# SOLUTIONS 

# PHASE TEST-1 RB-1810 TO 1812 RBK-1805 (JEE ADVANCED PATTERN) Test Date: 15-10-2017 

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## CHEMISTRY

1. (B)
(III) is most acidic due to ortho effect and (I) is least acidic due to $-\mathrm{NO}_{2}$ do not participated in resonance with benzene ring. Therefore acidic strength of (III) $>$ (IV) $>$ (II) $>$ (I)
2. (B)

Acidic strength $\propto \mathrm{k}_{\mathrm{a}} \alpha \frac{1}{\mathrm{P}_{\mathrm{k}_{\mathrm{a}}}} \alpha \frac{1}{\mathrm{pH}}$
3. $(A)$
+M group increase the $\mathrm{e}^{-}$density
4. (B)

The moles of the gas in the bubble remains constant, so that $n_{1}=n_{2}$. To calculate the final volume, $\mathrm{V}_{2}$,

$$
V_{2}=V_{1} \times \frac{p_{1}}{p_{2}} \times \frac{T_{2}}{T_{1}}
$$

$=2.0 \mathrm{~mL} \times \frac{6.0 \mathrm{~atm}}{1.0 \mathrm{~atm}} \times \frac{298 \mathrm{~K}}{281 \mathrm{~K}}$
$=12.72 \mathrm{~mL}$.
5. (A)

Let wt. of $\mathrm{NH}_{4} \mathrm{NO}_{3}$ and $\left(\mathrm{NH}_{4}\right)_{2} \mathrm{HPO}_{4}$ are $x$ and $y$ gram prespectively

$$
\frac{\frac{x}{80} \times 2 \times 14+\frac{y}{132} \times 2 \times 14}{x+y} \times 100=30.4
$$

$\Rightarrow x: y=2: 1$
6. (D)
$\mathrm{X}_{2} \mathrm{O}_{3}+3 \mathrm{H}_{2} \longrightarrow 2 \mathrm{X}+3 \mathrm{H}_{2} \mathrm{O}$
1 mol 3 mol
$(2 a+48) g 6 g$
$0.006 \mathrm{~g} \mathrm{H}_{2}$ is required by 0.1596 g oxide.
$\therefore 6 \mathrm{~g} \mathrm{H}_{2}$ will be required by 159.6 g oxide.
$\therefore \quad 2 a+48=159.6 \Rightarrow a=55.8$
where, $\mathrm{a}=$ atomic mass of metal M .
7. (C)

$$
\Delta \mathrm{En}<1.7 \text {-ionic bond(polar) }
$$

The reason is that $\mathrm{BaCl}_{2}$ has the biggest difference in electronegativity, which gives ionic character, we can tell this since electronegativity increases up and to the right on the periodic table and decreases down and to the left. Since Barium is farthest down and to the left. It has the lowest electronegativity which gives at the most ionic character.
8. (B)
$\mathrm{NaOH} \rightarrow \mathrm{Na}+: \stackrel{\ominus}{\mathrm{O}}-\mathrm{H}$
9. (A), (D)
10. (B, D)
$\therefore$ both are liquid \& $\mathrm{CH}_{3} \mathrm{OH}$ is solute (less amount)
Mass of $\mathrm{CH}_{3} \mathrm{OH}=30 \times 0.8=24 \mathrm{~g}$,
Mass of $\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}=60 \times 0.92=55.2 \mathrm{~g}$
Mass of solution $=24+55.2=79.2 \mathrm{~g}$
Volume of solution $=\frac{79.2}{0.88}=90 \mathrm{~mL}$
Molarity $=\frac{\mathrm{nCH}_{3} \mathrm{OH}}{\mathrm{V}(\mathrm{L})}=\frac{24 / 32}{90} \times 1000=8.33 \mathrm{~mol} \mathrm{~L}^{-1}$
Molality $=\frac{\mathrm{n}_{\text {solute }}}{\mathrm{w}_{\text {solvent }}(\mathrm{kg})}=\frac{24 / 32}{55.2} \times 1000=13.59$

Mole fraction of solute $=\frac{\frac{24}{32}}{\frac{24}{32}+\frac{55.2}{46}}=0.385$
Mole fraction of solvent $=1-0.385=0.615$
11. (A), (B)
12. $(A, B, C)$


No back bonding due to carbon has no vacant arbital.

(A) Nitrogen changing the hybridisation from $s p^{3}$ to $s p^{2}$ to acheive [ $\left.2 p-3 d\right] \pi$ effective back bonding.
(B) Trisilylamine has less basic due to back bonding tendency and Trimethylamine has no back bonding.
(C) Trisilylamine has $p \pi-d \pi$ back bonding.
13. (B)

