SOLUTIONS

WEEKLY TEST-15

GRS-1801 & GRKS-1801

[TOP 170 STUDENTS]

(JEE MAIN PATTERN)

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PHYSICS

1. (B)

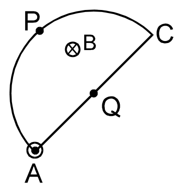
$B = \frac{\mu_0 i}{2\pi r}$ $r = \sqrt{2^2 + 3^2 + 6^2} = 7$

$\therefore\ B = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{7}{7} = 2 \times 10^{-7}\,T$.

2. (A) **3. (C)**

4. (B)

We connect a conducting wire from A to C and complete the semicircular loop.



The loop emf in the semicircular loop is zero because its magnetic flux does not change.

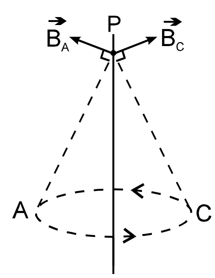
$\therefore$ emf of section APC + emf of section CQA = 0

or emf of section APC = emf of section AQC = $2Ba^2\omega$

$2B\omega a^2$.

5. (C)

The point charge moves in circle as shown in figure. The magnetic field vectors at a point P on axis of circle are $\vec{B}_A$ and $\vec{B}_C$ at the instants the point charge is at A and C respectively as shown in the figure.



Hence as the particles rotates in circle, only magnitude of magnetic field remains constant at the point on axis P but its direction changes.

Alternate solution

The magnetic field at point on the axis due to charged particle moving along a circular path is given by

$$\frac{\mu_o}{4\pi}\frac{q\vec{v} \times \vec{r}}{r^3}$$

It can be seen that he magnitude of the magnetic field at an point on the axis remains constant. But the direction of the of the field keeps on changing.

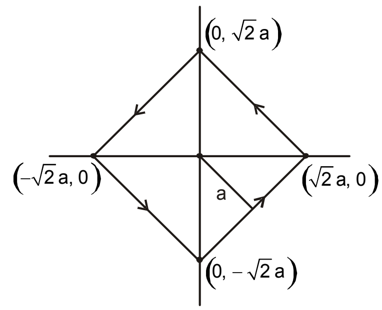
6. (B)

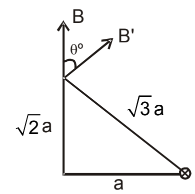
$$B' = \frac{\mu_0 i}{4\pi a\sqrt{3}}(\sin 30^\circ + \sin 30^\circ)$$

$$= \frac{\mu_0 i}{4\pi a\sqrt{3}}$$

$$B_{||} = B'\cos\theta = \frac{\mu_0 i}{4\pi a\sqrt{3}} \cdot \frac{1}{\sqrt{3}}$$

$$B_{net} = 4B_{||} = \frac{\mu_0 i}{3\pi a}$$



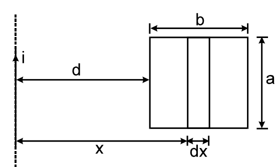


7. (A) **8. (C)** **9. (B)** **10. (B)**

11. (A)

12. (A)

The flux in rectangular loop due to current i in wire is



$$\phi = \int_d^{d+b} \frac{\mu_0 i}{2\pi x}\ a\, dx = \frac{\mu_0 i a}{2\pi}\ \ell n\ \frac{b+d}{d}$$

Mutual inductance is

$$M = \frac{\phi}{i} = \frac{\mu_0 a}{2\pi}\ \ell n\ \frac{b+d}{d}$$

$\therefore$ Mutual inductance is proportional to 'a'.

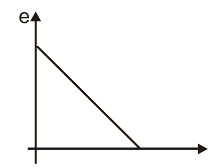
13. (A)

Sol. When switch K_1 is opended and K_2 is closed it becomes L–C circuit so applying energy conservation :

$$\frac{Q_0^2}{2C} = \frac{1}{2} Li^2 ;\ Q_0 = C_{eq}V = \frac{C_1C_2}{C_1+C_2} \cdot V = (20 \times 10^{-6})$$

$$\frac{(20\times10^{-6})^2}{2\times2\times10^{-6}} = \frac{1}{2}(0.2 \times 10^{-3})\, i^2 \Rightarrow i = 1\text{ A}.$$

14. (A)



The potential difference across the inductor is $e = E - iR$.

