$

14. (4)

Molarity = xM and molality = xm

Molarity = xM → n_{urea} = x mole

→ $v_{solution}$ = 1000 mL

$d_{solution}$ = d g/mL

$w_{solution}$ = 1000 dg

w_{urea} = 60 x

$w_{solvent}$ = (1000 – 60x)g

$$\therefore \text{ Molality } = \frac{x}{(1000d - 60x)} \times 1000 = xm$$

If x = 1, 1000 = 1000 d – 60

$$d = \frac{1060}{1060} = 1.06\,g/mL \qquad \text{or,} \quad \frac{1.06}{6.265} = 4$$

15. (4)

$$\underset{xmL,0.8M}{AlCl_3} + \underset{50mL,0.2M}{CaCl_2}$$

$$[d–] = \frac{3 \times 0.8 \times x + 2 \times 0.2 \times 50}{50 + x} = 0.6$$

$$2.4x + 20 = 30 + 0.6x$$

1.8 x = 10 ; x = 5.56

$$\frac{5.56}{1.39} = 4$$

16. (2)

HCF of charges = $2 \times 10^{-18} = X \times 10^{-18}$

$\therefore$ X = 2

17. (4)

$$E = \frac{nhc}{\lambda}$$

$$\text{or, } 10^{-17} = \frac{n \times 6.63 \times 10^{-34} \times 3 \times 10^{8}}{550 \times 10^{-9}}$$

$$\therefore n \approx 28$$

$$\therefore \text{Ans } = \frac{28}{7} = 4$$

18. (5)

In 100 gm of dried sample

$\Rightarrow$ 10gm H_2O, 50 gm silica and (100-50-10)=40 gm non volatile impurity

If W gm of original sample contained 50 gm silica and 40 gm non volatile impurities then

W = 0.28 W + 40 + 50

$\therefore W = 125\,gm$

$\therefore \%\text{ of Silica} = \frac{50}{125}\times 100 = 40 = 8X$

$\therefore X = 5$

19. (A) – (Q), (T); (B) – (Q), (S); (C) – (P), (T); (D) – (R), (S)

20. (A) – (Q), (T); (B) – (R); (C) – (S); (D) – (P)

PHYSICS

21. In condition (i), $20g - T = 20a$, $N = 20a$

$T - N = 40a \quad \Rightarrow \quad a = \frac{20g}{80} = \frac{g}{4}$

Net acceleration = $a_1 = a\sqrt{2}$, $\sqrt{2}a = \frac{\sqrt{2}g}{4} = \frac{g}{2\sqrt{2}}$

In condition (ii) $20g - T = 20a$, $T = 40a$, $a = \frac{g}{3}$, $a_2 = \frac{g}{3}$

$\frac{a_1}{a_2} = \frac{g/2\sqrt{2}}{g/3} = \frac{3}{2\sqrt{2}}$

$\therefore$ **(A)**

22. (D)

$a^2 + b^2 - 2ab\cos\theta = 25$ .....(i)

$a^2 + 9b^2 - 6ab\cos\theta = 25$ .....(ii)

and $a^2 + 4b^2 - 4ab\cos\theta = 16$ ....(iii)

$(i)\times 3 - (ii)$

$2a^2 - 6b^2 = 50$ .....(iv)

$(i)\times 2 - (iii)$

$a^2 - 2b^2 = 34$ .....(v)

$(iv) - 2 \times (v)$

$-2b^2 = -18$

$\Rightarrow b^2 = 9 \Rightarrow b = 3$

Putting the value of b^2 is (v)

$a^2 = 52 \Rightarrow a = 2\sqrt{13}$

Now, put the value of a and b in equation (i)

$52 + 9 - 2 \times 2\sqrt{13} \cdot 3\cos\theta = 25$

$\Rightarrow 36 = 12\sqrt{13}\cos\theta$

$\Rightarrow \cos\theta = \frac{3}{\sqrt{13}} \Rightarrow \tan\theta = \frac{2}{3} \qquad \Rightarrow \theta = \tan^{-1}\frac{2}{3}$

23. At equilibrium, let tension in each spring be *T*. Then

$2T\cos 60^\circ = Mg$

$T = Mg$

When right spring breaks, the net force on the block is *T*.