At constant temperature $\mathrm{T}, \mathrm{V} \propto \frac{1}{\mathrm{P}}$ (for 1 mol gas).
Thus, correct sequence of volume will be :

$$
V_{1}<V_{2}<V_{3}<V_{4}
$$

14. (B)

$$
M=\frac{V}{11.2}=\frac{11.2}{11.2}=1
$$

$\therefore \mathrm{M} \Rightarrow 1$ and mol. mass of $\mathrm{H}_{2} \mathrm{O}_{2}=34$
$\therefore 34 \mathrm{~g} \mathrm{H}_{2} \mathrm{O}_{2}$ persent per litre of solution of
$3.4 \mathrm{~g} \mathrm{H}_{2} \mathrm{O}_{2}$ present per 100 mL of solution.
15. (B)
m.eq. of $\left(\mathrm{MnO}_{4}\right)_{2}=m . e q$. of $\mathrm{H}_{2} \mathrm{O}_{2}$

$$
\begin{aligned}
& \left(\therefore \mathrm{M}=\frac{33.6}{11.2} \Rightarrow 3\right) \\
& \frac{\mathrm{w}}{375} \times 10 \times 1000=3 \times 125 \times 2 ; \\
& \quad \% \text { purity }=\frac{\mathrm{w}}{40} \times 100 \\
& \quad=\frac{28.125}{40} \times 100=70.31
\end{aligned}
$$

16. (A)
17. (C)
18. (C)
19. (3)

In HPh, Eq. of $\mathrm{NaOH}+$ Eq. of $\mathrm{Na}_{2} \mathrm{CO}_{3}=$ Eq. of HCl
$(0.5)(1)+(0.5)(1)=(x)(1) x=1$
In MeOH , Eq. of $\mathrm{NaOH}+$ Eq. of $\mathrm{Na}_{2} \mathrm{CO}_{3}+$ Eq. of $\mathrm{NaHCO}_{3}=\mathrm{Eq}$. of HCl
$(0.5)(1)+(0.5)(2)+(0.5)(1)=(y)(1)=2$

$$
x+y=3
$$

20. (7)

Balanced redox reaction

$$
28 \mathrm{NO}_{3}^{-}+3 \mathrm{As}_{2} \mathrm{~S}_{3}+4 \mathrm{H}_{2} \mathrm{O} \longrightarrow 6 \mathrm{AsO}_{4}^{3-}+28 \mathrm{NO}+9 \mathrm{SO}_{4}^{2-}+8 \mathrm{H}^{+}
$$

21. (9)
$p_{f}=1+\frac{36}{76}=\frac{112}{76}$ atm. Final height $=19 \mathrm{~cm}$
$p_{f}=1$ atm, initial length $=h_{i} \mathrm{~cm}$
$\therefore$ Boyle's law $\quad \mathrm{P}_{\mathrm{i}} \mathrm{V}_{\mathrm{i}}=\mathrm{p}_{\mathrm{f}} \mathrm{V}_{\mathrm{f}}$

$$
\begin{aligned}
& 1 \times h_{i} A=\frac{112}{76} \times 19 A \\
& h_{i}=28 \mathrm{~cm}
\end{aligned}
$$

$\therefore$ The length by which the Hg column shifts down $=\mathrm{h}_{\mathrm{i}}-\mathrm{h}_{\mathrm{f}}=9$
22. (7)

| 4 | 3 | 2 | 2 |
| :--- | :--- | :--- | :--- |

23. (6)


The negative charge is delocalised on the marked carbon atoms (1-6).
24. (9)
(b) $\xrightarrow{1,2 \mathrm{H} \text { shift }} \stackrel{+}{+}$
(c) $\xrightarrow{\text { 1,2 } \sigma \text { bond shift }}$

$(\mathrm{d}) \xrightarrow{1,2 \mathrm{H} \text { shift }}$

$(\mathrm{e}) \xrightarrow{1,2 \mathrm{H} \text { shift }}$

(g)

(i) $\xrightarrow{1,2 \mathrm{H} \text { shift }}$

(j)

(k)


$(\mathrm{n}) \xrightarrow{1,2 \mathrm{H} \text { shift }}$

25. (1)
26. (4)

Automic size
(1) $\mathrm{Kr}>\mathrm{Ne}$
(2) $\mathrm{Na}>\mathrm{Na}^{+}$
(3) $\mathrm{I}^{-}>\mathrm{Cl}^{-}$
(4) $\mathrm{Li}_{(\mathrm{aq})}^{+}>\mathrm{Na}_{(\mathrm{a})}^{+}$
27. (4)
$E_{N}=\frac{I P+E A}{2 \times 2.80}($ When IP $/ E A$ are in electron volt $)$
$3.05=\frac{13.0+\mathrm{E}_{\mathrm{A}}}{2 \times 2.80}$
$=5.60 \times 3.05=13-E_{A}$
$E_{A}=4$
28. (5)

By POAC on 'By' - atom

$$
\begin{aligned}
& 2 \times \mathrm{nBr}_{2}=1 \times \mathrm{n}_{\text {rr }_{n}} \\
& 2 \times \frac{1 \times 0.423}{0.0821 \times 423}=1 \times \frac{4.2}{(80+19 n)} \quad \Rightarrow n=5
\end{aligned}
$$

## MATHEMATICS

29. (D)

The image of $A$ in $y=x$ will lie on $B C$
$A^{\prime}=(5,4)$
$A D \perp B C$
$2\left(\frac{4-k}{5-h}\right)=-1 \Rightarrow 8-2 k=-5+h$

$\because h=k$
$\therefore \mathrm{h}=\mathrm{k}=\frac{13}{3}$
30. (B)