Hence the plot of e versus i is a straight line with negative slope.

15. (B)

$\frac{1}{2}Li^2 = 5 \quad \Rightarrow L = 5 \times 2$

$i^2 R = 10 \quad \Rightarrow R = 10$

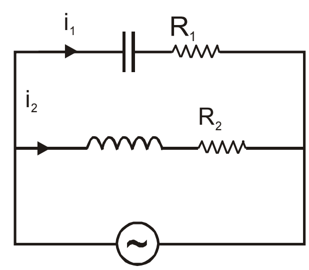
$\Rightarrow \tau = 1$ sec.

16. (A)

It is apparant from the graph that emf attains its maximum value before the current does, therefore current lags behind emf in the circuit. Nature of the circuit is inductive.

Value of power factor cos ϕ increases by either decreasing L of increasing C.

17. (C)



$$i_{1_{rms}} = \frac{E_{rms}}{\sqrt{X_C^2 + R_1^2}} = \frac{130}{13} = 10\ A$$

$$i_{2_{rms}} = \frac{E_{rms}}{\sqrt{X_L^2 + R_2^2}} = 13\ A$$

Power dissipated = $i_{1rms}^2 R_1 + i_{2rms}^2 R_2 = 10^2 \times 5 + 13^2 \times 6$

= power delivered by battery

= 500 + 169 × 6

= 1514 watt.

18. (C)

19. (D)

Acceleration of block AB = $\frac{3mg}{3m+m} = \frac{3}{4}g$; acceleration of block CD = $\frac{2mg}{2m+m} = \frac{2g}{3}$

Acceleration of image in mirror AB

= 2 acceleration of mirror

$= 2.\left(\frac{-3g}{4}\right) = \frac{-3}{2}g$

Acceleration of image in mirror CD = $2.\left(\frac{2g}{3}\right) = \frac{4g}{3}$

$\therefore$ Acceleration of the two image w.r.t. each other = $\frac{4g}{3} - \left(\frac{-3g}{2}\right) = \frac{17g}{6}$.

20. (B)

21. (D)

Conceptual.

22. (B)

$$E \propto \rho \Rightarrow \frac{\partial V}{\partial x} \propto \rho$$

Hence slope will be more in second wire.

23. (B)

The equivalent circuit is

C_{AB} = 60 μF.

24. (D)

When switch S_2 is closed, due to symmetry no charge will flow through S_2.

25. (C)

$F \propto q^2$. With insertion of dielectric 'q' increases 'k' times.

26. (C)

Charge on outer surface of C = – charge on inner surface of C

Hence potential at B due to charge an conductor C = 0

charge on outer surface of dielectric = – charge an inner surface of dielectric

$\therefore$ Potential at B due to charge an dielectric = 0

Potential at B due to charge on A $= \frac{Q}{4\pi \in_0 b}$

$\therefore$ net potential at B $= \frac{Q}{4\pi \in_0 b}$.

27. (C)

Charge flown = $q_f - q_i$ = 15 – 6 = 9 μC.

28. (D)

$$E = \frac{-dv}{dx} = \frac{-d(5+4x^2)}{dx} = -8x$$

$$F = -qE = 8qx = 8 \times 2 \times 10^{-6} \times 0.5\,N = 8 \times 10^{-6}\,N.$$

29. (B)

$E = 6t$

$$\frac{dH}{dt} = \frac{E^2}{R} = \frac{36t^2}{12} = 3t^2$$

$$H_{lib} = \int_0^4 3t^2 dt = 3\left(\frac{t^3}{3}\right)_0^4 = 64\ J.$$

30. (D)

$$i = \frac{E_{eff}}{r_{eff}} = \frac{nE}{nr} = \frac{E}{r}.$$

CHEMISTRY

31. (A)

$$\frac{(0.5+2)\times 20\times 1000}{40\times 500} = \Delta T_b + \Delta T_f = 2.5$$

$\therefore$ Required difference $= 80 + 2.5 = 82.5^\circ C = 82.5K$

32. (B)

$y = 1 - x$ ...(1)

$\Delta T_f = 1.85(2\times x + 3\times y) = (2x + 3(1-x)) = 1.85(3-x)$

when $x = 0$. $\Delta T_f = 5.55°C$

when $x = 1$. $\Delta T_f = 3.7°C$.