$\therefore \quad a = \frac{T}{M} = 10\ m/s^2$

$\therefore$ **(A)**

24. **(A)**

Let the unit vector is

$x\hat{i} + y\hat{j} + z\hat{k}$

From question

$\cos 45^\circ = \frac{(\hat{i} + \hat{j}) \cdot (x\hat{i} + y\hat{j} + 2\hat{k})}{\sqrt{1^2 + 1^2} \cdot \sqrt{x^2 + y^2 + z^2}}$ and $\cos 60^\circ = \frac{(3\hat{i} - 4\hat{j}) \cdot (x\hat{i} + y\hat{j} + z\hat{k})}{\sqrt{3^2 + 4^2} \cdot \sqrt{x^2 + y^2 + z^2}}$

$\sqrt{x^2 + y^2 + z^2} = 1$ as $x\hat{i} + y\hat{j} + z\hat{k}$ is a unit vector .

so,

$\cos 45^\circ = \frac{(\hat{i} + \hat{j}) \cdot (x\hat{i} + y\hat{j} + 2\hat{k})}{\sqrt{1^2 + 1^2} \cdot \sqrt{x^2 + y^2 + z^2}} \Rightarrow x + y = 1$ ...... (i)

$$\cos 60^{\circ} = \frac{\left(3\hat{i} - 4\hat{j}\right)\cdot\left(x\hat{i} + y\hat{j} + z\hat{k}\right)}{\sqrt{3^2 + 4^2}\cdot\sqrt{x^2 + y^2 + z^2}} \Rightarrow 3x - 4y = \frac{5}{2} \qquad \text{...... (ii)}$$

$(i) \times 4 + (ii)$

$$7x = \frac{13}{2} \Rightarrow x = \frac{13}{14}$$

Placeing, the value of x in (i)

$$y = \frac{1}{14}$$

$$\left(\frac{13}{14}\right)^2 + \left(\frac{1}{14}\right)^2 + z^2 = 1$$

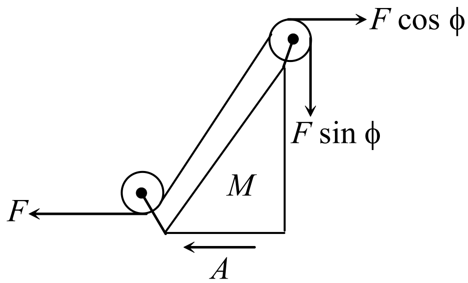
$$\Rightarrow z^2 = 1 - \frac{169}{196} - \frac{1}{196}$$

$$\Rightarrow z = \frac{\pm\sqrt{26}}{14}$$

So, the vector is

$$\frac{13}{14}\hat{i} + \frac{1}{14}\hat{j} \pm \frac{\sqrt{26}}{14}\hat{k}$$

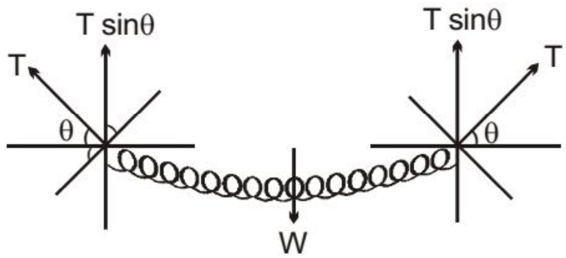
25. $F - F\cos\phi = MA$



$$A = \frac{F - F\cos\phi}{M}$$

$\therefore$ **(B)**

26. (A)

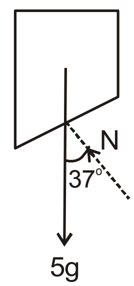


$2T\ \sin\theta = W$

$T = \frac{W}{2}\operatorname{cosec}\theta$

27. (B)

For equilibrium of 5 kg block

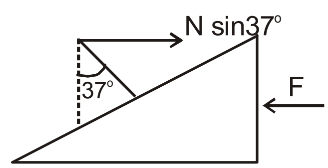


$N\cos 37^\circ = 50$

$\Rightarrow N = 50 \times \frac{5}{4} = 62.5N$

For equilibrium of 10 kg wedge

$N\sin 37^\circ = F$



$\Rightarrow\ 62.5 \times \frac{3}{5} = F$

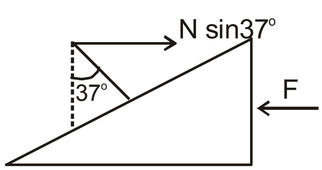
$\Rightarrow\ F = 37.5\ N$

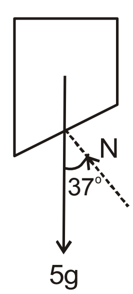
28. (B)

Let acceleration of 2kg block is a vertically upward. Then form wedge (surface) constraint relatin, acceleration of both surface along normal must be same.

