SOLUTIONS

MEAITS 2018

UNIT TEST-2

(MAIN & ADVANCED PATTERN)

Test Date: 04-11-2017

Mentors® Eduserv™

Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd.,
Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

JEE MAIN

PHYSICS

1. **(C)**

By Newton's 2nd law

$\vec{F} = m\vec{a}$

$\boxed{\hat{F} = \hat{a}}$

2. **(C)**

No force in horizontal direction so no component of Tension in horizontal direction

3. **(D)**

Work done by action-reaction pair of normal is zero

For two charges work done by action-reaction may be positive or negative

4. **(A)**

Mass of the system $= (50 + 25)\text{kg} = 75\text{kg}$

$\text{acc}^n = 2\text{m/s}^2$

$3F - 75g = 75 \times 2$

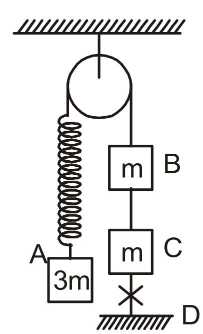
$F = \frac{900}{3} = 300\text{N}$

5. **(D)**

$F = \frac{-dU}{dx}$

At E, $\frac{dU}{dx} = (+)\text{ve}$ so F is (–)ve

6. **(D)**

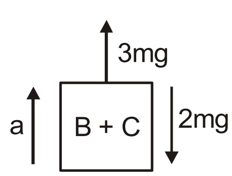


Tension in spring will remain constant and equal to 3mg.

So, $a_A = 0$

B & C are connected by a string so acceleration of both are same.

Tension in A – B is 3mg



$a = \frac{mg}{2m} = \frac{g}{2}$

7. (D)

Limiting friction between A and B = 60 N

Limiting friction between B and C = 90 N

Limiting friction between C and ground = 50 N

Since limiting friction is least between C and ground, slipping will occur at first between C and ground. This will occur when F = 50 N

8. (A)

$dw = \vec{F}.\vec{ds}$

$= \left(4y\hat{i} + 2x\hat{j}\right).\left(dx\hat{i} + dy\right)\hat{j}$

$\int dw = \int 4ydx + \int 2xdy$

$y = 2x + 1$

$dy = 2dx$

$w = \int 4y\dfrac{dy}{2} + \int 2x\ 2dx$

$= \int_1^5 2\ ydy + \int_0^2 4\ xdx$

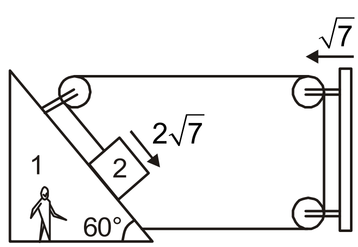
$= y^2\Big|_1^5 + 2x^2\Big|_0^2$

$= \left(25 - 1\right) + 2 \times 4$

$= 24 + 8 = 32\,J$

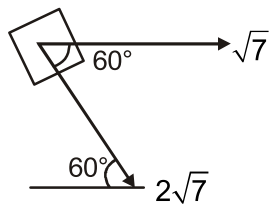
9. (A)

In frame of 1



$v_{2/1} = 2\sqrt{7}$

So, in ground in



$$v = \sqrt{\left(2\sqrt{7}\right)^2 + \left(\sqrt{7}\right)^2 + 2\times 2\sqrt{7}\times\sqrt{7}\cos 60^{o}}$$

$$= \sqrt{28+7+14}$$

$$= \sqrt{49} = 7\text{m/s}$$

So distance travelled in 1 s is 7m

10. (B)

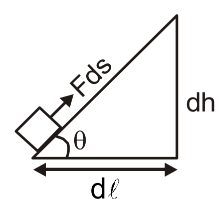
For any small section of path,

$$\Delta K = W_F + W_{gravity} + W_{friction}$$

$$O = dw_F - mg\sin\theta\, ds - \mu mg\cos\theta\, ds$$

$$= dw_F - mgdh - \mu mgd\ell$$

$$dw_I = mgdh + \mu mgdl$$



Work done by F from A to B is

$$w_1 = mgh + \mu_1 mg\ell_1$$

Work done by F from B to C is

$$w_2 = -mgh + \mu_2 mg\ell_2$$

$$w = w_1 + w_2 = \mu_1 mg\ell_1 + \mu_2 mg\ell_2$$

11. (C)

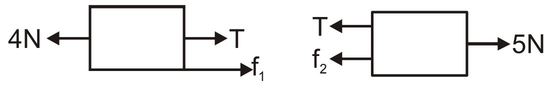
Consider a small element of spring of length dx at a distance of x from fixed end. The K.E. of this small element is given by

$$dK = \frac{1}{2}\left(\frac{m}{L}dx\right)\left(\frac{v_0 x}{L}\right)^2$$

$$K.E_{total} = \int dK = \int_0^L \frac{mdx}{2L}\left(\frac{x}{L}v_0\right)^2 = \frac{mv_0^2}{6}$$

12. (A)

The limiting friction for m_1 and m_2 are $f_{L1} = 6N$ and $f_{L2} = 10N$ respectively. At t = 2s the situation would be like as shown



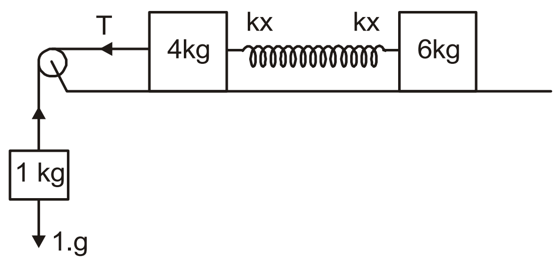
for $(m_1), 4 = f_1 + T$

As friction will be compensated first, so $f_1 = 4N$

And T = 0

For $m_2, f_2 = 5N$

13. (B)



F.B.D. of system is as shown in fig.

Limiting value of f_1 and f_2 are 4N and 6N resp. As $1.g = f_{L1} + f_{L2}$ the system is just one the verge sliding and friction are demitting in nature.

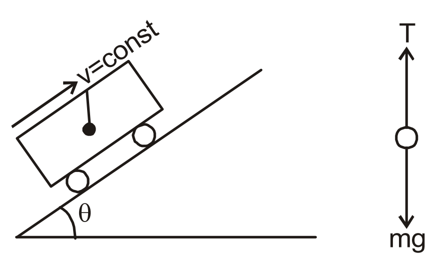
$T_1 = 1.g = 10N,\ kx + f_{L1} = T_1 \Rightarrow kx = 10 - 4 = 6N$

$x = .06m$

14. (A)

15. (A)

As the cart is moving up with constant velocity the FBD at sphere with respect to cart or with respect to ground would be same and would be like as shown in fig.



In this situation tension balances gravity force & hence string is vertical

16. (D)

in interfance redistribution of energy takes place .

17. (C)

$$I_R = I_1 + I_2 + 2\sqrt{I}\sqrt{I_2}\cos\Delta\phi$$

18. (A)

Wave front is locus of the points which are in same phase.

19. (B)

When I_0

$$I_{max} = 4I_0$$

$$I_{min} = 0$$

$$k_1 = 4$$

When

$$A \rightarrow \frac{I_0}{4},\ B = I_0$$

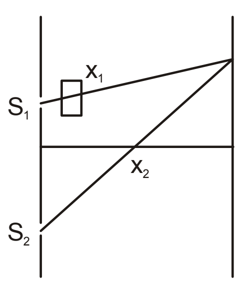
$$K_2 = \left(\sqrt{\frac{I_0}{4}} + \sqrt{I_0}\right)^2 - \left(\frac{\sqrt{I_0}}{\sqrt{4}} - \sqrt{I_0}\right)^2$$

$$= \frac{9}{4}I_0 - \frac{1}{4}I_0$$

$$= 2I_0$$

$$\frac{K_1}{K_2} = 2$$

20. (B)

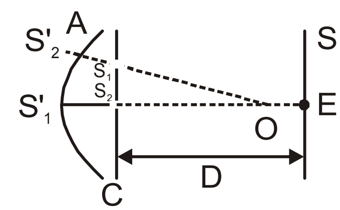


Optical path is more than real path

Central maxima will shift toward glass slab.

21. (A)

Let S'_1 and S'_2 are the points on the wave front from where perpendicular can be drawn through S_1 and S_2



$$S'_1S_2 + (\mu - 1)t + S_2E = S'_1S_1 + S_1E$$

$$\Rightarrow (OS'_2 - OS_2) + (\mu - 1)t + \sqrt{D^2 + d^2} = (OS'_1 - OS_1) + D$$

$$\Rightarrow (\mu - 1)t = (OS_2 - OS_1) - \left(\sqrt{D^2 + d^2} - D\right)$$

$$\Rightarrow 2\lambda - \frac{d^2}{2D} \text{ (Binomial approximation)}$$

$$\Rightarrow t = \frac{31\lambda}{8}$$

22. (D)

Huygen's principle can also prove snell's law & law of reflection.

23. (A)

Optical path = μt

$$\mu \times 4 = 6 \times \frac{4}{3}$$

$$\boxed{\mu = 2}$$

24. (D)

$$\beta = \frac{\lambda D}{d} \quad \Rightarrow \beta_1 = \frac{400 \times 1 \times 10^{-9}}{0.1 \times 10^{-3}} = 4\,mm$$

$$\beta_2 = \frac{560 \times 1 \times 10^{-9}}{0.1 \times 10^{-3}} = 5.6mm$$

25. (A)

If unpolarised light is passed through a Polaroid P_1, its intensity will become half So, $I_1 = \frac{I_0}{2}$ with vibrations parallel to the axis of P_1

Now this light will pass though the Analyzer

$$I_R = \left(\frac{I_0}{2}\right)\cos^2 30^\circ = \frac{I_0}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8}I_0$$

So the fractional transmitted $= \frac{I_2}{I_0} = \frac{3}{8} = 37.5\%$

26. (A)

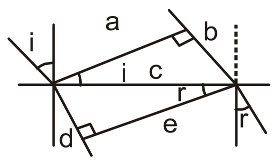
Since the reflected light is very highly polarized, it implies that incident light falls at polarizing angle of incidence (from Brewster's law.)

$$\mu = \tan\theta_P$$

$$\therefore \quad \theta_P = \tan^{-1}\mu = \tan^{-1}\left(\frac{4}{3}\right) = 53^\circ$$

Since θ_P is the angle which the rays from sun make with the normal of the interface, angle with the interface will be $90^\circ - 53^\circ = 37^\circ$

27. (C)

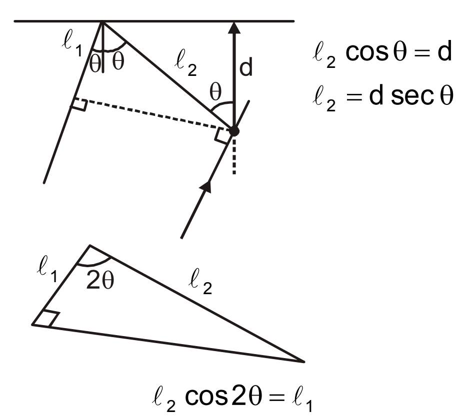


$$C\cos i = a \quad \Big| \quad C\sin i = b$$
$$C\cos r = e \quad \Big| \quad C\sin r = d$$

$$\frac{\sin i}{\sin r} = \frac{b}{d}$$

$$\mu_a \sin i = \mu_w \sin r \qquad \boxed{\frac{\mu_w}{\mu_a} = \frac{b}{d}}$$

28. (B)



$$\ell_1 + \ell_2 = \ell_2\cos 2\theta + \ell_2$$

$$= \ell_2(1+\cos 2\theta)$$

$$= d\sec\theta\left(1+2\cos^2\theta - 1\right)$$

$$= 2d\cos\theta$$

$\ell_1 + \ell_2 = \frac{\lambda}{2}$ for constructive interference at P

$$2d\cos\theta = \frac{\lambda}{2}$$

$$\cos\theta = \frac{\lambda}{4d}$$

(due to reflection from denser medium there is a phase different of π)

29. (A)

Let us first calculate the limiting and kinetic friction between various surfaces.