33. (C)

Number of Faraday = number of equivalents

$$\frac{2\times 5\times 60\times 60}{96500} = n\times\frac{22.2}{177}$$

$\therefore$ n = **3**.

34. (C)

Total charge = $\frac{15\times 3860}{96500}$ = 0.6 F

Fe^{3+}	+ e	$\longrightarrow$	Fe^{2+}
0.3 mol	0.6 F		
0	0.3 F		0.3 mol
Fe^{2+}	+ 2e	$\longrightarrow$	Fe
0.3 mol	0.3 F		
0.15 mol	0		0.15 mol

35. (C)

36. (B)

$$A = 4x \quad e^{-k_1 t}$$

$$B = x \quad e^{-k_2 t}$$

$$4 = e^{(k_2 - k_1)t}$$

$$2 \ln 2 = (k_2 - k_1)t$$

$$2 \ln 2 = \left[\frac{\ln 2}{5} - \frac{\ln 2}{15}\right]t$$

$$2 = \frac{15-5}{75}t$$

$$t = 15$$

37. (D)

38. (C)

Number of A atoms = 7

Contribution of each = $\frac{1}{8}$

$\therefore$ Net contribution of A atoms = $\frac{7}{8}$

B atoms at the face centres

$\therefore$ Net contribution = $\frac{6}{2}$ = 3

Formula = $A_{7/8}B_3$ = A_7B_{24}.

39. (B)

Mass of one fcc unit cell = $\frac{4 \times M}{N_A}$.

$\rho = \frac{4 \times M}{N_A\ a^3}$ $\therefore$ $\frac{4M}{N_A} = \rho\ a^3$.

Total number of fcc unit cells = $\frac{100}{\rho\ a^3} = \frac{100}{10 \times (10^{-8})^3} = 10^{25}$

Each fcc unit cell contains 4 atoms. So number of atom is 100 g of fcc unit cell = 4×10^{25}.

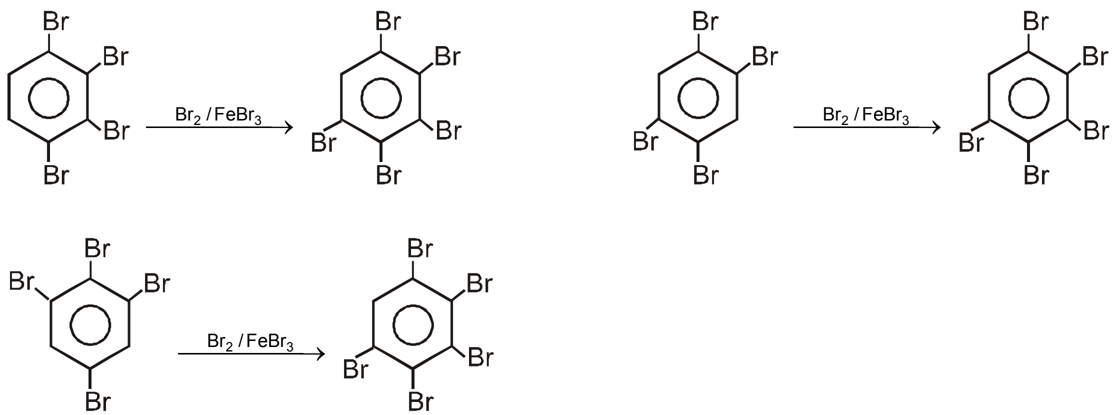
40. (D)

Packing fraction in one layer of square close packing = $\dfrac{\left(\frac{4}{3}\pi r^3\right)}{(2r)^3} = \dfrac{\pi}{6}$

Percentage of occupied space in one layer = $\dfrac{\pi}{6}\times 100$

$\therefore$ Percentage of vacant space in one layer = $100 - \dfrac{\pi}{6}\times 100$.

41. (C)



42. (A)

Rate of electrophilic substitution $\propto$ Stability of arenium ion.

43. (A)

Electrophile attack on that ring which have more +M effect.

44. (D)

If both +M group are present of benzene ring then electrophilic attack in the influence of more +M group.

45. (A)

46. (D)

47. (B)

$C^1 – C^2$ - is shorter because it is double bond in two of three resonance strucutre ; C_2–C_3 is a single bond in two of three resonance strucutres.