Slope of $A B=-2 ;$ slope of $A C=\frac{-1}{2} ;$ slope of $B C=m$
$\frac{m+2}{1-2 m}=\frac{-\frac{1}{2}-m}{1-\frac{1}{2} m} \Rightarrow 4-m^{2}=-\left(1-4 m^{2}\right)=4 m^{2}-1$
$5 \mathrm{~m}^{2}=5$

$$
\Rightarrow \quad m= \pm 1
$$

$(y-2)=1(x-1)$ or $(y-2)=-1(x-1)$

x-intercept $x=-1 \quad x=3$ Ans.
31. (A)

Clearly lines intersect at $P(20,7)$

$\therefore$ Area of shaded region $=\frac{1}{2}\left(42-40 \frac{1}{3}\right) 20=\frac{1}{2}\left(\frac{5}{3}\right) 20=\frac{50}{3}$ (square units)
32. (D)

$$
\text { Let } P \equiv(a \cos \theta, a \sin \theta)
$$

and centroid of $\triangle A P B$ be $(h, k)$.
Then $\mathrm{h}=\frac{\mathrm{a} \cos \theta+0+\mathrm{a}}{3}, \mathrm{k}=\frac{\mathrm{a} \sin \theta+\mathrm{a}+0}{3}$

$$
\Rightarrow \cos \theta=\frac{3 \mathrm{~h}}{\mathrm{a}}-1, \sin \theta=\frac{3 \mathrm{k}}{\mathrm{a}}-1
$$

$\because \sin ^{2} \theta+\cos ^{2} \theta=1$

$$
\Rightarrow\left(\frac{3 h}{a}-1\right)^{2}+\left(\frac{3 k}{a}-1\right)^{2}=1 \quad \Rightarrow 9 h^{2}+9 k^{2}-6 a h-6 a k+a^{2}=0
$$


so locus of centroid is

$$
9 x^{2}+9 y^{2}-6 a x-6 a y+a^{2}=0
$$

33. (C)

We have, $\tan ^{-1} x+\cos ^{-1} \frac{y}{\sqrt{1+y^{2}}}=\sin ^{-1} \frac{3}{\sqrt{10}}$
$\Rightarrow \tan ^{-1} x+\tan ^{-1} \frac{1}{y}=\tan ^{-1} 3 \Rightarrow \tan ^{-1}\left(\frac{x+\frac{1}{y}}{1-\frac{x}{y}}\right)=\tan ^{-1} 3$
$\Rightarrow \frac{x y+1}{y-x}=3 \Rightarrow y=\frac{3 x+1}{3-x}$
Now, $y>0 \Rightarrow \frac{3 x+1}{3-x}>0 \Rightarrow-\frac{1}{3}<x<3 \Rightarrow x=1,2$
Hence, the required solutions are $(1,2)$ and $(2,7)$.
34. (A)

Graph of $f(x)$ is given by


$$
\text { Therefore period of } f(x) \text { is } 6 \text { and }|f(x)| \text { is } 1 \quad \Rightarrow T_{1}^{2}+T_{2}^{2}=37
$$

35. (D)

$$
x^{2}+4 x+\alpha^{2}-\alpha \geq 0 \forall x \in R
$$

$$
\text { According to given condition we must have } D=0 \Rightarrow \alpha=\frac{1 \pm \sqrt{17}}{2}
$$

36. (D)

$$
\begin{aligned}
& x=\frac{10[x]-14}{[x]+1} \\
& \because \quad[x] \leq x<[x]+1 \\
& \therefore \quad[x]=2,6,7
\end{aligned}
$$

37. $(A, C)$
$2 x^{2}+5 x y+2 y^{2}+4 x+5 y+a=0$ represents a pair of line.
$\therefore \Delta=0 \quad \Rightarrow a=2$
$\therefore$ the seperate straight lines are $2 \mathrm{x}+\mathrm{y}+2=0$ and $\mathrm{x}+2 \mathrm{y}+1=0$.
If $b=2$, then two of the three lines are parallel and then $n=2$.
If $b=1 / 2$, then again two lines are parallel and then $n=2$.
If $b=5$, then the lines are concurent, and then $n=0$.
38. (A, D)

$$
f(x)=\left[\begin{array}{ll}
\frac{a x^{2}+b x}{} & \text { for }-1<x<1 \\
\frac{a-b-1}{2} & x=-1 \\
\frac{a+b+1}{2} & x=1 \\
\frac{1}{x} & \text { for } x>1 \text { or } x<-1
\end{array}\right.
$$

for continuity at $\mathrm{x}=1$

$$
\begin{equation*}
a+b=1 \tag{1}
\end{equation*}
$$

for continuity at $x=-1$

$$
\begin{equation*}
a-b=-1 \Rightarrow a-b=-1 \tag{2}
\end{equation*}
$$

hence $a=0$ and $b=1$
39. (A, C)

$$
\begin{aligned}
& 4^{\mathrm{x}}-2^{\mathrm{x}+2}+5+||\mathrm{b}-1|-3|=|\sin y| \\
& =4^{\mathrm{x}}-2^{\mathrm{x}} \cdot 4+4+1+||\mathrm{b}-1|-3|=|\sin y| \\
& \Rightarrow\left(2^{\mathrm{x}}-2\right)^{2}+1+||\mathrm{b}-1|-3|=|\sin \mathrm{y}|
\end{aligned}
$$