So,

$a\cos 37^\circ = 4\sin 37^\circ$





$\Rightarrow a = 4 \times \tan 37^\circ = 3m/s^2$

Now, applying $\vec{F} = \overrightarrow{ma}$ to block and wedge

$N\cos 37^\circ - 2g = 2\times 3$

& $F - N\sin 37^\circ = 5\times 4$

on solving

$F = 39.5N$

29. (B, C) **30. (A, B, D)** **31. (A, B, C)** **32. (A, B, D)**

33. (3)

For the angle 'θ' normal reaction between A & B becomes zero, they ready to seprate. So, solve 'θ' for $N_{AB} = 0$

34. (2)

The resultant of $\vec{a}, \vec{b}$ and $\vec{c}$ is of magnitude $\frac{x}{\sqrt{2}} + x + \frac{x}{\sqrt{2}}$ whcih is equal to the resultant of $\vec{d}$ and $\vec{e}$.

So,

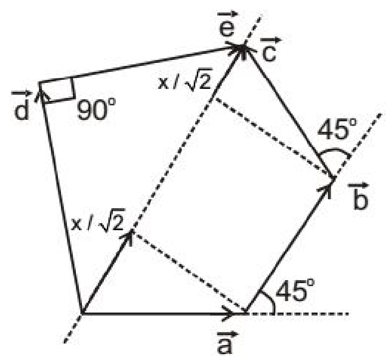
$\sqrt{2}x + x = \sqrt{2}y$

$\Rightarrow y = \left(1 + \frac{1}{\sqrt{2}}x\right)$

$\Rightarrow y = \left(1 + \frac{\sqrt{2}}{2}\right)$

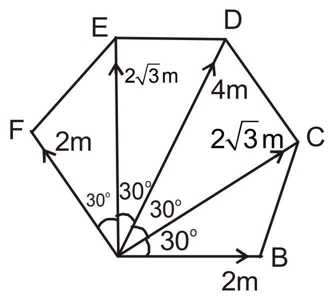
so,

k = 2



35. (8)

$\overrightarrow{AB}\cdot\overrightarrow{AC} + \overrightarrow{AB}\cdot\overrightarrow{AD} + \overrightarrow{AB}\cdot\overrightarrow{AE} + \overrightarrow{AB}\cdot\overrightarrow{AF}$

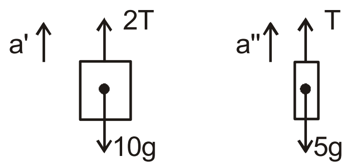


$= 2\times 2\sqrt{3}\times\cos 30^\circ + 2\times 4\times\cos 60^\circ + 2\times 2\sqrt{3}\times\cos 90^\circ + 2\times 2\times\cos/20^\circ$

$= 6 + 4 + 0 - 2 = 8$ m^2

36. (1)

FBD of 'A' and 'B'



As, froces and mass all are in ration 2 : 1 for blck ' A' and 'B'. So, their acceleration will be equal From constain realatio

$a' + a'' = a$

$a' + 2a' = a$

$\Rightarrow a' = 1\ \text{m/s}^2$

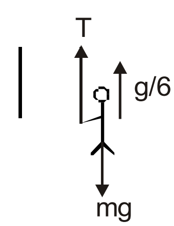
37. (4)

Let the block moves up with acceleration of 'a' m/s^2, then as the person is moving up will acceleration $\frac{g}{6}$ relative to string. Acceleration of person in ground frame will be

$\vec{a}_{pg} = \vec{a}_{ps} + \vec{a}_{sg}$

$\Rightarrow a_{pg} = \left(\frac{-g}{6} + a\right)\downarrow$

So, $mg - T = m\left(a - \frac{g}{6}\right)$ .... (i)



and, for block

$T - \frac{mg}{2} = \frac{m}{2}a$ ....(ii)

(i) + (ii)

$\frac{mg}{2} = \frac{3ma}{2} - \frac{mg}{6}$

$\Rightarrow mg\left(\frac{1}{2} + \frac{1}{6}\right) = \frac{3ma}{2} \Rightarrow a = \frac{4g}{9}$