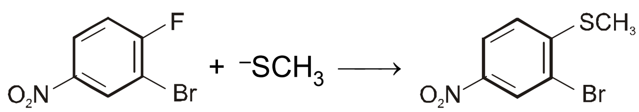
48. (C)

The + M of Cl stabilizes the intermediate carbocation.

49. (A)

– I and – M group increases rate of ArS_{N2} reaction.

50. (B)



51. (D)

$$\left.\begin{array}{l} X = 5 \\ Y = 0 \end{array}\right\}$$

52. (D)

$$KMnO_4 \xrightarrow{\Delta} \underset{\text{(green)}}{K_2MnO_4} + \underset{\text{(black)}}{MnO_2} + O_2$$

$$K_2Cr_2O_7 \xrightarrow{\Delta} \underset{\text{yellow}}{K_2CrO_4} + \underset{\text{green}}{Cr_2O_3} + O_2$$

$$KNO_3 + K \xrightarrow{\Delta} K_2O + N_2$$

$$Pb_3O_4 \xrightarrow{\Delta} PbO + O_2$$

53. (D)

54. (A)

55. (C)

56. (A)

X - trivalent ; Y - pentavalent ; Z - monovalent

Hence only A is possible.

57. (C)

$$Cr_2O_7^{--} \xrightarrow[\substack{\downarrow \\ \text{Alkaline med} \\ (pH>7)}]{pH=x} CrO_4^{--} \xrightarrow[\substack{\downarrow \\ \text{acidic med} \\ (pH<7)}]{pH=y}$$

58. (A)

$$FeO + SiO_2 \rightarrow FeSiO_3$$

59. (B)

$$Co + Cl_2 \xrightarrow{h\nu} CoCl_2$$

60. (B)

MATHEMATICS

61. (A)

As $\sum_{i=1}^{6}(\sin^{-1}x_i+\cos^{-1}y_i)=9\pi$

$=(\sin^{-1}x_1+\sin^{-1}x_2+....+\sin^{-1}x_6)+(\cos^{-1}y_1+\cos^{-1}y_2+...+\cos^{-1}y_6)=9\pi$

Which is possible only when $\sin^{-1}x_i=\frac{\pi}{2}$ & $\cos^{-1}y_i=\pi\ (\forall i=1,2,...6)$

$\therefore x_i=1$ & $y_i=-1\ \forall i=1,2,....6.$

$\sum_{i=1}^{6}x_i=6$ and $\sum_{i=1}^{6}y_i=-6$

Now $\int_{6}^{-6}x\,\ell n(1+x^2)\left(\frac{1}{e^x+e^{-x}}\right)dx=0.$ (Given function is odd function)

62. (C)

$xy^2dx-y(x^2-y^2)^2\,dx=y^3\,dy-x(x^2-y^2)^2\,dy$

$y^2(x\,dx-ydy)=(x^2-y^2)^2(ydx-xdy)$

$\frac{d(x^2-y^2)}{(x^2-y^2)^2}=2d\left(\frac{x}{y}\right)$

$\frac{2x}{y}+\frac{1}{x^2-y^2}=c$

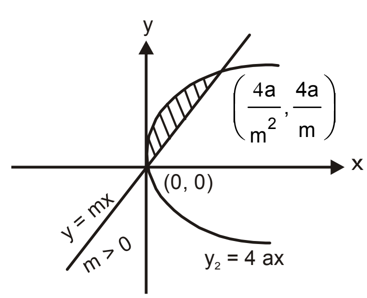
63. (B)

Required area

$=\int_0^{4a/m^2}\left[y(\text{parabola})\,dx-y(\text{line})\right]dx$

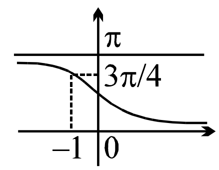
$=\int_0^{4a/m^2}\left(2\sqrt{a}\sqrt{x}-mx\right)dx$

$=\frac{4}{3}\sqrt{a}\left(\frac{4a}{m^2}\right)^{3/2}-\frac{m}{2}\left(\frac{4a}{m^2}\right)^2$



$$= \frac{32}{3}\frac{a^2}{m^3} - \frac{8a^2}{m^3} = \frac{8a^2}{3m^3}$$

64. (B)



$y = (x^2 - 1)^2 + 2 \Rightarrow \quad y_{min} = 2$

$\Rightarrow \; \log_{0.5}(x^4 - 2x^2 + 3) \le -1 \; \Rightarrow$ range $\left[\frac{3\pi}{4}, \pi\right)$

65. (B)

$$\lim_{x\to 0^-}\left[\frac{4f(x)-12}{\tan(2f(x)-6)}\right] = \lim_{x\to 0^-}\left[\frac{2(2f(x)-6)}{\tan(2f(x)-6)}\right] = 1$$

66. (C)

In a skew symmetric matrix,

diagonal elements are zero.