Now, LHS $\geq 1$ and $R H S \leq 1$, equality is possible only when LHS $=$ RHS $=1$.
$\therefore||b-1|-3|=0$
$\Rightarrow|\mathrm{b}-1|=3 \Rightarrow \mathrm{~b}-1= \pm 3$
$\therefore \mathrm{b}=4,-2$
40. (A, B, C,D)


If $\mathrm{CL} \| \mathrm{AP}$ then $\mathrm{OL}=\mathrm{OA}-\mathrm{AL}=\mathrm{OA}-\mathrm{CQ}=36-9=27$
$\sin \theta=\frac{\mathrm{OL}}{\mathrm{OC}}=\frac{27}{45}=\frac{3}{5} \Rightarrow \tan \theta=\frac{3}{4}$
$\because \quad \angle \mathrm{APB}=2 \theta=2 \tan ^{-1} \frac{3}{4}=\sin ^{-1}\left(\frac{24}{25}\right)$
Further, length of common tangent is distance between points of contact = AQ
Here,

$$
\mathrm{AQ}=\mathrm{CI}=\sqrt{45^{2}-27^{2}}=36
$$

Also, area

$$
(\triangle \mathrm{OAP})=\frac{1}{2} \mathrm{OA} \times \mathrm{AP}=\frac{1}{2} \times 36 \times 48=864
$$

41. $(A, D)$

We have line $x+\lambda y=1+k \lambda$

$$
\Rightarrow(x-1)+\lambda(y-k)=0 \quad\left(L_{1}+\lambda I_{2}=0\right)
$$

hence $(1, k)$ lies on circle $C_{1}$
$\Rightarrow \mathrm{k}^{2}+1=4 \Rightarrow \mathrm{k}= \pm \sqrt{3}$
$\Rightarrow$ length of tangent from $(1, \pm 3)$ to the circle $C_{2}$ is $\sqrt{1+9-1}=3$
42. (B)
43. (A)
44. (D)

$$
\begin{aligned}
& \mathrm{x}=3^{\log 5-\log 7} \\
& \mathrm{y}=5^{\log 7-\log 3} \\
& \mathrm{z}=7^{\log 3-\log 5} \\
& \therefore \quad \mathrm{x} \cdot \mathrm{y} \cdot \mathrm{z}=1 \quad \therefore \quad \mathrm{~A}=1
\end{aligned}
$$

$$
\begin{aligned}
& \log _{2}\left(6 \log _{2}|\mathrm{x}|-3\right)-\log _{2}\left(4 \log _{2}|\mathrm{x}|-5\right)=\log _{2} 3 \\
& \\
& \quad \frac{6 \log _{2}|\mathrm{x}|-3}{4 \log _{2}|\mathrm{x}|-5}=3 \quad \text { let } \quad \log _{2}|\mathrm{x}|=\mathrm{t} \quad \therefore \quad \frac{6 \mathrm{t}-3}{4 \mathrm{t}-5}=3 \\
& 6 \mathrm{t}-3=121-15,6 \mathrm{t}=12 \quad \therefore \quad \mathrm{t}=2, \log _{2}|\mathrm{x}|=2, \quad|\mathrm{x}|=4 \quad \therefore \mathrm{x}= \pm 4 \\
& \mathrm{~B}=16+16=32 \\
& \log _{2}\left(\log _{2} 3\right)+\log _{2}\left(\log _{3} 4\right)+\log _{2}\left(\log _{4} 5\right)+\log _{2}\left(\log _{5} 6\right)+\log _{2}\left(\log _{6} 7\right)+\log _{2}\left(\log _{7} 8\right) \\
& \quad=\log _{2}\left(\log _{2} 8\right)=\log _{2} 3 \\
& \therefore \quad C=1 \text { Ans.] }
\end{aligned}
$$

45. (D)
46. (C)

$$
\begin{equation*}
f(x f(y))=x^{2} y^{n}, \quad(n \in R) \tag{1}
\end{equation*}
$$

putting $x=1, f(f(y))=y^{n}$
Now putting $f(y)=\frac{1}{x}$ in
$f(1)=\frac{y^{n}}{\left(f(y)^{2}\right.} \Rightarrow$ put $y=1$

$f(1)=\frac{1}{(f(1))^{2}} \Rightarrow(f(1))^{3}=1 \Rightarrow f(1)=1$
$\Rightarrow \therefore \mathrm{f}(\mathrm{x})=\mathrm{x}^{2}$
$2 f(x)=e^{x}$
$2 x^{2}=e^{x}$
for $x>0$
47. (1)

$$
\begin{gathered}
a=\log _{12} 18 \& b=\log _{24} 54 \\
a=\frac{\log _{2} 18}{\log _{2} 12}=\left(\frac{2 \log _{2} 3+1}{\log _{2} 3+2}\right), b=\frac{\log _{2} 54}{\log _{2} 24}=\left(\frac{3 \log _{2} 3+1}{\log _{2} 3+3}\right) \\
\text { Let } \log _{2} 3=x \Rightarrow a=\left(\frac{2 x+1}{x+2}\right) b=\left(\frac{3 x+1}{x+3}\right) \\
a b+5(a-b)=\frac{(2 x+1)(3 x+1)}{(x+2)(x+3)}+5\left(\frac{2 x+1}{x+2}-\frac{3 x+1}{x+3}\right)
\end{gathered}
$$

$$
\begin{aligned}
& =\frac{6 x^{2}+5 x+1+5\left(2 x^{2}+7 x+3-3 x^{2}-7 x-2\right)}{(x+2)(x+3)} \\
& =\frac{x^{2}+5 x+6}{(x+2)(x+3)}=\frac{(x+2)(x+3)}{(x+2)(x+3)}=1
\end{aligned}
$$