38. (4)

$$T = \frac{4m_1m_2\,m_3\,g}{4m_1m_2 + m_2m_3 + m_1m_3}$$

39. In condition (A), by constraint relation $\frac{a_1}{a_2} = 4$

In condition (B), by constraint relations $\frac{a_1}{a_2} = \frac{1}{3}$

In condition (A), tension in string of m_2 is $T = \frac{16m_1m_2g}{(16m_1 + m_2)} = 32$ N

In condition (B), tension in string connecting m_2 is $T = \frac{4m_1m_2g}{(m_1 + 9m_2)} = \frac{160}{37}$ N

$\therefore$ **(A – q); (B – p) ; (C – r) ; (D – t)**

40. (A – p) ; (B – r) ; (C – q) ; (D – p)

MATHEMATICS

41. (C)

a, ar, ar^2, ar^3 (G.P.)

$a - 2, ar - 7, ar^2 - 9, ar^3 - 5$ (A.P.)

$\therefore\ 2(ar - 7) = (a - 2) + (ar^2 - 9)$

$\Rightarrow\ 2ar - 14 = a(1 + r^2) - 11$

$\Rightarrow\ a(1 - r)(r - 1) = 3$ ....(i)

Also $2(ar^2 - 9) = (ar - 7) + (ar^3 - 5)$

$\Rightarrow\ 2ar^2 - 18 = ar(1 + r^2) - 12$

$\Rightarrow\ a.r(r - 1)(1 - r) = 6$ ....(ii)

From (i) & (ii), $r = 2$ and $a = -3$

$\therefore$ third term of A. P. $= ar^2 - 9 = (-3).(2)^2 - 9 = -12 - 9 = -21$

42. (B)

$$x = \frac{z^{1/3}}{2}, y = \frac{z^{1/6}}{5}; \text{if } xy = z^{3/2}; \frac{z^{1/3}}{2}.\frac{z^{1/6}}{5} = z^{3/2} \Rightarrow z = \frac{1}{10}$$

43. (C)

$$\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7} - \cos\left(\frac{\pi}{2} - \frac{\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{5\pi}{14}\right)$$

$$= \cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7} - \cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cdot\cos\frac{\pi}{7}$$

$$=2\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$$

$$=2\frac{\sin\frac{8\pi}{7}}{2^3\sin\frac{\pi}{7}}=-\frac{1}{4}$$

44. (B)

$B\cap(A\cup B)=B$

45. (C)

AP I = 12, 15, 18, ... (common difference d_1 = 3)

AP II = 17, 21, 25 ... (common difference d_2 = 4)

First term of the series of common numbers = 21

Here a = 21, common difference of the series of common number = L.C.M of d_1 and d_2 = 12

$\therefore$ Required sum of first hundred terms

$$=\frac{100}{2}[2\times 21+(100-1)\,12]=100\,[21+594]=61500$$

46. (D)

We have, tanA – tanB = x

$$\Rightarrow \frac{1}{\cot A}-\frac{1}{\cot B}=x$$

$$\Rightarrow \frac{\cot B-\cot A}{\cot A\cot B}=x \quad \Rightarrow \quad \cot A\cot B=\frac{y}{x}$$

Now, $\cot(A-B)=\dfrac{\cot A\cdot\cot B+1}{\cot B-\cot A}$

$$=\frac{\frac{y}{x}+1}{y}=\frac{x+y}{xy}=\frac{1}{x}+\frac{1}{y}$$

47. (D)

if a, b, c are in A.P.

$\Rightarrow$ 2b = a + c . ...(i)

$\Rightarrow$ (a + 2b – c) (2b + c – a) (c + a – b)

$\Rightarrow$ (a + a + c – c) (a + c + c – a) (2b – b)

$\Rightarrow$ (2a) (2c)b

$\Rightarrow$ 4abc

48. (D)

We must have $x^2 - 2 > 0, \frac{3}{2}|x| - 1 > 0$ and $x^2 - 2 < \frac{3}{2}|x| - 1$

$\Rightarrow \quad x^2 > 2, |x| > \frac{2}{3}$ and $|x|^2 - \frac{3}{2}|x| - 1 < 0$

$\Rightarrow \quad |x| > \sqrt{2}$ and $\quad |x| < 2$

$\Rightarrow \quad x \in \left(-2, -\sqrt{2}\right) \cup \left(\sqrt{2}, 2\right)$

Hence, (D) is the correct answer.