Between 3kg and ground

$$f_{\ell_1} = f_{k_1} = \mu_1(2+3)g = 0.1 \times 5g = 5N$$

Between 2kg & 3kg

$$f_{\ell_2} = f_{k_2} = \mu_2 2g = .2 \times 2 \times 10 = 4N$$

Let us 1st assume that both blocks move together with Common acceleration a

$$a = \frac{5+10-f_{\ell_1}}{5} = \frac{15-5}{5} = 2m/s^2$$

Now let us see how much friction force is required between 2kg & 3kg for common acceleration a

$$5 - f_2 = 2 \times 2$$

$$f_2 = 1$$

$$f_2 < f_{\ell_2}$$

It means both block move together.

30. (A)

Output power = η. Input power

$$= 0.75 \times 400 = 300w$$

Now $P = \frac{dw}{dt} = \frac{d(mgh)}{dt}$

$$\frac{dm}{dt} = \frac{P}{gh} = \frac{300}{10 \times 40} = \frac{3}{4}\frac{kg}{s}$$

$$= \frac{3}{4} \times 60kg/m^{-1} = 45kg\,min^{-1}$$

Hence, water drawn in $10\,min = 45 \times 10 = 450\,kg\,min^{-1}$

Since density of water is 100 kg m^{-3}, therefore

$$m = \rho v$$

$$v = \frac{450}{1000} = 0.45m^3$$

CHEMISTRY

31. (B)

$A \rightarrow B$, V = constant or $P \alpha T$, So, $PV \alpha T$

$B \rightarrow C$, T = constant or PV = constant

$C \rightarrow A$, $V \alpha T$, or P = constant

32. (A)

$$N_2 + 3H_2 \rightleftharpoons 2NH_3$$

Moles $\frac{4}{RT}-\frac{x}{RT}$ $\quad \frac{12}{RT}-\frac{3x}{RT}$ $\quad \frac{2x}{RT}$

for $NH_3, P \times 3 = \frac{2x}{RT} \times RT$

or, 3P = 2x or 3×2 = 2x or x = 3

So, mole of $H_2 = \frac{3}{RT}$

So, $P \times 3 = \frac{3}{RT} \times RT$ or $P = 1atm$

33. (D)

Initially, $n_{He} = 1, n_{O_2} = 2$

$n_{H_2} = 2, n_{SO_2} = 1$

After long time, in compartment A

$n_{He} = \frac{1}{2}; n_{H_2} = 1, n_{O_2} = 2$

In compartment B,

$n_{He} = \frac{1}{2}; n_{H_2} = 1, n_{SO_2} = 1$

as, V,T, constant

So, $p \alpha n$

So, $\frac{P_A}{P_B} = \frac{3.5}{2.5} = \frac{7}{5}$

34. (C)

At 0ºC and 2 atm 1 mole of an ideal gas occupy 11.2 litre.

So, n mole gas occupy 11.2 n litre at 0ºC and 2 atm i.e. $V_o = 11.2\,n$

Now, $\Delta V = \frac{\Delta t}{273.15} V_o$

or, $112 = \frac{546.3}{273.15} \times 11.2n$ or, $112 = 2 \times 11.2n$

or, n = 5

35. (A)

$P_{N_2} + 10\,\text{cm of Hg} + \frac{380.8}{13.6} = 76$

or, $P_{N_2} = 38\,\text{cm of Hg} = \frac{1}{2}\text{atm}$

Now, PV = nRT for N_2

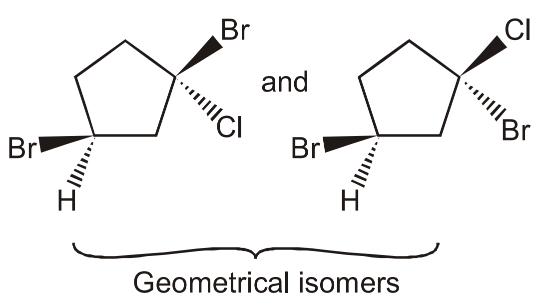
$\frac{1}{2} \times 1 = n \times \frac{1}{12} \times 360$

or, $n = \frac{1}{60}$ mole

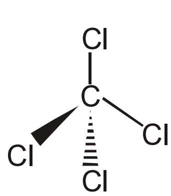
mass of nitrogen $= \frac{1}{60} \times 28 = \frac{14}{30}$g

$\% = \frac{14}{30 \times 1.4} \times 100 = 33.33\%$

36. (D)



37. (B)



It posses 6σ.

38. (D)

Stability order of various conformers follows Anti > gauche > partially eclipsed > fully eclipsed

(A) - gauche

(B) - fully eclipsed

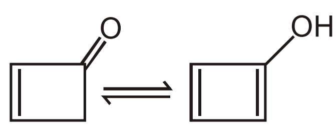
(C) - partially eclipsed

(D) - Anti

39. (B)

Although the meso compounds containing asymmetric c-atoms (must be similar) but they are optically inactive due to superimposable mirror image or due to the presence of any element of symmetry

40. (A)



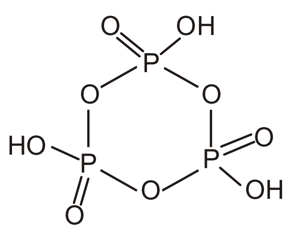
0% anti-aromatic

41. (C)

White vitriol $\rightarrow ZnSO_4.7H_2O$

Epsomite $\rightarrow MgSO_4.7H_2O$

42. (B)



43. (C)

Bond order α Bond energy

	B.O.
$NO^+ \longrightarrow$	3.00
$NO \longrightarrow$	2.5
$NO^- \longrightarrow$	2.00

44. (D)

$$\text{charge(q)} = \frac{\text{D.M.}}{\text{d}} = \frac{1.2\times10^{-18}\,\text{esu}-\text{cm}}{1\times10^{-8}\,\text{cm}} = 1.2\times10^{-10}\,\text{esu}$$

$$\therefore \text{ Fraction of charge } = \frac{1.2\times10^{-10}}{4.8\times10^{-10}} = 0.25$$

45. (D)

No in liquid state exist as dimer.

46. (A)

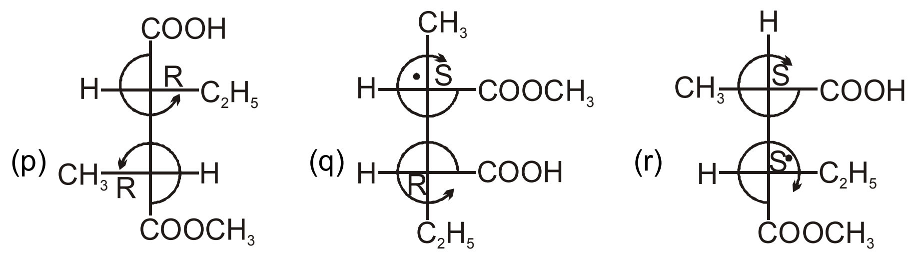
$\because$ 10 cm = 1dm

$$[\alpha] = \frac{+20^\circ}{1 \times 0.4} = +50^\circ$$

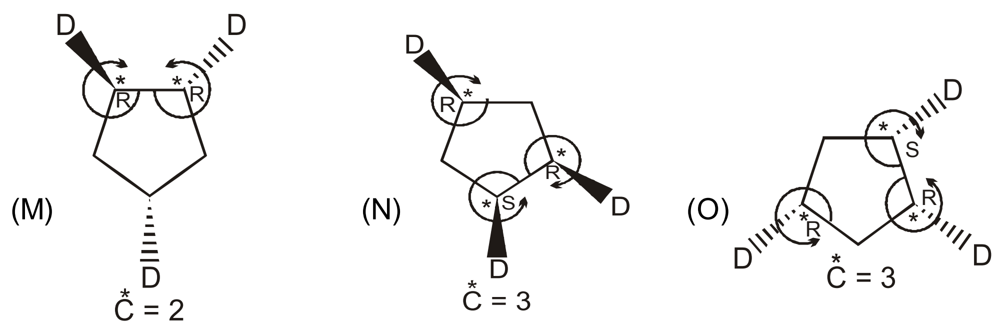
$$\because [\alpha] = \frac{\theta}{\ell \times c}$$

$\because$ c = concentration of solution (i.e density of solution)

47. (B)



48. (C)

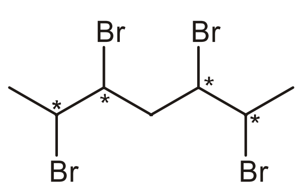


N & O : Mesomers

M & N : Diastereomers

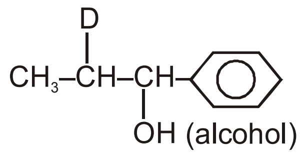
M & O : Diastereomers

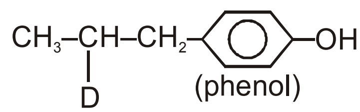
49. (C)



When molecule is symmetrical ans has even number of chiral centres then total number of isomers $= 2^{n-1} + 2^{\frac{n}{2}-1} = 2^{4-1} + 2^{\frac{4}{2}-1} = 2^3 + 2 = 10$

50. **(D)**

The given compounds are 

and 

51. **(D)**

$V_c = 3b$

or, 3 = 3b or b=1 litre/mol

Now, $b = 4 \times \frac{4}{3}\pi r^3 N_A$

or, $\sqrt[3]{\frac{1000 \times 3}{16\pi N_A}} = r$

or, $r = \left(\frac{3000}{16\pi N_A}\right)^{1/3}$ cm

52. **(C)**

For ideal gas $p \,\alpha\, \frac{1}{v}$ at constant T, n

53. **(D)**

At same T, K.E. is equal and P.E. of gas > P.E. of liq.