48. (1)

Any line through $A(10,-8)$ is $\frac{x-10}{\cos \theta}=\frac{y+8}{\sin \theta}=r$

$$
\Rightarrow x=10+r \cos \theta, \quad y=-8+r \sin \theta
$$

If the point lies on the curve $x^{2}-x y+y^{2}+4 x-5 y-2=0$, then

$$
\begin{aligned}
& (10+r \cos \theta)^{2}-(10+r \cos \theta)(r \sin \theta-8)+(r \sin \theta-8)^{2}+4(10+r \cos \theta)-5(r \sin \theta-8)-2=0 \\
& \Rightarrow r^{2}(1-\sin \theta \cos \theta)+r(32 \cos \theta-31 \sin \theta)+322=0 \\
& \therefore r_{1}+r_{2}=\frac{31 \sin \theta-32 \cos \theta}{1-\sin \theta \cos \theta}, \quad r_{1} r_{2}=\frac{322}{1-\sin \theta \cos \theta}
\end{aligned}
$$

Let $A B=r_{1}, A C=r_{2}$, and $A P=r$, then $\frac{2}{r}=\frac{1}{r_{1}}+\frac{1}{r_{2}}$
$\Rightarrow 644=31 r \sin \theta-32 r \cos \theta=31(y+8)-32(x-10)$
$\Rightarrow 32 x-31 y+76=0$
$\therefore a=32, b=-31$
49. (5)

$$
\begin{array}{ll}
3 x-7 \leq x^{2}-3 x+2<3 x-7+1 \quad \& 3 x \in Z \\
\Rightarrow 0 \leq x^{2}-6 x+9<1 \quad \& 3 x \in Z & \\
\Rightarrow 2<x<4 \quad \& 3 x=n \text { for some } n \in Z & \\
\Rightarrow 2<\frac{n}{3}<4 \quad \& x=\frac{n}{3}, n \in Z & \Rightarrow 6<n<12 \quad \& x=\frac{n}{3}, n \in Z \\
\Rightarrow n \in\{7,8,9,10,11\} \& x=\frac{n}{3}, n \in Z & \Rightarrow x \in\left\{\frac{7}{3}, \frac{8}{3}, 3, \frac{10}{3}, \frac{11}{3}\right\}
\end{array}
$$

50. 

(4)

The period of the function is 8
$\therefore \sum_{r=0}^{\infty}(f(1+8 r))^{r}=5$
$\Rightarrow 1+\mathrm{f}(1)+(\mathrm{f}(1))^{2}+\ldots \infty$ terms $=5$
$\frac{1}{1-f(1)}=5$
$5 f(1)=4$
51. (8)

By symmetry, the quadrilateral is a rectangle having $y=x$ and $y=-x$ as axis of symmetry.
Let $(a, b)$ be one of the vertex then
Area $=2\left|a^{2}-b^{2}\right|$
$=2 \sqrt{\left(a^{4}+b^{4}-2 a^{2} b^{2}\right)}=16$
52. (2)

$$
\left[\cot ^{-1} x\right]+2\left[\tan ^{-1} x\right]=0, \text { will be satisfied only when }
$$

$\left[\cot ^{-1} x\right]=0 \&\left[\tan ^{-1} x\right]=0$
or $\left[\cot ^{-1} x\right]=2 \&\left[\tan ^{-1} x\right]=-1$
Case-I $\left[\cot ^{-1} \mathrm{x}\right]=0 \Rightarrow \mathrm{x} \in(\cot 1, \infty)$
$\&\left[\tan ^{-1} x\right]=0 \Rightarrow x \in[0, \tan 1)$
$\therefore \mathrm{x} \in(\cot 1, \tan 1)$
Case-II $\left[\cot ^{-1} x\right]=2 \Rightarrow x \in(\cot 3, \cot 2]$

$$
\begin{aligned}
& \&\left[\tan ^{-1} x\right]=-1 \Rightarrow x \in[-\tan 1,0) \\
& \therefore x \in[-\tan 1, \cot 2]
\end{aligned}
$$

Thus the solution set for the given equation is

$$
[-\tan 1, \cot 2] \cup(\cot 1, \tan 1) \Rightarrow x=1,-1
$$

53. (7)

$$
\text { Let } \mathrm{x}=\mathrm{I}+\mathrm{f} \quad 0 \leq \mathrm{f}<1
$$

$73 I+\left[f+\frac{1}{19}\right]+\left[f+\frac{1}{20}\right]+\ldots+\left[f+\frac{1}{91}\right]=546$
Now $546=7 \times 73+35$

$$
\Rightarrow I=7
$$

54. (4)