49. (A, B, D)

$P(x) = \left(1 + \cos\frac{\pi}{6x}\right)\left(1 + \sin\frac{\pi}{6x}\right)\left(1 - \sin\frac{\pi}{6x}\right)\left(1 - \cos\frac{\pi}{6x}\right)$

$P(x) = \frac{1}{4}\sin^2\left(\frac{\pi}{3x}\right)$

50. (A, B)

a = a ; b = ar ; c = ar^2

alsoa, x, b in A.P and b y c in A.P.

2x = a + b = a(1 + r)....(1)

2y = b + c = ar(1 + r)....(2)

now $\frac{1}{x} + \frac{1}{y} = \frac{2}{a(1+r)} + \frac{2}{ar(1+r)} = \frac{2(1+r)}{ar(1+r)} = \frac{2}{ar} = \frac{2}{b}$

again $\frac{a}{x} + \frac{c}{y} = \frac{2}{1+r} + \frac{(ar^2)2}{ar(1+r)}$ [from 1 and 2]

$= \frac{2}{1+r} + \frac{2r}{(1+r)} = 2$]

51. (A, B,C)

If a, b, c, d are in AP

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ will be in HP

$\Rightarrow \frac{a^2bc}{a}, \frac{a^2bc}{b}, \frac{a^2bc}{c}, \frac{a^2bc}{d}$ in HP $\therefore$ (A) is true

again a, b, c, d in AP

$\Rightarrow$ a(abc + bcd + acd + abd), b(abc + bcd + acd + abd), c(abc + bcd + acd + abd), d(abc + bcd, acd + abd) in AP

$\Rightarrow$ a^2(bc + cd + bd), b^2(ac + cd + ad), c^2(ab + bd + ad), d^2(ac + ab + bc) in AP $\therefore$ (B) is true

once again a, b, c, d in AP

$\Rightarrow \frac{a}{abcd}, \frac{b}{abcd}, \frac{c}{abcd}, \frac{d}{abcd}$ in AP

$\Rightarrow$ bcd, acd, abd, abc in HP $\therefore$ (C) is true

52. (A,B,C,D)

$\sin^2x - \cos^2x = -\cos 2x \le 1$

$\sqrt{\frac{6}{5}}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{3}}\cos x\right) = \frac{\sqrt{3}}{\sqrt{5}}\sin x + \frac{\sqrt{2}}{\sqrt{5}}\cos x$

$= \sin x \,.\, \sin\phi + \cos x \cos\phi$ where $\sin\phi = \frac{\sqrt{3}}{\sqrt{5}}$, $\cos\phi = \frac{\sqrt{2}}{\sqrt{5}}$

$= \cos(x - \phi) \le 1$

$= \cos^6x + \sin^6x = (\cos^2x)^3 + (\sin^2x)^3$

$= 1 - 3\sin^2x\cos^2x = 1 - \frac{3}{4}(\sin 2x)^2$

$= \le 1$

$= \cos^2x + \sin^4x = 1 - \frac{(\sin 2x)^2}{4} \le 1$

53. (1)

$x = \frac{1}{9}(999....9) = \frac{1}{9}\left(10^{20} - 1\right)$; $y = \frac{1}{3}(999....9) = \frac{1}{3}\left(10^{10} - 1\right)$; $z = \frac{2}{9}(999....9) = \frac{2}{9}\left(10^{10} - 1\right)$

54. (1)

3x, 4y, 5z are in G.P.

$\therefore$ $16y^2 = 15xz$ ....(1)

and $y = \dfrac{2xz}{x+z}$ ....(2)

using (2) in (1)

$16 \times 4x^2z^2 = 15(x+z)^2xz$

$\dfrac{(x+z)^2}{xz} = \dfrac{64}{15}$; $\dfrac{x}{z} + \dfrac{z}{x} + 2 = \dfrac{64}{15}$; $\therefore \dfrac{x}{z} + \dfrac{z}{x} = \dfrac{34}{15}$