Also $a_{ij} + a_{ji} = 0$

$$\begin{bmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{bmatrix}$$

Hence number of matrices = 2 × 2 × 2 = 8

67. (A)

$$e^{f(x)}\, f(x) = x$$

$$e^{f(x)} \cdot f'(x) + f(x) e^{f(x)} . f'(x) = 1$$

$$f'(x) = \frac{1}{e^{f(x)} + f(x).e^{f(x)}} = \frac{1}{x + e^{f(x)}} > 0 \uparrow$$

68. (A)

$3 \le |[x]| + |[y]| \le 6$

$|[x]| = 0 \;\&\; 3 \le |[y]| \le 6 \quad \Rightarrow \; 0 \le x < 1 \;\&\; 3 \le y < 7, -6 \le y < -2$

Hence area = 1 × 8 = 8.

$|[x]| = 1 \;\&\; 2 \le |[y]| \le 5 \quad \Rightarrow \; 1 \le x < 2, -1 \le x < 0 \;\&\; 2 \le y < 6, -5 \le y < -1$

Area = 2 × 8

$|[x]| = 2 \;\&\; 1 \le |[y]| \le 4 \quad \Rightarrow \; 2 \le x < 3, -2 \le x < -1 \;\&\; 1 \le y < 5, -4 \le y < 0$

Area = 2 × 8

For $|[x]| = 3,\ 0 \le |[y]| \le 3$, Area $= 2 \times 7$

For $|[x]| = 4,\ |[y]| \le 2$, Area $= 2 \times 5$

For $|[x]| = 5, |[y]| \le 1$, Area $= 2 \times 3$

For $|[x]| = 6, |[y]| = 0$, Area $= 2 \times 1$

Hence total area = 8 + 16 + 16 + 14 + 10 + 6 + 2 = 72

$\Rightarrow \alpha = 7, \beta = 2$

Now, $I = \int_\beta^\alpha \frac{\sqrt{9-x}}{\sqrt{x}+\sqrt{9-x}} dx = \int_2^7 \frac{\sqrt{9-x}}{\sqrt{x}+\sqrt{9-x}} dx = \int_2^7 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} dx$

Hence, $2I = \int_2^7 dx = 5 \Rightarrow I = \frac{5}{2}$

69. (A)

$$I_n = 2\int_0^\pi \left(\frac{\pi}{2} - x\right)^{n+1} \cos nx \, dx$$

$$= 2\int_0^\pi \left(\frac{\pi}{2} - (\pi - x)\right)^{n+1} \cos(n(\pi - x)) \, dx$$

$$= 2\int_0^\pi (-1)^{n+1}\left(\frac{\pi}{2} - x\right)^{n+1} (-1)^n \cos nx \, dx = -I_n$$

$\therefore \quad I_n = 0 \;\; \forall\, n \in N$

70. (D)

$$A^2 = \begin{bmatrix} 3 & 1 \\ -6 & -2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -6 & -2 \end{bmatrix} \Rightarrow A^2 = A$$

$\therefore$ A is an idempotent matrix.

Now, $(I + A)^{99} = {}^{99}C_0 I + {}^{99}C_1 A + {}^{99}C_2 A^2 + ... + {}^{99}C_{99} A^{99}$

$= I + A({}^{99}C_1 + {}^{99}C_2 + ... + {}^{99}C_{99}) \quad \{\because \quad A = A^2 = A^3 = ... = A^{99}\}$

$= I + (2^{99} - 1)A$]

71. (B)

Using LMVT, $\frac{\tan^{-1}\beta - \tan^{-1}\alpha}{\beta - \alpha} = \frac{1}{1+c^2}$

where, $0 < \alpha < c < \beta < \sqrt{3}$

So, $\frac{1}{4} < \frac{1}{1+c^2} < 1$

$$\Rightarrow \frac{1}{4} < \frac{\tan^{-1}\beta - \tan^{-1}\alpha}{\beta - \alpha} < 1 \Rightarrow \frac{1}{4} < \frac{\tan^{-1}\left(\frac{\beta-\alpha}{1+\alpha\beta}\right)}{\beta-\alpha} < 1 \Rightarrow 1 < \frac{\beta-\alpha}{\cot^{-1}\left(\frac{1+\alpha\beta}{\beta-\alpha}\right)} < 4$$