54. **(C)**

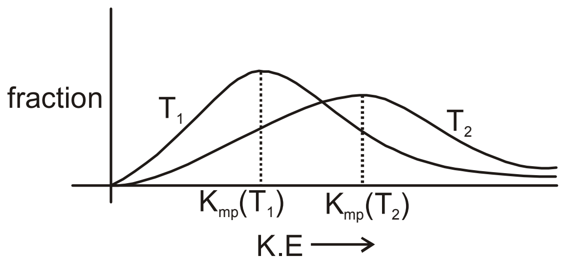
	$A_n(g) \rightleftharpoons$	$nA(g)$
t=0	1 mole	O
t	$1 - \alpha$	$n\alpha$
$p \,\alpha\, n$		[at constant V,T]

or, or, $\frac{p_f}{p_i} = \frac{T_f n_f}{T_i n_i}$

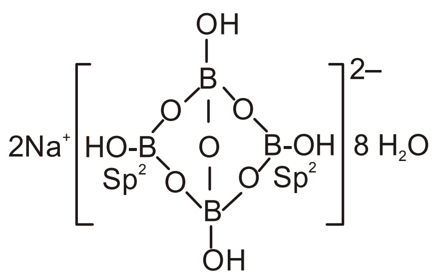
$$\frac{3.5}{2} = \frac{1 - \alpha + n\alpha}{1} = 1 + \alpha(n-1)$$

or, $\frac{3.5}{2} - 1 = \frac{1}{4}(n-1)$ or, $\frac{1.5}{2} = \frac{1}{4}(n-1)$ or, n = 4

55. (D)



56. (C)



57. (B)

π - bond strength is high when π -bond is formed between small orbitals.

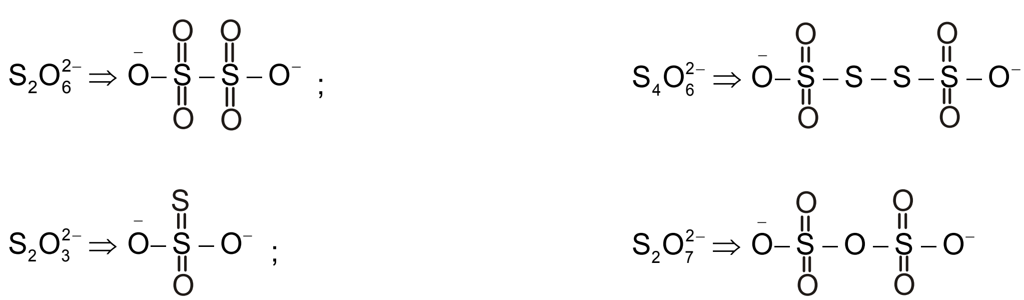
58. (A)

3dxz and 3dyz orbitals are suitable for π-bonding (3dπ - 2pπ)

59. (C)

Isoster : Same number of atom as well as no. of electrons.

60. (D)



MATHEMATICS

61. (B)

in the neighbourhood of x= $\sqrt[3]{\frac{\pi}{2}}$

$$f(x)=\cos\left(2\pi+\frac{\pi}{2}[x]-x^3\right)$$

$$\Rightarrow f(x)=\cos\left(\frac{\pi}{2}-x^3\right)$$

$\Rightarrow f(x)=\sin x^3$ in the neighbourhood of x= $\sqrt[3]{\frac{\pi}{2}}$

$$\Rightarrow f'(x)=3x^2\cdot\cos x^3$$

$$\therefore f'\left(\sqrt[3]{\frac{\pi}{2}}\right)=3\left(\frac{\pi}{2}\right)^{\frac{2}{3}}\cdot\cos\frac{\pi}{2}=0$$

$$\Rightarrow f'\left(\sqrt[3]{\frac{\pi}{2}}\right)=0$$

62. (C)

We have

$x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$

or, $\left(x^2+y^2\right)^2=t^2+\frac{1}{t^2}-2$

$$\Rightarrow\left(x^2+y^2\right)^2=x^4+y^4-2$$

$$\Rightarrow x^4+y^4+2x^2y^2=x^4+y^4-2$$

$$\Rightarrow x^2y^2=-1$$

$$\Rightarrow x^3y\frac{dy}{dx}=1$$

63. **(D)**

Let $t = xy + yz + zx,$

So $-\frac{1}{2} \le t \le 1$

$\therefore x^3 + y^3 + z^3 - 3xyz$

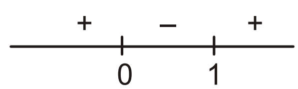
$= (x + y + z)\left(x^2 + y^2 + z^2 - xy - yz - zx\right)$

$= \sqrt{(1+2t)}(1-t)$

Let $f(t) = (1+2t)(1-t)^2$

$f'(t) = 6t(t-1) = 0$

Clearly,



$f(t)_{max} = f(0) = 1$

Hence (D) is correct answer.

64. **(B)**

f(x) = $8x^3 + 4ax^2 + 2x + a$

$f'(x) = 24x^2 + 8ax + 2$

$\Rightarrow f'(x) = 2\left(12x^2 + 4ax + 1\right)$

For f(x) to be non-monotonic, $f'(x) = 0$ must have distinct roots.

hence,

$D > 0$

i.e $16a^2 - 48 > 0$

$\Rightarrow a^2 > 3$

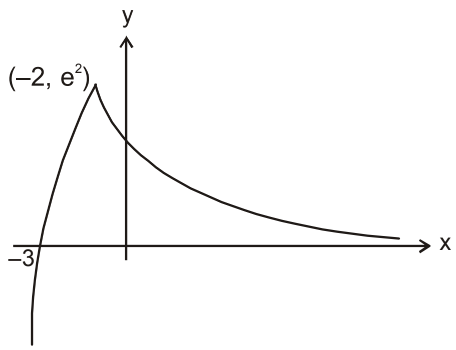
$\therefore a > \sqrt{3}$ or $a < -\sqrt{3}$

$\therefore a \in 2, 3, 4, ...$

$\therefore \text{sum} = 5050 - 1 = 5049$

65. (D)

Consider graph of $f(x) = \dfrac{x+3}{e^x}$



For one solution $k \in (-\infty, 0] \cup \{e^2\}$

$\therefore k = e^2$

66. (C)

$$\frac{x^4}{(x-1)(x-2)} = x^2 + 3x + 7 + \frac{16}{x-2} - \frac{1}{x-1}$$

Third derivative $= 0 + \dfrac{(-16)(3!)}{(x-2)^4} + \dfrac{3!}{(x-1)^4}$

$\therefore -12k = -96$

k = 8

67. (B)

put x = y = 1

3 f(2) = $f(1)^2$

$\Rightarrow$ f(2) = 12

f(1) = ab = 6

$\Rightarrow$ ab = 6 ... (i)

$\Rightarrow$ f(2) = ab^2 = 12

$\Rightarrow$ ab^2 = 4 ... (ii)

From (i) & (ii)

a = 3, b = 2

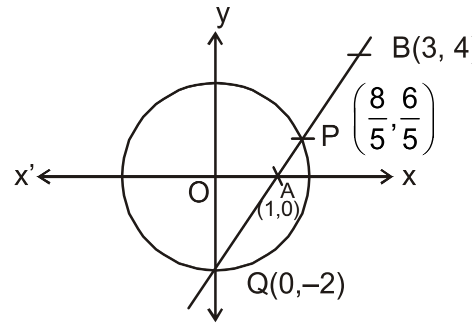
$\Rightarrow \left[\dfrac{3}{2}\right] = 1$

68. (D)

The equation of the line joining A(1, 0) and B(3, 4) is y = 2x – 2.

This line cuts the circle $x^2 + y^2 = 4$ at $Q(0,-2)$ and $P\left(\frac{8}{5},\frac{6}{5}\right)$.

We have $BQ = 3\sqrt{5}, QA = \sqrt{5}, BP = \frac{7}{\sqrt{5}}$ and $PA = \frac{3}{\sqrt{5}}$.



Therefore, $\alpha = \frac{BP}{PA} = \frac{\frac{7}{\sqrt{5}}}{\frac{3}{\sqrt{5}}} = \frac{7}{3}$

and $\beta = \frac{BQ}{QA} = \frac{3\sqrt{5}}{\sqrt{5}} = 3$

$\because$ α and β are the roots of the equation.

So, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - x\left(\frac{7}{3} + 3\right) + \frac{7}{3}(3) = 0$

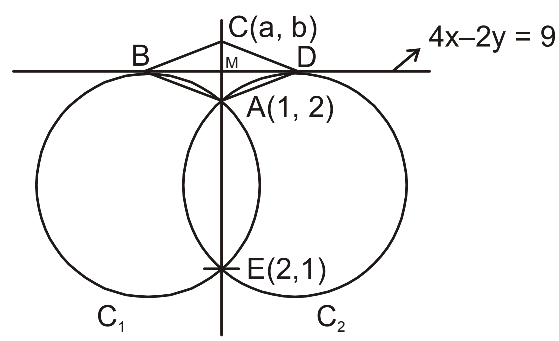
$\Rightarrow 3x^2 - 16x + 21 = 0$

69. (C)

The radical axis bisects the common tangent BD. Hence, M is the Mid-point of BD.

Let $C \equiv (a,b)$

Now, C(a, b) lies on common chord AE which is



$y - 2 = -1(x - 1)$

or, x+ y = 3

$\therefore a + b = 3$ ..... (i)

Also $M \equiv \left(\frac{a+1}{2},\frac{b+2}{2}\right)$ lies on $4x - 2y = 9$

$\therefore 4\left(\frac{a+1}{2}\right) - 2\left(\frac{b+2}{2}\right) = 9$

$\Rightarrow 2a + 2 - b - 2 = 9$

$\Rightarrow 2a - b = 9$ ..... (ii)

Solving (i) and (ii) we get

a = 4, b = –1

70. (C)

$x^2 + y^2 - 6x - 10y + \lambda = 0$

$S: (x-3)^2 + (y-5)^2 = \left(\sqrt{34-\lambda}\right)^2$

since point P(1,4) lies inside the circle

$S_1 < 0$

$\Rightarrow 1 + 16 - 6 - 40 + \lambda < 0$

$\Rightarrow \lambda < 29$ ...... (i)

Also, circle neither touches nor cuts the axes, then

$3 > \sqrt{34-\lambda}$ or $\lambda > 25$ ...... (ii)

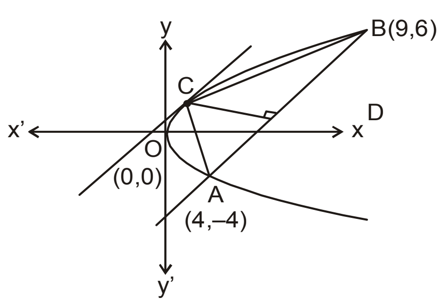
and $5 > \sqrt{34-\lambda}$ or $\lambda > 9$ .... (iii)

From (i), (ii) and (iii)

$25 < \lambda < 29$

Hence Difference = 29–25 = 4

71. (D)



The Area of triangle ABC is maximum if CD is maximum because AB is fixed.