Centre $\equiv C_{n}=1+(n-1) \cdot 3 \Rightarrow C_{n}=3 n-2$

$$
C_{5}=(13,0), C_{3}=(7,0)
$$

Radius $\equiv R_{n}=a r^{n-1}=2^{n-1} \Rightarrow R_{3}=4$
tangents from $C_{5}(13,0)$ to $C_{3}$ are given by $y-0=m(x-13) \Rightarrow m= \pm \frac{2}{\sqrt{5}} \Rightarrow m_{1} \cdot m_{2}=\frac{4}{5}$
55. (1)

$$
\begin{aligned}
& \lim _{x \rightarrow \infty}\left\{x^{3 c}\left(1+\frac{4}{x}+\frac{1}{x^{3}}\right)^{c}-x\right\} \\
& \lim _{x \rightarrow \infty} x\left[x^{3 c-1}\left\{1+\left(\frac{4}{x}+\frac{1}{x^{3}}\right)\right\}^{c}-1\right] \\
& \lim _{x \rightarrow \infty} x\left[x^{3 c-1}\left\{1+c\left(\frac{4}{x}+\frac{1}{x^{3}}\right)\right\}+\ldots .-1\right] \\
& 3 c=1
\end{aligned}
$$

56. (4)

The equation of circle can be taken as $(x+r)^{2}+(y-r)^{2}=r^{2}$
If it passes through $(2 \alpha, 3 \beta)$ then $4 \alpha^{2}+9 \beta^{2}+4 \alpha r-6 \beta r+r^{2}=0$
$\Rightarrow r^{2}+(4 \alpha-6 \beta) r+4 \alpha^{2}+9 \beta^{2}=0$ is a quadratic equation in ' $r$ '
If radii are $r_{1}$ and $r_{2}$ of the circles then $r_{1}+r_{2}=6 \beta-4 \alpha, \quad r_{1} r_{2}=4 \alpha^{2}+9 \beta^{2}$
Now two circles intersect each other orthogonally,
therefore $\left(r_{1}-r_{2}\right)^{2}+\left(r_{1}-r_{2}\right)^{2}=r_{1}^{2}+r_{2}^{2}$
$\Rightarrow r_{1}^{2}+r_{2}^{2}=4 r_{1} r_{2}$
$\Rightarrow(6 \beta-4 \alpha)^{2}=6\left(4 \alpha^{2}+9 \beta^{2}\right)$
$\Rightarrow 2\left(9 \beta^{2}+4 \alpha^{2}-12 \alpha \beta\right)=3\left(4 \alpha^{2}+9 \beta^{2}\right)$
$\Rightarrow 4 \alpha^{2}+9 \beta^{2}+24 \alpha \beta=0$

$$
(2 \alpha+3 \beta)^{2}=-12 \alpha \beta
$$

## PHYSICS

57. (C)
58. (C)

Friction between $P$ and $Q$ will retard $P$ (and accelerate $Q$ ) till slipping is stopped
Masses of the blocks are same so
$\therefore \quad$ Retardation of $P=$ acceleration of $Q=\mu \mathrm{g}$


Thus $\mathrm{v}_{\mathrm{p}}=\mathrm{u}-\mu \mathrm{gt}$ and $\mathrm{v}_{\mathrm{q}}=\mu \mathrm{gt}$
Once slipping is stopped both blocks will move with same velocity (i.e. $\frac{\mathrm{u}}{2}$ ). Graph (C) depicts this treatment.
59. (A)

Focal length of the convex lens
$\frac{1}{f}=\left(\frac{\mu_{2}-\mu_{1}}{\mu_{1}}\right)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$
$\frac{1}{f}=\left(\frac{1.5-1}{1}\right)\left(\frac{1}{R}-\frac{1}{\infty}\right)=\frac{1}{2 R} \Rightarrow f=2 R$
So the ray would become parallel to the principal axis after the refraction and fall $\perp^{r}$ to the mirror and hence would get reflected back along the same path.
60. (C)

In the case of minimum deviation, ray inside the prism is parallel to base.
Therefore, ray is deviated equally from both refracting faces
If, $\delta=34^{\circ}, \delta^{\prime}=\frac{\delta}{2}=17^{\circ}$
61. Net force on any charge $=0$.Force on any charge $Q$ at end
$F=K \frac{Q^{2}}{4 x^{2}}+\frac{K q Q}{x^{2}}=0$. Hence, $q=\frac{-Q}{4}$
$\therefore \quad(\mathrm{A})$
62. (C)


$$
\frac{1}{20}=(\mu-1)\left(\frac{1}{R}-\frac{1}{-R}\right)
$$

$$
=\frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)
$$

$$
f=40 \mathrm{~cm}
$$

63. $\Sigma F_{y}=0, R=m a$
$m g=\mu R=\mu m a$
$\mu=\frac{g}{a}=0.5$
$\therefore \quad$ (C)

64. For charge $+q$ at $A$ to come down, $\mathrm{F}_{\mathrm{e}}<\mathrm{mg}$
$\therefore \quad \frac{q^{2}}{4 \pi \varepsilon_{0} h^{2}}<\mathrm{mg}$
$\therefore \quad$ (C)
65. (B,D)
66. (A, B)