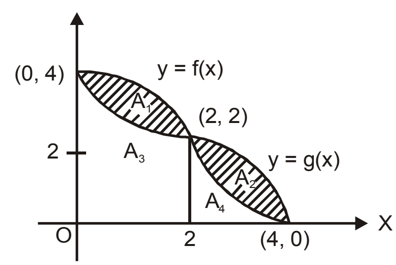
72. (C)

Given $\int_0^4 f(x)\,dx - \int_0^4 g(x)\,dx = 10$

$(A_1 + A_3 + A_4) - (A_2 + A_3 + A_4) = 10$

$A_1 - A_2 = 10$ ....(i)

Again $\int_2^4 g(x)\,dx - \int_2^4 f(x)\,dx = 5$



$(A_2 + A_4) - A_4 = 5$ ....(ii)

$A_2 = 5$

$\therefore$ (i) + (ii)

$A_1 = 15$

73. (A)

$$I = \int_{\pi/4}^{\pi/2} 2\sin t\cos t\,dt + \underbrace{\int_{\pi/2}^{\pi} (-\sin t\cos t) + (\sin t\cos t)\,dt}_{\text{zero}} + \int_{\pi}^{5\pi/4} -2\sin t\cos t\,dt$$

$$= \int_{\pi/4}^{\pi/2} \sin 2t\,dt - \int_{\pi}^{5\pi/4} \sin 2t\,dt = 0$$

74. (A)

$$u = \int_0^{\pi/2} \cos\left(\frac{2\pi}{3}\sin^2 x\right)dx \Rightarrow u = \int_0^{\pi/2} \cos\left(\frac{2\pi}{3}\cos^2 x\right)dx$$

$$\Rightarrow 2u = \int_0^{\pi/2} \left(\cos\left(\frac{2\pi}{3}\sin^2 x\right) + \cos\left(\frac{2\pi}{3}\cdot\cos^2 x\right)\right)dx$$

$\Rightarrow \quad 2u = \int_0^{\pi/2} 2\cos\frac{\pi}{3}\cdot\cos\left(\frac{\pi}{3}\cos 2x\right)dx = \frac{1}{2}\int_0^{\pi}\cos\left(\frac{\pi}{3}\cos t\right)dt$ [Put 2x = t]

$= \int_0^{\pi/2}\cos\left(\frac{\pi}{3}\cos t\right)dt = \int_0^{\pi/2}\cos\left(\frac{\pi}{3}\sin t\right)dt = v$

75. (C)

$A^2 = \begin{bmatrix}\cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha\end{bmatrix}, \; A^3 = \begin{bmatrix}\cos 3\alpha & \sin 3\alpha \\ -\sin 3\alpha & \cos 3\alpha\end{bmatrix}, \; A^4 = \begin{bmatrix}\cos 4\alpha & \sin 4\alpha \\ -\sin 4\alpha & \cos 4\alpha\end{bmatrix}$

Now, $\cos\alpha + \cos2\alpha + \cos3\alpha + \cos4\alpha = \cos\alpha + \cos2\alpha + \cos(\pi - 2\alpha) + \cos(\pi - \alpha) = 0$

$\sin\alpha + \sin2\alpha + \sin3\alpha + \sin4\alpha$

$= \sin\alpha + \sin2\alpha + \sin(\pi - 2\alpha) + \sin(\pi - \alpha) = 2(\sin\alpha + \sin2\alpha)$

$= 4\sin\frac{3\alpha}{2}\cos\frac{\alpha}{2} = 4\sin\frac{3\pi}{10}\cos\frac{\pi}{10} = a$

$\therefore B = \begin{bmatrix}0 & a \\ -a & 0\end{bmatrix}$

76. (B)

$g(x) = \lambda\sin x$

where $\lambda = 1 - 2\int_0^{\pi/2}(\cos t)g(t)dt$

$\Rightarrow \lambda = 1 - 2\int_0^{\pi/2}\cos t.\lambda\sin t\,dt \quad \Rightarrow \lambda = 1 - \lambda\int_0^{\pi/2}\sin 2t\,dt \quad \Rightarrow \lambda = 1 - 2\lambda\int_0^{\pi/4}\sin 2t\,dt$