It is clear that tangent drawn to the parabola at C should be parallel to AB.

For $y^2 = 4x$

$\therefore 2y\frac{dy}{dx} = 4$ or, $\frac{dy}{dx} = \frac{2}{y}$

$\Rightarrow \frac{2}{\beta} = \frac{6+4}{9-4} = 2$ or, $\beta = 1$

then $\alpha = \frac{1}{4}$

$\therefore$ coordinates of C are $\left(\frac{1}{4}, 1\right)$

$\therefore 4\alpha + \beta + 3 = 1 + 1 + 3 = 5$

72. **(A)**

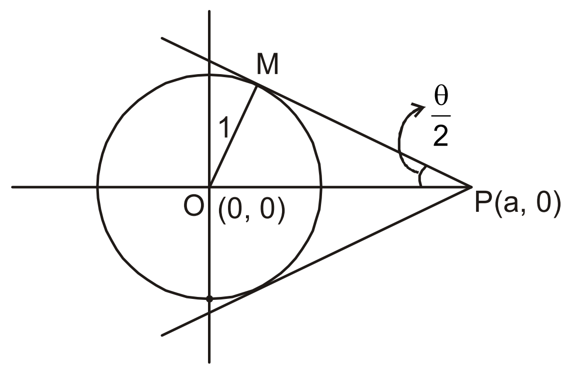
$\frac{\pi}{3} < \theta < \pi$

$\Rightarrow \frac{\pi}{6} < \frac{\theta}{2} < \frac{\pi}{2}$

$\Rightarrow \frac{1}{2} < \frac{1}{a} < 1$

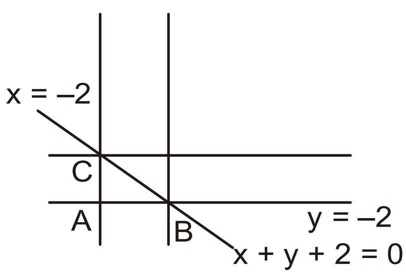
$\Rightarrow 1 < a < 2$

$\therefore a \in (-2, -1) \cup (1, 2)$

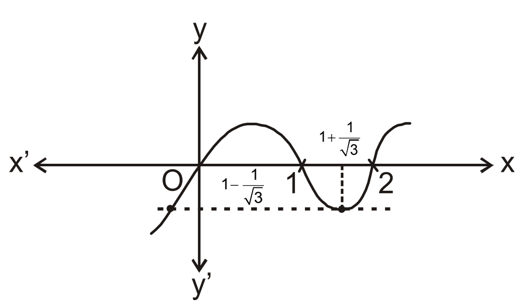


73. **(B)**

Given lines (x + 2) (y + 2) = 0 and x + y + 2 = 0 form a right angle triangle with right angle at (–2, –2) with other vertices as (–2, 0) and (0, –2).



74. **(A)**



Given $f(x) = x\left(x^2 - 3x + 2\right)$

$\Rightarrow f(x) = x(x-2)(x-1)$

Graph of y = f(x) is shown as

Now for exactly one positive and one Negative solution of the equation f(x) = K we should have

$$K = f\left(1 + \frac{1}{\sqrt{3}}\right)$$

$$\therefore\ K = \left(1 + \frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}} - 1\right)\left(\frac{1}{\sqrt{3}}\right)$$

$$\left(\frac{1}{3} - 1\right)\left(\frac{1}{\sqrt{3}}\right) = \frac{-2}{3\sqrt{3}} = -\frac{2\sqrt{3}}{9}$$

75. (D)

$$y_1 = \frac{1}{x}$$

$$y_2 = -\frac{1}{x^2}$$

$$y_3 = \frac{2}{x^3}$$

$$y_n = \frac{(-)^{n-1}(n-1)!}{x^n}$$

76. (D)

Given, two circles are

$S_1 : x^2 + y^2 - 16 = 0$

$C_1(0,0), r_1 = 4$

and $S_2 :\ x^2 + y^2 + 6x + 8y + n^2 = 0$

$C_2 : (-3,-4);\ r_2 = \sqrt{9 + 16 - n^2}$

$r_2 = \sqrt{25 - n^2}$

Here, $25 - n^2 > 0 \Rightarrow -5 < n < 5$ .... (i)

For exactly two common tangents

$r_1 + r_2 > c_1c_2 > |r_1 - r_2|$

$\Rightarrow 4 + \sqrt{25-n^2} > 5 > |4 - \sqrt{25-n^2}|$ $\Rightarrow \sqrt{25-n^2} > 1$

$\Rightarrow 25 - n^2 > 1$

$\Rightarrow n^2 < 24 \quad \Rightarrow -\sqrt{24} < n < \sqrt{24}$ .... (ii)

But $n \in I$, So

n = –4, –3, –2, –1, 0, 1, 2, 3, 4.

Hence possible values of n is 9

77. **(A)**

Let coordinates of B be $\left(\frac{t^2}{2}, t\right)$

Now area of ΔAOB = $\frac{1}{2}\left|\begin{vmatrix} 0 & 0 & 1 \\ 2 & -2 & 1 \\ \frac{t^2}{2} & t & 1 \end{vmatrix}\right| = \frac{3}{2}$

$\Rightarrow (2t + t^2) = \pm 3$

$\therefore t = 1, -3$

$\therefore \left(\frac{1}{2}, 1\right)$ and $\left(\frac{9}{2}, -3\right)$

78. **(D)**

Normal at point t_1 cuts the parabola again at t_2. Then

$t_2 = -t_1 - \frac{2}{t_1} \Rightarrow t_1^2 + t_1t_2 + 2 = 0$

Since t is real, So

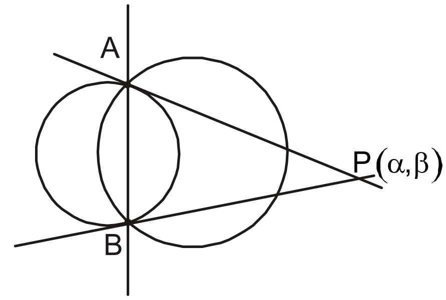
$t_2^2 - 4 \times 1 \times 2 \geq 0$

$t_2^2 \geq 8$

minimum value of t_2^2 = 8

79. (A)

Given circles are



$S_1 : x^2 + y^2 - 12 = 0$ .... (i)

and $S_2 : x^2 + y^2 - 5x + 3y - 6 = 0$ .... (ii)

Now, equation of common chord of circle (i) and (ii) is

$S_1 - S_2 = 0$

$\Rightarrow 5x - 3y - 6 = 0$ .... (iii)

Let tangents to circle at A and B meet at $P(\alpha, \beta)$ then AB will be chord of contact of the tangents to the circle from P. Therefore equation of AB will be

$x\alpha + y\beta - 12 = 0$ .... (iv)

Now, lines (iii) and (iv) are same, so

$\frac{\alpha}{5} = \frac{\beta}{-3} = \frac{-12}{-6}$

$\Rightarrow \alpha = 10$

$\beta = -6$

$\therefore \; P \equiv (10, -6)$

80. (B)

$f(x) = \sqrt{1 + \sin^2(x^2)}$

$f'(x) = \frac{1}{2\sqrt{1+\sin^2(x^2)}}\left(\sin(2x^2)(2x)\right)$

$\Rightarrow f'(x) = \frac{x\sin(2x^2)}{\sqrt{1+\sin^2(x^2)}} \qquad \Rightarrow f'\left(\frac{\sqrt{\pi}}{2}\right) = \frac{\frac{\sqrt{\pi}}{2}}{\sqrt{1+\frac{1}{2}}} = \sqrt{\frac{\pi}{6}}$

81. (A)

The required circle will be $x(x+5) + y(y+5) = 0$

82. (C)

Given,

$\lim_{x \to 0}\left(\frac{p(x)}{x^5} - 2\right) = 5$

$\therefore \lim_{x\to 0} \frac{p(x)}{x^5} = 7$

Consider $p(x) = ax^7 + bx^6 + 7x^5$

$\therefore p'(x) = 7ax^6 + 6bx^5 + 35x^4$

Now, $p'(-1) = 0$

$\Rightarrow 7a - 6b + 35 = 0$ .... (i)

and $p'(1) = 0$

$\Rightarrow 7a + 6b + 35 = 0$ .... (ii)

From (i) and (ii)

14a + 70 = 0

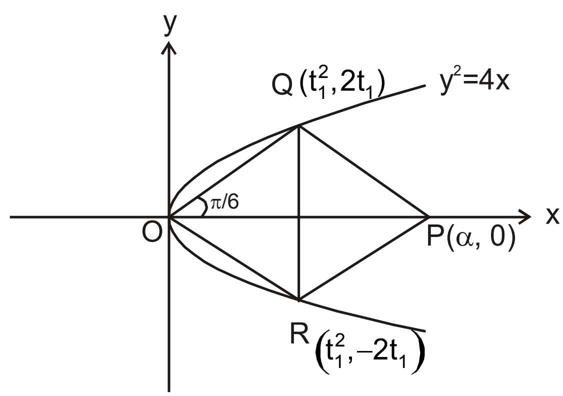
a = –5 and b = 0

$\therefore p(x) = -5x^7 + 7x^5$ $\therefore p(1) = -5 + 7 = 2$

83. (C)

Let $Q(t_1^2, 2t_1)$

$\Rightarrow R(t_1^2, -2t_1)$



clearly, $\angle QOP = \frac{\pi}{6}$

$\tan\frac{\pi}{6} = \frac{2}{t_1}$

$\therefore \frac{2}{t_1} = \frac{1}{\sqrt{3}}$

or, $t_1 = 2\sqrt{3}$

The equation of normal at Q is

$y = -t_1 x + 2t_1 + t_1^3$

$\because$ It passes through $P(\alpha, 0)$, So

$\Rightarrow 0 = -t_1\alpha + 2t_1 + t_1^3$

$\Rightarrow \alpha = 2 + t_1^2$

$\Rightarrow \alpha = 2 + 12 = 14$

84. (C)

Given equation of parabola

$y^2 = 16x$, here $a = 4$

Since, two tangents passes through the points $(-4, -\alpha)$ which lies on the directrix x = –4

Then tangents are perpendicular

$$\text{i.e } m_1 \cdot m_2 = -1$$

85. (A)

$f(x) = \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x$

$$\Rightarrow f(x) = \frac{1}{32}\frac{\sin 32x}{\sin x}$$

$$f'(x) = \frac{1}{32}\left[\frac{32\sin x \cdot \cos 32x - \sin 32x \cdot \cos x}{\sin^2 x}\right] \qquad \Rightarrow f'\left(\frac{\pi}{4}\right) = \frac{\frac{1}{32}\left[32 \times \frac{1}{\sqrt{2}} \times 1 - 0\right]}{\frac{1}{2}}$$

$$f'\left(\frac{\pi}{4}\right) = \sqrt{2}$$

86. (A)

We know that normals at $\left(at_1^2, 2at_1\right)$ and $\left(at_2^2, 2at_2\right)$ meet again on the parabola.