$\therefore$ m = 34 and n = 15

$\therefore$ m + n = 34 + 15 = 49

55. (3)

$$\tan\frac{\pi}{3} = \frac{3\tan\frac{\pi}{9} - \tan^3\frac{\pi}{9}}{1 - 3\tan^2\frac{\pi}{9}}$$

squaring both sides and then rearranging
we get,

$$\tan^6\frac{\pi}{9} - 33\tan^4\frac{\pi}{9} + 27\tan^2\frac{\pi}{9} = 3$$

56. (4)

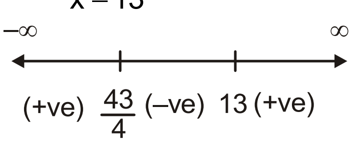
$x \neq 13$

$\dfrac{2}{x-13} < -\dfrac{8}{9}$ or $\dfrac{2}{x-13} > \dfrac{8}{9}$

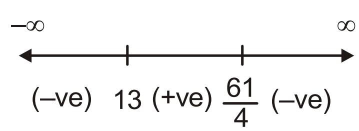
$\Rightarrow \dfrac{2}{x-13} + \dfrac{8}{9} < 0$ or $\dfrac{2}{x-13} - \dfrac{8}{9} > 0$

$\Rightarrow \dfrac{18 + 8x - 104}{9(x-13)} < 0$ or $\dfrac{18 - 8x + 104}{9(x-13)} > 0$

$\Rightarrow \dfrac{4x-43}{x-13} < 0$ or $\dfrac{61-4x}{x-13} > 0$



or



$\Rightarrow x \in \left(\dfrac{43}{4}, 13\right)$ or $x \in \left(13, \dfrac{61}{4}\right)$

$\Rightarrow x \in \left(\dfrac{43}{4}, 13\right) \cup \left(13, \dfrac{61}{4}\right)$

$\Rightarrow$ x = 11, 12, 14, 15.i.e., Four integers.

57. (4)

$$\sum_{\alpha=4}^{n+3} 4(\alpha-3) = An^2 + Bn + C \quad \Rightarrow \quad \sum_{\alpha=1}^{n} 4\alpha = An^2 + Bn + C$$

$\Rightarrow \quad 2n(n + 1) = An^2 + Bn + C \quad \Rightarrow \quad A = 2,\ B = 2,\ C = 0$

$\Rightarrow \quad A + B - C = 4$ **Ans.**

58. (2)

$|x^2 - 3x - 1| < |3x^2 + 2x + 1| + |2x^2 + 5x + 2|,\ x^2 - 3x - 1 \neq 0$

$\Leftrightarrow \ |(3x^2 + 2x + 1) - (2x^2 + 5x + 2)| < |3x^2 + 2x + 1| + |2x^2 + 5x + 2|,\ x^2 - 3x - 1 \neq 0$

The inequality holds if and only if

$(3x^2 + 2x + 1)(2x^2 + 5x + 2) > 0$

i.e. $\quad 2x^2 + 5x + 2 > 0$

i.e $\quad (2x + 1)(x + 2) > 0$

i.e. $\quad x \in (-\infty, -2) \cup (-1/2, \infty) \quad \Rightarrow \quad a = 2$ and $b = \frac{1}{2}$

$\therefore \quad a + \log ab = 2$ **Ans.**

59. (A) $\rightarrow$ (q) ; (B) $\rightarrow$ (p) ; (C) $\rightarrow$ (s); (D) $\rightarrow$ (r)

$a\cos\theta + b\sin\theta = c$

$$\Rightarrow a\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right) + b\left(\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right) = c$$

$\Rightarrow (a+c)\tan^2\frac{\theta}{2} - 2b\tan\frac{\theta}{2} + (c-a) = 0$ has two roots α and β

$$\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} = \frac{2b}{a+c}$$

$$\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{c-a}{a+c}$$

60. (A) → (s) ; (B) → (r) ; (C) → (q); (D) → (p)

(B) $\left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x \geq 1$

$\therefore \cos^x \alpha + \sin^x \alpha \geq 1$, where $\cos\alpha = \frac{5}{13}$

Equally holds for x = 2. If x < 2, both cos α sin α increase (being positive fractions). So, $\cos^x \alpha + \sin^x \alpha > 1$ if x < 2. Thus $x \leq 2$.

(C) $\frac{x^2 - 4x + 3}{x+1} > 0 \Rightarrow (x-1)(x-3)(x+1) > 0, x \neq -1.$

(D) Solve the inequation $x^2 - 3x + 2 \leq 0$ and $2x^2 - 3x - 5 \geq 0$. and take intersection of both.