$\Rightarrow \lambda = 1 + 2\lambda\left[\frac{\cos 2t}{2}\right]_0^{\pi/4} \Rightarrow \lambda = 1 + 2\lambda\cdot\left(-\frac{1}{2}\right) \quad \Rightarrow \quad 2\lambda = 1 \;\Rightarrow\; \lambda = \frac{1}{2}$

$g'(x) = \lambda\cos x \Rightarrow g'(\pi/4) = \frac{\lambda}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$

77. (B)

$I = \int\frac{dx}{x^{29}\left(1 - \frac{6}{x^7}\right)}$

Let $\left(1-\frac{6}{x^7}\right)=p \Rightarrow \frac{42}{x^8}dx = dp$ and $x^7=\left(\frac{6}{1-p}\right)$

$$I=\frac{1}{42}\int\frac{(1-p)^3}{(6)^3 p}dp=\frac{1}{(42)(216)}\int\frac{1-p^3-3p+3p^2}{p}$$

$$=\frac{1}{54432}\ [\ln p^6 + 9p^2 - 2p^3 - 18p] + c$$

78. (A)

Equation of tangent at $P(x_1, f(x_1))$

$$y-f(x_1)=f'(x_1)(x-x_1) \ \Rightarrow B\equiv(x_1,0),\ A\equiv\left(x_1-\frac{f(x_1)}{f'(x_1)},0\right)$$

According to equation, $2x_1-\frac{f(x_1)}{f'(x_1)}=4$

on generalising, $f'(x)=\frac{f(x)}{2(x-2)}$

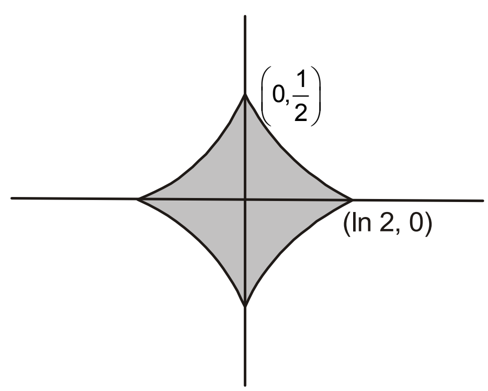
$$\frac{dy}{dx}=\frac{y}{2(x-2)} \ \Rightarrow \int 2\frac{dy}{y}=\int\frac{dx}{x-2} \Rightarrow 2\ln y=\ln(x-2)+k \Rightarrow y^2=c(x-2)$$

79. (D)

$|y|\le e^{-|x|}-\frac{1}{2}$

Graph is symmetric about x-axis and y-axis,

and $e^{-|x|}-\frac{1}{2}\ge 0$



$$\text{Area}=4\int_0^{\ln 2}\left(e^{-x}-\frac{1}{2}\right)dx$$

$= 2 - 2\ln 2$

80. (A)

$$\left(\frac{dy}{dx}-e^{-x}\right)\left(\frac{dy}{dx}-e^{x}\right)=0$$

$\Rightarrow \frac{dy}{dx} = e^{-x}$ or $\frac{dy}{dx} = e^{x}$

$\Rightarrow y = -e^{-x} + c$ or $y = e^{x} + c$

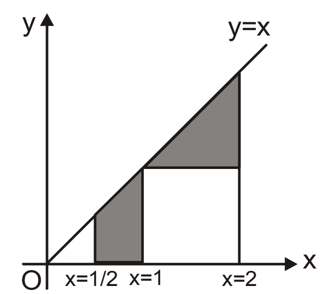
$\Rightarrow y + e^{-x} = c$ or $y = e^{x} + c$

81. (A)

Case –1 : Let $\frac{1}{2} \le x < 1$

$\Rightarrow 0 < \log_y x \le 1$

$\Rightarrow 1 < x \le y$, if y > 1(not possible) and $1 > x \ge y$, if 0<y<1



Case –2 : Let $1 < x \le 2$

$\Rightarrow \log_y x \ge 1 \Rightarrow x \ge y$, if y > 1

and $x \le y$, if y < 1(not possible)

So, the possibilities are $x \ge y$, if $\frac{1}{2} \le x < 1$ and 0<y<1 and $x \ge y$, if $1 < x \le 2$ and y > 1

So, required area is 7/8.