Then $t_1 t_2 = 2$

Here, a = 2

Given $2at_1 + 2at_2 = 8$

$\Rightarrow 4(t_1 + t_2) = 8$

$t_1 + t_2 = 2$

So, $\alpha + \gamma = at_1^2 + at_2^2$

$$= a\left(t_1^2 + t_2^2\right)$$

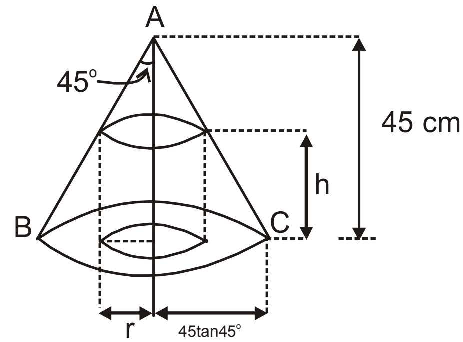
$$= 2\left[(t_1 + t_2)^2 - 2t_1 t_2\right]$$

$$= 2[4 - 2 \times 2]$$

$$\alpha + \gamma = 0$$

87. (B)

From Geometry, we have



$$\frac{r}{45\tan 45^o} = \frac{45-h}{45}$$

$\Rightarrow h = 45 - r$ .... (i)

Now, volume of cylinder

$v = \pi r^2 h \quad = \pi r^2(45-r)$

Now, $\frac{dv}{dr} = 0$

$\Rightarrow \pi(90r - 3r^2) = 0 \quad \Rightarrow r = 30$

$\therefore V_{max} = \pi(30)^2(15)$

$= \pi(900)(15)$

$= 13500\pi \text{ cm}^3$

88. (A)

Let $f(x) = ax^2 + bx + c$

Then $f'(x) = 2ax + b$

Also, $f(1) = f(-1)$

$\Rightarrow a + b + c = a - b + c \Rightarrow b = 0$

$f'(x) = 2ax$

$\therefore \quad f'(a_1) = 2aa_1$

$f'(a_2) = 2aa_2$

$f'(a_3) = 2aa_3$

As a_1, a_2, a_3 are in A.P.

$\therefore f'(a_1), f'(a_2), f(a_3)$ are in A.P.

89. (B)

Let x = $\cos\theta$

Then y = $\operatorname{cosec}^{-1}\left(\frac{1}{2x^2-1}\right) = \operatorname{cosec}^{-1}(\sec 2\theta) = \operatorname{cosec}^{-1}\left(\operatorname{cosec}\left(\frac{\pi}{2} - 2\theta\right)\right)$

$y = \frac{\pi}{2} - 2\theta$

Also, $z = \sqrt{1-\cos^2\theta} = \sin\theta$

$\therefore \frac{dy}{dz} = \frac{\frac{dy}{d\theta}}{\frac{dz}{d\theta}} = \frac{-2}{\cos\theta} = \frac{-2}{x}$

$\therefore \left.\frac{dy}{dz}\right|_{x=\frac{1}{2}} = -4$

90. (B)

Let $f(x) = x + ax^{-1} - 3$

$\therefore f'(x) = 1 - ax^{-2} = 0$

$\Rightarrow 1 = \frac{a}{x^2} \quad \Rightarrow x^2 = a$

$\therefore x = (a)^{\frac{1}{2}}$

Also, $f''(x) = 2ax^{-3}$ or, $f''\left(a^{\frac{1}{2}}\right) > 0$

Thus $x = a^{\frac{1}{2}}$ is the point of minima.

For $x + ax^{-1} - 3 > 0\ \forall x \in (0, \infty)$

we must have

$f\left(a^{\frac{1}{2}}\right) > 0$

or, $a^{\frac{1}{2}} + a \cdot a^{-\frac{1}{2}} - 3 > 0$

$\Rightarrow 2a^{\frac{1}{2}} > 3 \qquad \Rightarrow a^{\frac{1}{2}} > \frac{3}{2}$

$\Rightarrow a > \frac{9}{4}$

$\therefore$ least value of a is 3.

JEE ADVANCED

PHYSICS

1. **(A)**

$s = L\sin\theta\sqrt{1+\mu^2} = 0.3\,cm$

Friction is not sufficient to prevent slipping between the block and the bar & normal reaction between the bar & the block is a constant, therefore a constant total contact force (resultant of the normal reaction from the bar & kinetic friction) acts on the block by the bar. In addition, starting from rest, the block moves in a straight line.

2. **(A)**

$\frac{dm}{dt} = K\left(4\pi r^2\right)$, where K is a constant and r is the radius of the drop at any instant.

But $m = \frac{4}{3}\pi r^3\rho = \frac{4}{3}\pi r^3$, since $\rho =$ density of water = 1 gm/cc.

$$\frac{dm}{dt} = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt} = K4\pi r^2$$

$\because K = \frac{dr}{dt} \Rightarrow r = Kt$ (since it starts with zero radius)

Since, $m\frac{dv}{dt} + v\frac{dm}{dt} = mg$

$$\Rightarrow \frac{4}{3}\pi r^3\frac{dv}{dt} + vK4\pi r^2 = \frac{4}{3}\pi r^3 g$$

$\Rightarrow \frac{dv}{dt} + vk\frac{3}{r} = g$; But $\frac{dv}{dt} = a$ & $v = at$

$$\Rightarrow a + atK\left(\frac{3}{Kt}\right) = g$$

$$\Rightarrow 4a = g$$

$$\therefore a = \left(\frac{g}{4}\right)$$

3. (A)

$$\text{Force} = ma = m\frac{dv}{dt} = (P - Kt)g$$

$$\Rightarrow mv = \int (P - Kt)g\,dt = \left(Pt - \frac{Kt^2}{2}\right)g$$

Maximum velocity is attained when $\frac{d}{dt}\left(Pt - \frac{Kt^2}{2}\right) = 0$

$$\Rightarrow (P - Kt) = 0$$

$$\Rightarrow t = T = \frac{P}{K} \quad \text{But } v = \frac{ds}{dt}$$

$$\Rightarrow m\frac{ds}{dt} = \left(Pt - \frac{Kt^2}{2}\right)$$

$$\Rightarrow ms = \left(\frac{Pt^2}{2} - \frac{Kt^3}{6}\right)g$$

$$\Rightarrow s = \frac{1}{m}\left(\frac{P}{2}\frac{P^2}{K^2} - \frac{K}{6}\frac{P^3}{K^3}\right)g$$

$$\because \quad s = \frac{gKT^3}{3m}$$

4. (B)

Path difference $\Delta x = \{(S_2P - t_2) + \mu_2 t_2\} - \{(S_1P - t_1) + \mu_1 t_1\}$

$$= S_2P - S_1P + (\mu_2 - 1)t_2 - (\mu_1 - 1)t_1$$

For n^{th} order maxima $n\lambda = \{(\mu_2 - 1)t_2 - (\mu_1 - 1)t_1\} + \frac{dy}{D}$

For zero order maxima $y_0 = \frac{D}{d}\{(\mu_2 - 1)t_2 - (\mu_1 - 1)t_1\}$

(a) When both sheets have same average thickness $\frac{t_1 + t_2}{2}$ and R.I. μ_1 and μ_2

$$y_1 = \frac{D}{d}\left\{(\mu_2 - \mu_1)\frac{t_1 + t_2}{2}\right\}$$

$\Rightarrow \quad \dfrac{5\times10^{-3}\times1\times10^{-3}}{1}=(1.6-1.4)\dfrac{t_1+t_2}{2}$

$\therefore \quad t_1+t_2=5\times10^{-5}$ …(i)

(b) When both sheets have same R.I. = $\dfrac{\mu_1+\mu_2}{2}$ and thickness t_1 and t_2

$$y_2=\frac{D}{d}\left\{\frac{\mu_2+\mu_1}{2}-1\right\}(t_1-t_2)$$

$\Rightarrow \quad \dfrac{8\times10^{-3}\times1\times10^{-3}}{1(1.5-1)}=t_1-t_2$

$\Rightarrow \quad t_1-t_2 = 1.6\times10^{-5}$ …(ii)

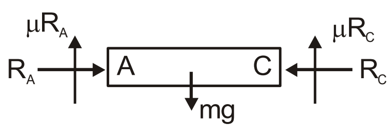
From (i) and (ii)

$$2t_1=6.6\times10^{-5}$$

$\Rightarrow \quad t_1=3.3\times10^{-5}$ m and $t_2=1.7\times10^{-5}$ m

5. (D)

For sphere 'A' : $mg=2kx-\mu R_A$



For block :

$$mg=\mu(R_A+R_C)$$

But $R_A=R_C\Rightarrow R_A=\dfrac{mg}{2\mu}$

$$\Rightarrow R_A=\frac{2kx-\mu R_A}{2\mu}$$

$$\Rightarrow R_A=\frac{kx}{\mu}-\frac{R_A}{2}$$

$$\because R_A=\frac{2Kx}{3\mu}$$

6. (A)

The detector receives direct as well as reflected waves. Distance moved between two consecutive positions of maxima $=\frac{\lambda}{2}$

For fourteen successive maxima $=14\times\frac{\lambda}{2}$

This is given to be 0.14 m

$\therefore \lambda = 2\times10^{-2}\,\text{m}$

$\nu = \frac{C}{\lambda} = \frac{3\times10^{8}}{2\times10^{-2}}$

$=1.5\times10^{10}\,\text{Hz}$

7. (B)

$\beta_{red} = \frac{\lambda D}{d} = \frac{675\times10^{-9}\times2}{1.5\times10^{-3}}$

$= 0.9\,\text{mm}$

$\beta_{violet} = \frac{450\times10^{-9}\times2}{1.5\times10^{-3}} = 0.6\,\text{mm}$

Let mth order red bright fringe coincide with (m + n)th violet bright fringe

$m\times0.9 = (m+n)0.6$

$\frac{m+n}{m} = \frac{0.9}{0.6} = 1.5$

$m = 2n$

When n = 1

$m = 2$

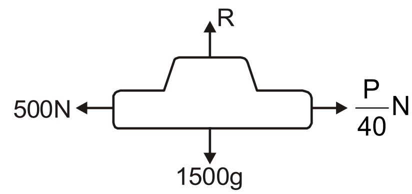
$\therefore$ minimum distance $= 2\times0.9$

$= 1.8$ mm

8. (B)

Let P be power of the engine.