67. $v_{x}=3 \mathrm{~m} / \mathrm{s}$
$a_{x}=-1.0 \mathrm{~m} / \mathrm{s}^{2}$
$\therefore \quad v_{x}^{2}=u_{x}^{2}+2 a_{x} \cdot x$
or $\quad 0=(3)^{2}+2(-1)(x)$ or $x=4.5 \mathrm{~m}$
Also $v_{x}=u_{x}+a_{x} t$

$$
0=3-(1.0) t \text { or } \mathrm{t}=3 \mathrm{~s}
$$

$y=u_{y} t+\frac{1}{2} a_{y} t^{2}=0+\frac{1}{2}(-0.5)(3)^{2}=-2.25 \mathrm{~m}$
and $v_{y}=a_{y} t=(-0.5)(3)=-1.5 \mathrm{~m} / \mathrm{s}$
$\therefore \quad \vec{v}=v_{x} \hat{i}+v_{y} \hat{j}=0-1.5 \hat{j}=(-1.5 \hat{j}) \mathrm{m} / \mathrm{s}$
and $\vec{r}=x \hat{i}+y \hat{j}=(4.5 \hat{i}-2.25 \hat{j}) \mathrm{m}$
$\therefore \quad(B)$ and (C)
68. If the image is real and magnified means object is between $f$ and $2 f$.

When lens immersed in water focal length, $\quad f_{1}=\frac{(\mu-1)}{\left(\frac{\mu}{\mu_{r}}-1\right)} f=4 f$
Now object is between pole and focus so image is virtual and magnified.
$\therefore \quad(A)$ and (C)
69. Friction maximum $=24 \mathrm{~N}$

So net applied force on $P$ is less than $f_{\text {max }}$.
Hence acceleration is zero and $T_{A}=20 \mathrm{~N}, T_{B}=40 \mathrm{~N}$
Contact force $=\sqrt{N^{2}+(f)^{2}}=\sqrt{(40)^{2}+(20)^{2}}=20 \sqrt{5} \mathrm{~N}$
$\therefore \quad(A)(B)(C)$ and (D)
70. $F \cos \theta=m a, \quad 13 \cos \theta=5, \quad \cos \theta=\frac{5}{13}$
$\therefore \quad$ (B)
71. $F \sin \theta-\mu m g=m a_{1}$
$13 \times \frac{12}{13}-0.6 \times 10=m a_{1} \quad\left(a_{1}=\right.$ acceleration of the particle with respect to train $)$
$a_{1}=6 \mathrm{~m} / \mathrm{s}^{2}$
$a_{\text {net }}=\sqrt{36+25}=\sqrt{61} \mathrm{~m} / \mathrm{s}^{2}$
$\therefore \quad(A)$
72. K.E. $=\frac{1}{2} \times 1 \times 100^{2}=5 \times 10^{3} \mathrm{~J}$
$\therefore \quad(A)$
73. (A)


Here
(A) $\Delta x=d \sin \theta=d \tan \theta=d \times \frac{h}{l}$

For green light to be missing

$$
\Delta \mathrm{x}=\frac{\lambda_{\mathrm{g}}}{2} \quad \Rightarrow \mathrm{~h}=\frac{\lambda_{\mathrm{g}} \mathrm{l}}{2 \mathrm{~d}}
$$

for minimum $\mathrm{h}, \mathrm{n}$ should be equal to 1
or $h_{\text {min }}=\frac{\lambda_{g} l}{2 d}=\frac{5 \times 10^{-7} \times 0.5}{2 \times 10^{-3}}=1.25 \times 10^{-4} \mathrm{~m}$
Here fringe width $\beta=\frac{\lambda_{g} D}{d}=\frac{5 \times 10^{-7} \times 1}{10^{-3}}=5 \times 10^{-4} \mathrm{~m}$
74. (B)

If intensity due to $S_{2}$ any point on the screen is, $I_{2}=4 I_{0} \cos ^{2} \frac{\phi}{2}$, then intensity due to $S_{1}$ at the same point $I_{1}=4 I_{0} \cos ^{2}\left[\frac{\phi+\phi_{1}}{2}\right]$
where, $\phi_{1}=\left(\frac{h d}{l}\right) \times \frac{2 \pi}{\lambda}=2 \times \frac{\lambda l}{2 d} \times \frac{d}{l} \times \frac{2 \pi}{\lambda}=2 \pi$
i.e. $I_{1}=4 I_{0} \cos ^{2}\left[\frac{\phi+2 \pi}{2}\right]=4 I_{0} \cos ^{2} \frac{\phi}{2}=I_{2}$
$\therefore \quad$ total intensity $I=I_{1}+I_{2}=8 I_{0} \cos ^{2} \frac{\phi}{2}$
$\therefore \quad$ minimum distance of maximum from $O=\frac{\beta}{2}=\mathbf{2 . 5} \times \mathbf{1 0}^{-4} \mathbf{m}$
75. (6)