82. (B)

As the matrix is skew-symmetric, hence |A| = 0, if n is odd

$\Rightarrow$ A is not an invertible matrix when n is odd.

83. (B)

(i) AB is symmetric $(AB)^T = B^T A^T = AB \Rightarrow BA = AB$

(II) $(B^T AB)^T = B^T A^T (B^T)^T = B^T A^T B = B^T AB$

(iii) and (iv)

Let A be skew symmetric, then $A^T = -A$

and $(A^n)^T = (A^T)^n, \forall n \in N$

$$\Rightarrow (A^n)^T = \begin{cases} A^n & \text{If n is even} \\ -A^n & \text{If n is odd} \end{cases}$$

Hence A^n is symmetric if n is even

Hence Answer is B.

84. (D)

Rearranging the given differential eqn.

$$xdx + \frac{ydx - xdy}{y^4} = 0 \Rightarrow x^3dx + \frac{x^2}{y^2}.\frac{ydx - xdy}{y^2} = 0 \Rightarrow x^3dx + \left(\frac{x}{y}\right)^2 .d\left(\frac{x}{y}\right) = 0$$

on integration, $\frac{x^4}{4} + \frac{1}{3}\left(\frac{x}{y}\right)^3 = c$

85. (C)

$$x^2(4ydx + xdy) = \frac{xdy - 2ydx}{x^4 + y^2}$$

$$4x^2ydx + x^3dy = \frac{xdy - 2ydx}{x^4 + y^2}$$

$$4x^3ydx + x^4dy = \frac{x^2dy - 2xydx}{x^4 + y^2}$$

$$d(x^4y) = d\left(\tan^{-1}\frac{y}{x^2}\right)$$

$$\Rightarrow x^4y = \tan^{-1}\frac{y}{x^2} + c$$

86. (C)

$y^2 = kx \qquad 2y.\frac{dy}{dx} = k = \frac{y^2}{x} \qquad \Rightarrow \frac{dy}{dx} = \frac{y}{2x}$

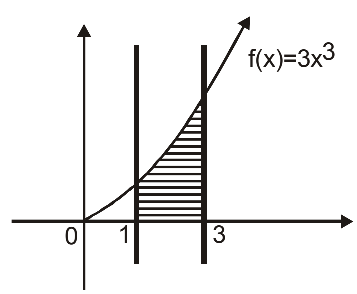
now, $\tan\frac{\pi}{4} = \left|\frac{\frac{dy}{dx} - \frac{y}{2x}}{1 + \frac{y}{2x}.\frac{dy}{dx}}\right| \qquad \Rightarrow \frac{dy}{dx} = \frac{2x + y}{2x - y}$ or $\frac{y - 2x}{y + 2x}$

87. (A)

$f(x) = ax^3 + bx$

Now $\lim_{x \to 0} \frac{f(x)}{x} = 0 \Rightarrow b = 0$

$$\lim_{x \to 0}\left(\frac{f(x)}{x} + 1\right)^{1/x^2} = e^3 \Rightarrow e^a = e^3 \Rightarrow a = 3$$



$\therefore\ f(x) = 3x^3$

$$A = \int_1^3 3x^3 dx = 60$$

88. (C)

$$x\,dy = y\,dx + \sqrt{x^2 + y^2}dx \Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sqrt{1 + (y/x)^2}$$

Put y/x = v $\qquad \dfrac{dy}{dx} = x\dfrac{dv}{dx} + v$

$$\therefore \int \frac{1}{\sqrt{1+v^2}}dv = \int \frac{dx}{x} \Rightarrow \ell n(v + \sqrt{1+v^2}) = \log x + \log c \Rightarrow y + \sqrt{x^2 + y^2} = cx^2$$

89. (D)

$xy(x) = x^2 y'(x) + y(x).2x$

$xy(x) + x^2 y'(x) = 0$

$$x\frac{dy}{dx} + y = 0$$

x y = 6

90. (B)

$$\frac{dx}{dy} = \frac{x}{y} + 2y^2$$

$$\frac{dx}{dy} - \frac{x}{y} = 2y^2$$

$$\frac{x}{y} = \int 2y dy$$

$$\frac{x}{y} = y^2 + c$$