At maximum speed there is no acceleration.

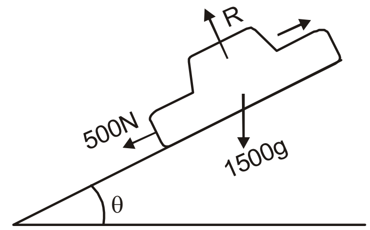


$$\frac{P}{40}=500$$

$$P=2\times10^{4}\ W$$

On the incline, let the maximum speed be v.

At maximum speed there is no acceleration



$$\frac{P}{v}=500+1500g\ \sin\theta=500+1500\times9.8\times\frac{1}{49}=800$$

$$v=\frac{2\times10^{4}}{800}=25\ m/s$$

9. (A)

Path difference at point $O=t(\mu_2-\mu_1)=\frac{10}{6}\mu m$

$\therefore$ Phase difference $\Delta\phi=\frac{2\pi}{\lambda}\Delta x\approx\pi$

Hence intensity will be minimum i.e. dark.

10. (C)

For point O to become dark

$$t(\mu_2\sim\mu_1)=(2n+1)\frac{\lambda}{2}$$

$$t=\frac{(2n+1)\lambda}{2(\mu_2\sim\mu_1)}$$

$$\Rightarrow t_{min}=\frac{\lambda}{2(\mu_2\sim\mu_1)}=9.9\,\mu m$$

11. (C)

Resistive force given by the cable system $F=\left(9\times10^{5}t-4.5\times10^{5}t^{2}\right)N$

For $F_{max},\frac{dF}{dt}=0,\frac{d^2F}{dt^2}=-ve$

$\frac{dF}{dt} = 0$, for $t = 1s$

$\therefore F_{max} = 4.5 \times 10^5 N$

Total resistive force by the brakes and cable system

$R_{max} = 600kN + 4.5 \times 10^5 N$

$= 10.5 \times 10^5 N$

Maximum acceleration experienced by the pilot $= \frac{10.5 \times 10^5}{1.5 \times 10^4} = 70 m/s^2$

12. (C)

At any instant, the force opposing the motion, $F_{rest} = +m\frac{dv}{dt}$

$$\int_{120}^{0} dv = -\int_{0}^{t} \frac{\left(9 \times 10^5 t - 4.5 \times 10^5 t^2 + 6 \times 10^5\right)}{1.5 \times 10^4} dt$$

$10t^3 - 30t^2 - 40t + 120 = 0$

$10t^2(t-3) - 40(t-3) = 0$

$\Rightarrow t = \pm 2 s$ or $+3 s$ (-ve sign is not possible)

The plane stops at t = 2s

13. (C)

$\frac{1}{2}kx^2 = mgh = h \propto x^2$

$\therefore \frac{h_2}{h_1} = \frac{x_2^2}{x_1^2}$

$h_1 = \left(\frac{11}{10}\right)^2 3m = 3.63m$

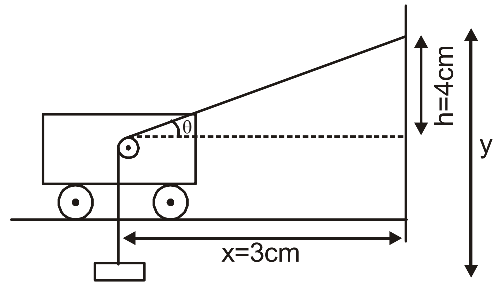
14. (D)

$\frac{1}{2}kx^2 = mgh \Rightarrow h = \frac{kx^2}{2mg} \Rightarrow h \propto \frac{1}{m}$

$\frac{h_2}{h_1} = \frac{m_1}{m_2} = \frac{m}{m + \frac{m}{10}} = \frac{10}{11}$

$\therefore h_2 = \frac{10}{11} \times 3.3 = 3m$

15. (C, D)



$$\left(y-h\right)+\sqrt{x^2+h^2}=\ell$$

$$\frac{dy}{dt}+\frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt}=0$$

$$\frac{dy}{dt}=-\frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt}$$

$$\frac{dy}{dt}=-\frac{3}{5}v_A$$

$$|v_B|=\frac{3}{5}v_A \qquad \text{....(i)}$$

$$\frac{d^2y}{dt^2}=v_A\frac{h^2}{\left(x^2+h^2\right)^{3/2}}$$

$$a_B=v_A\frac{16}{(5)^3}$$

$$a_B=\frac{16}{125}v_A \qquad \text{.....(ii)}$$

16. (A, B, C, D)

$$F=-\frac{dU}{dx}=-5\left(2x-4\right)$$

At mean position $F=0\Rightarrow x=2m$

$U_{min}=-20\,J$

$a=-50\times 2\left(x-2\right)$

$\omega=10\text{ rad / sec}$

$T=\pi/5\,\text{sec}$

17. (B, D)

There is a dark fringe at O if the path difference $\delta=ABO-AO'O=\frac{\lambda}{2}$

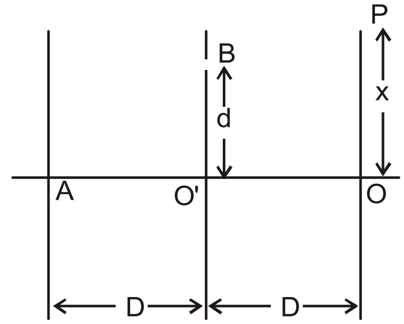
$$=2\sqrt{D^2+d^2}-2D=\frac{2d^2}{2D}=\frac{d^2}{D}=\frac{\lambda}{2} \qquad \Rightarrow d_{min}=\sqrt{\frac{\lambda D}{2}}$$

The bright fringe is formed at P if the path difference

$\delta' = AO'P - ABP = \lambda$

$= D + \sqrt{D^2 + x^2} - \sqrt{D^2 + d^2} - \sqrt{D^2 + (x-d)^2} = \lambda$

$= \frac{x^2}{2D} - \frac{d^2}{2D} - \frac{(x^2 + d^2 - 2xd)}{2D} = \lambda$



Given $d = d_{min}$

Solving $x = d_{min} = \sqrt{\frac{\lambda D}{2}}$

18. (A, D)

The centre-to-centre distance between the two slits, d, determines the width of the fringes. If this distance is kept the same while widening the adjustable slit, the fringe width will remain the same.

Intensity is the light energy per unit area. If this intensity is kept the same at I_0 when the wavelength is changed, the location of maxima will be changed but the maximum intensity in the fringes will be the same.

Since some light energy is absorbed by the plate, only 80% of the light energy is transmitted to form interference fringes. Hence the intensity at the maxima will be reduced.

19. (A, D)

All brick are in identical condition. So, when normal contact force is slightly less than 50 N, all bricks fall simultaneously.

Here,

$2\mu F = nmg$

Or $n = \frac{2\mu F}{mg} = \frac{2 \times 0.5 \times 50}{10} = 5$

And contact force, $F_C = \sqrt{F^2 + (\mu F)^2}$

$= \sqrt{(50)^2 + (0.5 \times 50)^2} = 55.9N$

20. (A,B,C,D)

Maximum loss in $PE = mgh = \frac{1}{2}mv_{max}^2$

or, $v_{max} = \sqrt{2gh} = \sqrt{2 \times 10 \times 10}$

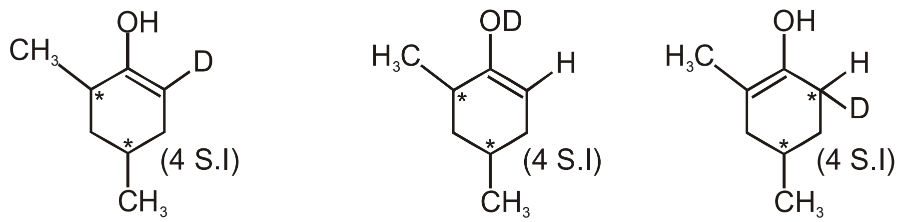
$= 10\sqrt{2}\, m/s$

For reaching at point B, minimum mechanical energy required is $mg \times 11 >$ initial mechanical energy. That is why, block never reach at the point B and repeats its path continuously.

CHEMISTRY

21. (D)

Possible enol of X:



22. (B)

Polarity of bond α critical temp.

23. (B)

$$\text{Bond length } \alpha \frac{1}{\text{Bond order}}$$

$$\left\{O_2^+ < O_2 < O_2^- < O_2^{2-}\right\}$$

24. (C)

Initially $P_{He}+P_{Ne}=950 - 50 = 900$ torr

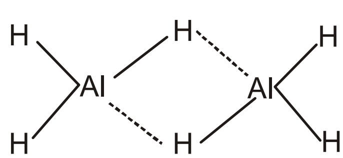
$$P_{He} = \frac{2}{3}\times 900 = 600\,\text{torr} \;;\quad P_{Ne} = \frac{1}{3}\times 900 = 300\,\text{torr}$$

In final condition, P_{He} = 1800 torr

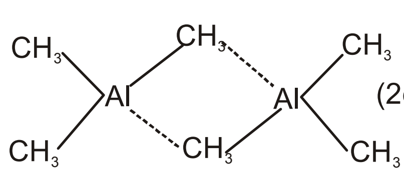
$$P_{H_2O}(g) = 100 \text{ torr}$$

$$\frac{n_{He}(g)}{n_{H_2O}(g)} = \frac{1800}{100} = \frac{18}{1}[n\,\alpha\,p \text{ at constant } V \text{ and } T]$$

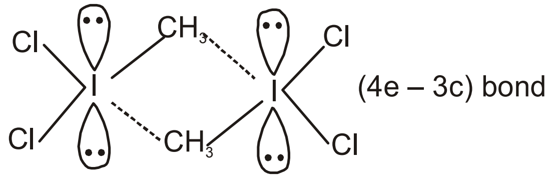
25. (C)