Let T be the tension in the string. The upward force exerted on the clamp $=\mathrm{T} \sin 30^{\circ}=\mathrm{T} / 2$
$\mathrm{T} / 2=40 \mathrm{~N} \Rightarrow \mathrm{~T}=80 \mathrm{~N}, \mathrm{a}=\frac{\mathrm{T}-\mathrm{mg}}{\mathrm{m}}=\frac{80-50}{5}=6 \mathrm{~m} / \mathrm{s}^{2}$
76. (1)

for $B, u=30+10=40 \mathrm{~cm}$
$\mathrm{f}=-20 \mathrm{~cm}$
$v=-40 \mathrm{~cm}$
$m=-\frac{v}{u}=-1$
for $A \quad u=-30 f=-20, v=-60$
$\mathrm{A}_{1} \mathrm{~B}_{1}=\sqrt{20^{2}+10^{2}}=\sqrt{500}=10 \sqrt{5} \mathrm{~cm}$
77. (3)

For image formed by lens
$\frac{1}{\mathrm{v}_{1}}-\frac{1}{-15}=\frac{1}{+10}$
$\Rightarrow \mathrm{v}_{1}=+30 \mathrm{~cm}$
i.e. 20 cm behind mirror

For mirror

$$
\begin{aligned}
& \frac{1}{v_{2}}+\frac{1}{20}=\frac{1}{-20} \\
& \Rightarrow v_{2}=-10 \mathrm{~cm}
\end{aligned}
$$

Overall magnification $=\left(\frac{30}{-15}\right) \times\left(\frac{-10}{20}\right)=1$
Length of image $=1 \times 3=3 \mathrm{~mm}$
78. (1)
79. (2)
80. (1)

The light escape is confined within a cone of apex angle ' $2 \theta_{c}{ }^{\prime}$ where $\theta_{c}$ is the critical angle. Imagine a sphere with source of light as its centre and the surface area $a b c$ is $A$.
here

$$
\begin{gathered}
A=\int_{0}^{\theta_{c}} 2 \pi R^{2} \sin \theta d \theta=2 \pi R^{2}\left(1-\cos \theta_{c}\right) \\
=\pi R^{2} \quad\left[\therefore \theta_{c}=\sin ^{-1}\left(\frac{\sqrt{3}}{2}\right)=60^{\circ}\right] \\
\therefore \quad \text { Power transfer }=P \times \frac{A}{4 \pi R^{2}} \\
=4 \times \frac{1}{4}=1 \mathrm{~W}
\end{gathered}
$$

81. (4)

Here $3^{\text {rd }}$ maxima is shifted by $3 \times 10^{-4} \mathrm{~m}$. It indicates fringe width increases by $1 \times 10^{-4} \mathrm{~m}$.
Hence $\beta=\frac{\lambda(D+0.5)}{d}=\frac{\lambda D}{d}+1 \times 10^{-4}$
or $\frac{0.5 \lambda}{d}=1 \times 10^{-4} \mathrm{~m} \quad$ or $\quad \lambda=\frac{2 \times 10^{-3} \times 1 \times 10^{-4}}{0.5}=4 \times 10^{-7} \mathrm{~m}=400 \mathrm{~nm}$
82. (6)

Let $A S=h$
Now, $\beta=\frac{\lambda D}{d}$
In the first case, $d=2 h$

$$
\begin{equation*}
\therefore \quad \beta=\frac{\lambda D}{2 h} \tag{i}
\end{equation*}
$$

In the second case, $d=2(h+\Delta x)$, where $\Delta x=$ shift in the source away from the mirror along $A B$.

$$
\begin{equation*}
\therefore \quad \beta^{\prime}=\frac{\lambda D}{2(h+\Delta x)} \tag{ii}
\end{equation*}
$$

Dividing equation (i) by equation (ii), we have,

$$
\frac{\beta}{\beta^{\prime}}=\frac{h+\Delta x}{h}=1+\frac{\Delta x}{h}
$$

or, $\quad \frac{\beta-\beta^{\prime}}{\beta^{\prime}}=\frac{\Delta x}{h} \Rightarrow h=\left(\frac{\Delta x \times \beta^{\prime}}{\beta-\beta^{\prime}}\right)=\frac{.6 \times \frac{1}{6}}{\frac{1}{4}-\frac{1}{6}}=1.2 \mathrm{~mm}$
Putting the value of $h$ in equation (1), we get

$$
\lambda=\frac{2 \times \beta \times h}{D}=\frac{2 \times 1.4 \times 10^{-3} \times 1.2 \times 10^{-3}}{1}=6 \times 10^{-\mathbf{7}} \mathbf{~ m}
$$

83. (7)

$$
\begin{equation*}
7 T-N-M g=M a \tag{i}
\end{equation*}
$$

$T+N-m g=m a$
Dividing the equation (1) and (2)
$\left(\frac{7 m-M}{m+M}\right) T=N>0, \frac{\mathrm{M}}{\mathrm{m}}>7$
84. (3)

Let $M_{1}$ be the mass of the rod.
$M_{1} g-N_{1} \cos \theta=M_{1} A_{1} \ldots$ (i)
$N_{1} \sin \theta=(M+M) A$
$A=g \tan \theta$
relation between $A_{1}$ and $A$

$A_{1}=A \tan \theta$
So by solving these equations $M_{1}=3 M$