(2e – 3c) bond



(2e – 3c) bond



26. (B)

Let volume of balloon is V litre

PV = nRT

$$3V = \frac{n \times 0.0821 \times 36}{0.0821}$$

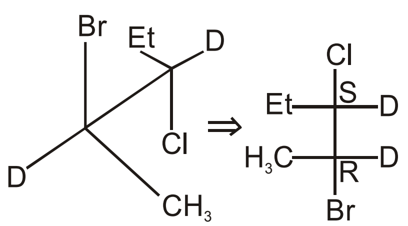
$$\text{or, } n = \frac{V}{12} \text{ mole He}$$

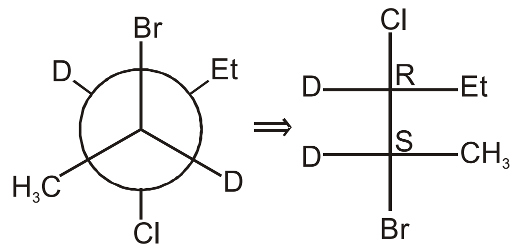
$$\text{mass of He} = \frac{V}{12} \times \frac{4}{1000} \text{kg} = \frac{V}{3000} \text{kg}$$

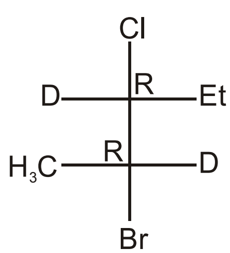
$$\text{Now, } \frac{V}{3000} + 50 + 450 = \frac{V}{1000}$$

or, V = 250 × 3000 litre = 250 × 3m^3 = 750 m^3

27. (C)

(I) 

(II) 

(III) 

I & II : enantiomers

I & III : diastreomers

II & III : diastreomers

28. (B)

When only attraction force works

$U(r) = \frac{K Q_1 Q_2}{r}$, rectangular hyperbola

29. (C)

$$DBE = \frac{\sum n(v-2)}{2} + 1$$

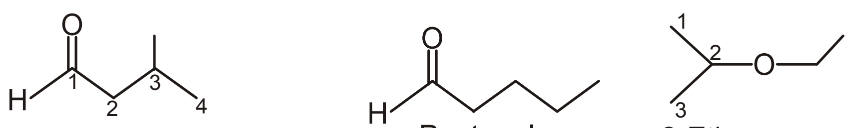
$$\text{DBE of } C_5H_{10}O = \frac{5(4-2)+10(1-2)}{2} + 1$$

$$= \frac{10-10}{2} + 1$$

= 0 + 1

= 1

It means open chain structural isomers with one $\pi-$bond in each or a ring without $\pi-$bond.



2-Methylbutanal (DBE = 1) Pentanal (DBE = 1) 2-Ethoxypropane (DBE = 0)

30. (D)

If degree of unsaturation = 3 then structural isomers may content-

(i) Three $\pi-$bonds without ring

(ii) Three rings without $\pi-$bonds

(iii) Two rings with one $\pi-$bond

(iv) Two $\pi-$bonds with one ring

31. (C)

For ideal gas, $P_{ideal} = \frac{Mu_{rms}^2}{3V}$

For 1 mole real gas $P_{real} + \frac{a}{V^2} = P_{ideal}$

So, $P_{real} + \frac{a}{V^2} = \frac{Mu_{rms}^2}{3V}$

or, $P_{real} = \frac{Mu_{rms}^2}{3V} - \frac{a}{V^2}$ [b is negligible]

or, $P_{real} = \frac{Mu_{rms}^2 V - 3a}{3V^2}$

32. (A)

Average kinetic energy energy depends only on temperature.

33. (A)

$R_3N–O$ has higher D.M as compared to $R_3P–O$ due to back bonding from O atom to P atom.

34. (D)

$NCl_3 > NH_3 > NF_3$ (Bond angle)

35. (A,B,C,D)

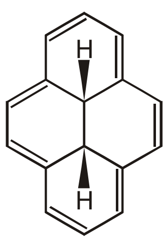
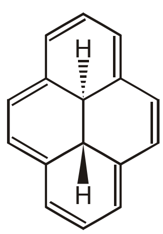
$$\eta \alpha \rho \bar{u} \lambda_{mean} \text{ or } \eta \alpha \frac{PM}{RT}\sqrt{\frac{8RT}{\pi M}} \quad \frac{1}{\sqrt{2}\,\pi\sigma^2 N^*}$$

$$\text{or, } \eta \alpha \frac{PM}{RT}\sqrt{\frac{8RT}{\pi M}} \quad \frac{KT}{\sqrt{2}\,\pi\sigma^2 P} \qquad \text{or, } \eta \alpha \frac{\sqrt{MT}}{\sigma^2}\text{ ; } P \alpha T \text{ at const. V}$$

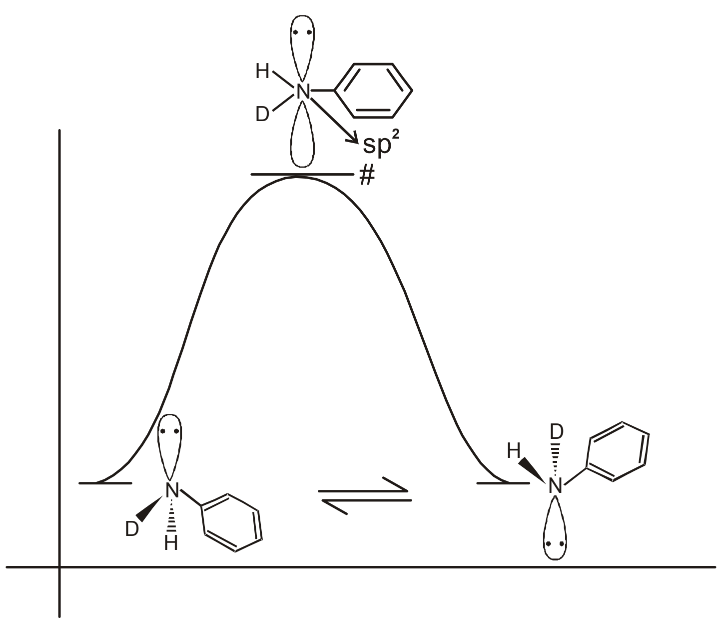
36. (B,D)

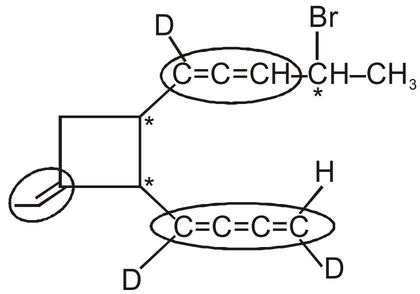
Factual

37. (A,C,D)

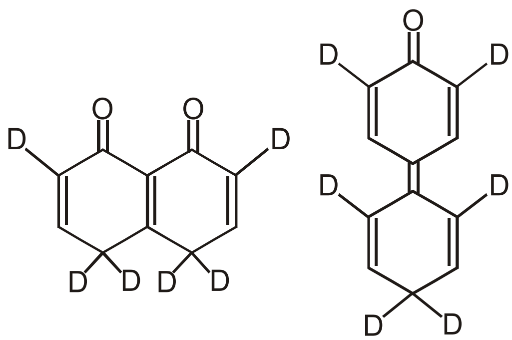
(A)  and  are geometrical isomers

(B)

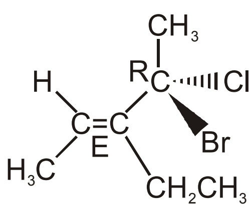


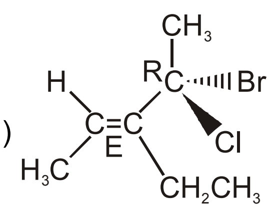
(C) 

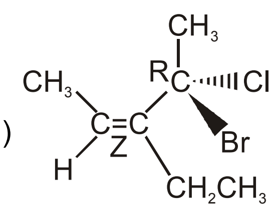
Total stereogenic unit = 6

(D) 

38. (B,C)

(I) 

(II) 

(III) 

I, II - Enantiomers, II, III-Diastereomers

39. (A,B)

I_3^- is linear so bond angle is 180°

40. (A,B,D)

When mixture is passed through hot graphite the following reaction will occur.

$CO_2 + C(s) \rightarrow 2CO(g)$

x ml 2x ml will formed

$\therefore$ Total volume of mixture = 160

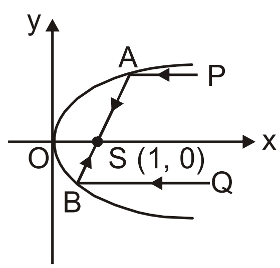
100 – x + 2x = 160

x = 60 ml

Volume of CO = 100 – x = 40 ml

MATHEMATICS

41. (B)



Let incidient rays be PA & QB After reflection, both rays passes through the focus (1, 0) there fore, AB is focal chord Let A be (t^2, 2t) then B be $\left(\frac{1}{t^2}, \frac{-2}{t}\right)$

given 2t = 3

$\therefore \quad t = \frac{3}{2}$

Hence distance of B rom Axis is $\left|\frac{-2}{t}\right| = \frac{4}{3}$

42. (B)

Given, x + y – ln (x + y) = 2x + 5

differentiate:-

$$1 + \frac{dy}{dx} - \frac{1}{x+y}\left(1 + \frac{dy}{dx}\right) = 2$$

$$\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$$

$$\left(\frac{dy}{dx}\right)_{\alpha,\beta} = \frac{\alpha+\beta+1}{\alpha+\beta-1} = \infty$$

$\alpha + \beta = 1$

43. (B)

$$\frac{a+2c}{b+3d} = \frac{-4}{3}$$

$\Rightarrow$ 3a + 4b + 6c + 12d = 0

$\Rightarrow \quad \frac{a}{4}+\frac{b}{3}+\frac{c}{2}+d=0$

Now, $f(x)=\frac{ax^4}{4}+\frac{bx^3}{3}+\frac{cx^2}{2}+dx+k$

then f(0) = 0 = f(1)

$\therefore$ f(x) satisfy the condition of Rolle's theorem in [0, 1]

Hence $f'(x)=0$ has at least one root in (0, 1)

44. (D)

$g(x)=f(\cot^2 x+2\cot x+2)$

$g'(x)=f'(\cot^2 x+2\cot x+2)\left\{-2\cot x\,\text{cosec}^2 x-x\text{cosec}^2 x\right\}$

For g(x) be decreasing $g'(x)<0$

$\Rightarrow \quad f'\left((\cot x+1)^2+1\right).\left(-2\text{cosec}^2 x\right)(\cot x+1)<0$; As $f''(x)>0$

i.e $f'(x)$ is increasing

$\Rightarrow$ (cot x +1) > 0 $\cot x>-1, \forall\, x\left(0,\frac{3\pi}{4}\right)$

45. (A)

Tangent to $y^2 = 4x$ in terms of m, is

$y=mx+\frac{1}{m}$ ...(i)

and normal to $x^2=4by$ in terms of m

$y = mx + 2b + \frac{b}{m^2}$ ...(ii)

$\because$ equation (i) and (ii) are same then

$\frac{1}{m}=2b+\frac{b}{m^2}$

$\Rightarrow 2bm^2 - m + b = 0$

For two different tangets

D > 0 $\Rightarrow 1 - 8b^2 > 0$

or $|b|<\frac{1}{2\sqrt{2}}$

46. (D)

Here $f'(x) = \dfrac{2}{x + f(x)} > 0$

$\Rightarrow$ f(x) is increasing function [1, 6]

$\therefore \int_1^6 f'(x)\,dx \le \int_1^6 \dfrac{2}{x}\,dx$

$f(x)\Big|_1^6 \le 2\ell nx\Big|_1^6$

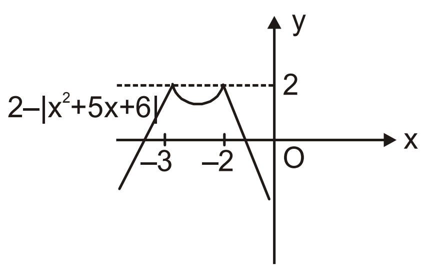
$f(6) - f(1) \le 2\ell n_6$

$f(6) \le 2\ell n_6$

47. (A)

f(x) will have maxima only, if $a^2 + 1 \ge 2$

$\Rightarrow |a| \ge 1$

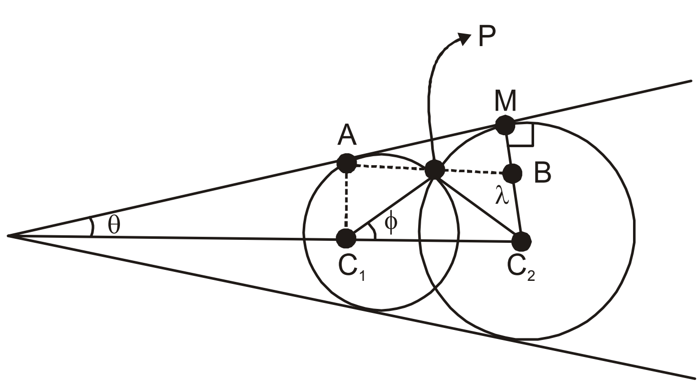


48. (C)

$f(x) = f'(x) \times f'''(x)$ is satisfied only by the polynomial of degree 4. Since f(x) = 0 satisfies x = 1, 2, 3 only. Now it is clear that one of the roots is repeated twice.

$\Rightarrow f'(1)\,f'(2)\,f'(3) = 0$

Solution for question no. 49 & 50.



Let 2d = length of common chord and $\angle PC_1C_2 = \phi$

then $\sin\phi = \dfrac{d}{3}$

and $\sin(90° - \phi) = \dfrac{d}{\lambda}$ or $\cos\phi = \dfrac{d}{\lambda}$

Now $\sin^2\phi + \cos^2\phi = \dfrac{d^2}{9} + \dfrac{d^2}{\lambda^2} = 1$

$\Rightarrow \sin^2\phi + \cos^2\phi = \dfrac{d^2}{9} + \dfrac{d^2}{\lambda^2} = 1$

$\therefore \lambda = 4$

49. (B)

$\lambda = r = 4$

50. (B)

$\because (C_1C_2)^2 = \lambda^2 + 3^2 = 16 + 9 = 25$

$\Delta ABM, \sin\theta = \dfrac{BM}{AB} = \dfrac{4-3}{C_1C_2} = \dfrac{1}{5}$

$\therefore \sin 2\theta = 2\sin\theta\cos\theta = 2\times\dfrac{1}{5}\times\sqrt{1-\left(\dfrac{1}{5}\right)^2} = \dfrac{4\sqrt{6}}{25}$

Solution for question no. 51 & 52.

$\because$ y = 3x is tangent to the parabola

$2y = ax^2 + b$ ...(i)

$\therefore 2(3x) = ax^2 + b$

$\Rightarrow ax^2 - 6x + b = 0$

D = 0

$\therefore 36 - 4ab = 0$

$\Rightarrow ab = 9$ ...(ii)

from (i) & (ii)

$2y = ax^2 + \dfrac{9}{a}$ ...(iii)

51. (C)

$\because \dfrac{a+b}{2} \geq \sqrt{ab} = 3 \Rightarrow a + b \geq 6$

52. (B)

$\because$ (2, 6) is point of contact

$2.6 = 4a + \frac{9}{a}$

$\Rightarrow$ 2a = 3

53. (B)

Let y = sin x

$f'(x) = \cos x$

$\therefore \cos x_1 = -\frac{1}{\cos x_2} = \frac{\sin x_2 - \sin x_1}{x_2 - x_1}$

i.e. $\cos x_1 \cos x_2 = -1$

$\therefore \sin x_1 = \sin x_2 = 0$

$\therefore$ No solution

54. (C)

$f(x) = y = |\ell nx|$

$\therefore f'(x_1) = -\frac{1}{f'(x_2)} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

$\Rightarrow \frac{\ell nx_1}{x_1|\ell n x_1|} = \frac{-x_2|\ell n x_2|}{|\ln x_2} = \frac{|\ell nx_2| - |(\ell nx_1)|}{x_2 - x_1}$ **...(i)**

$\Rightarrow \ell nx_1 \, \ell nx_2 < 0$

Let $0 < x_1 < 1$ then $1 < x_2$ and $x_1x_2 = 1$

$\frac{-1}{x_1} = -x_2 = \frac{\ell nx_2 + \ell nx_1}{x_2 - x_1} = \frac{\ell nx_1x_2}{x_2 - x_1} = 0 \Rightarrow$ since $x_1x_2 = 1$, $\ln x_1x_2 = 0$

where is Not possible

$\therefore$ No solution

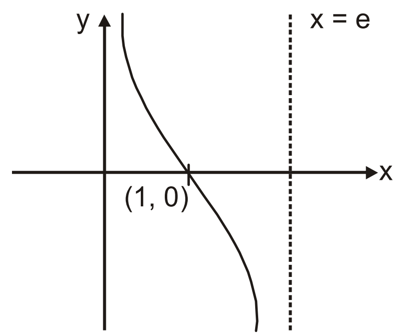
55. (A, D)

For condition of tangency

$\frac{|4k + 3k - 12|}{5} = k \Rightarrow 7k - 12 = \pm 5k$

$\therefore$ k = 1, 6

56. (A, B, C)



$f(x) = \ell n(1-\ell nx)$; Domain (0, e)

$f'(x) = -\frac{1}{1-\ell nx}\ \frac{1}{x} < 0$

$\Rightarrow$ Decreasing $\forall x$ in its domen

hence (A) and (B) are incorrect

$f'(1) = -1 \Rightarrow$ (C) is also incorrect

As shown in graph y – Axis & x = e are two assymptotes, hence (D) is correct

57. (A,D)

By intermediate mean value theorem, $\frac{f(0)+f(2)}{2} = f(C); 0 < C < 2$ ...(i)

By Lagrange Mean Value theorem, $f(1)-f(0) = f'(C_1); O < C_1 < 1$ ...(ii)

$f(2)-f(1) = f'(C_2); 1 < C_2 < 2$ ...(iii)

on substracting equation (ii) from (iii)

$f(2)+f(0)-2f(1) = f'(C_2)-f'(C_1)$ ...(iv)

Again by LMVT we get

$$f''(C_3) = \frac{f'(C_2)-f'(C_1)}{C_2-C_1}$$

$\Rightarrow f'(C_2)-f'(C_1) = (C_2-C_1)f''(C_3) < 0$ ... (v)

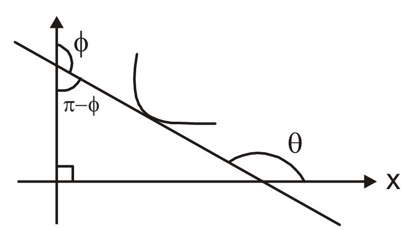
from equation (iv) & (v) we get

$f(2) + f(0) – 2 f(1) < 0$

$f(2) + f(0) < 2f(1)$

58. (B, C)

Here $\frac{dy}{dx} = k^2e^{kx}$



$\Rightarrow \left.\frac{dy}{dx}\right|_{x=0} = k^2 = \tan\theta$, where θ is angle made by X-Axis

Let ϕ be two angle made by Y-Axis

$$\tan\theta = \tan\left(\frac{3\pi}{2}-\phi\right) = \cot\phi$$

$\Rightarrow \cot\phi = k^2$

$$\phi = \sin^{-1}\left(\frac{1}{\sqrt{1+k^4}}\right)$$

59. (A,C,D)

Radical Axis of C_1 & C_2 is $C_1 - C_2 = 0$

$\Rightarrow$ x + 2y + 2 = 0

hence option (A) is correct

centre and radius of $C_1 = 0$ are (1, 2) & 3

$\because$ Length of perpendicular from (1, 2) on L = 0 is

$$\left|\frac{1+4+2}{\sqrt{1+4}}\right| = \frac{7}{\sqrt{3}} \neq \text{radius}$$

$\therefore$ (B) option is wrong

L is also common chord of C_1 and C_2

$\therefore$ (C) option is correct

equation of $C_1 = 0$ & $C_2 = 0$ are (1, 2) and (–1, 2)

$\therefore$ slope of line joining centre is

$$= \frac{-2-2}{-1-1} = 2 = m_1 \text{ (say)}$$

and slope of L = 0 is $\frac{-1}{2} = m_2$ (say)

$\therefore$ $m_1 m_2 = -1$

hence (D) is correct

60. (B,C)

Equation of tangent of parabola $y^2 = 40x$ is

$$y = mx + \frac{10}{m} \quad \text{...(i)}$$

Equation of tangent of circle $x^2 + y^2 = 50$

$$y = mx \pm 5\sqrt{2}\sqrt{1+m^2} \quad \text{...(ii)}$$

Equation constant from (i) & (ii) i.e.

$$\frac{10}{m} = \pm 5\sqrt{2}\sqrt{1+m^2}$$

$\Rightarrow$ $m^4 + m^2 - 2 = 0$ $\Rightarrow$ $(m^2+2)(m^2-1) = 0$

$\Rightarrow$ $m = \pm 1$

So equation of common tangent are

$\Rightarrow$ $x - y + 10 = 0$ $x + y + 10 = 0$