SOLUTIONS

WEEKLY TEST-15

GRA

(JEE MAIN PATTERN)

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PHYSICS

1. **(D)**

Conceptual.

2. **(B)**

$$E \propto \rho \Rightarrow \frac{\partial V}{\partial x} \propto \rho$$

Hence slope will be more in second wire.

3. **(B)**

The equivalent circuit is

C_{AB} = 60 μF.

4. **(D)**

When switch S_2 is closed, due to symmetry no charge will flow through S_2.

5. **(C)**

$F \propto q^2$. With insertion of dielectric 'q' increases 'k' times.

6. **(C)**

Charge on outer surface of C = – charge on inner surface of C

Hence potential at B due to charge an conductor C = 0

charge on outer surface of dielectric = – charge an inner surface of dielectric

$\therefore$ Potential at B due to charge an dielectric = 0

Potential at B due to charge on A $= \dfrac{Q}{4\pi \in_0 b}$

$\therefore$ net potential at B $= \dfrac{Q}{4\pi \in_0 b}$.

7. **(C)**

Charge flown = $q_f - q_i$ = 15 – 6 = 9 μC.

8. **(D)**

$$E = \frac{-dv}{dx} = \frac{-d(5+4x^2)}{dx} = -8x$$

$F = -qE = 8qx = 8 \times 2 \times 10^{-6} \times 0.5\,N = 8 \times 10^{-6}\,N$.

9. **(B)**

$E = 6t$

$$\frac{dH}{dt}=\frac{E^2}{R}=\frac{36t^2}{12}=3t^2$$

$$H_{lib}=\int_0^4 3t^2dt \;=3\left(\frac{t^3}{3}\right)_0^4=64\ J.$$

10. (D)

$$i=\frac{E_{eff}}{r_{eff}}=\frac{nE}{nr}=\frac{E}{r}.$$

11. (C)

Current through cells $i=\frac{8E}{8r}=\frac{E}{r}$

$\therefore$ Terminal potential = E – ir = 0.

12. (C)

13. (B)

$$\Delta R_1+\Delta R_1=0$$

$$\Rightarrow \quad \alpha_1 R_1=\alpha_2 R_2$$

$$\Rightarrow \quad \alpha_1\rho_1 L_1=\alpha_2\rho_2 L_2.$$

14. (B)

Let length of x^{th} fraction is increased y times.

$$\therefore \quad \ell(1-x)+\ell xy=\frac{3\ell}{2}$$

$$\Rightarrow \quad x(y-1)=\frac{1}{2} \qquad \text{.... (i)}$$

$$\frac{\rho\ell}{A}(1-x)+\frac{\rho\ell}{A}xy^2=\frac{4\rho\ell}{A}$$

$$\Rightarrow \quad x(y^2-1)=3 \qquad \text{.... (ii)}$$

(ii)/(i) $\Rightarrow$ y = 5 $\quad\therefore\quad x=\frac{1}{8}.$

15. (C)

$$U_i=\frac{kq^2}{a}\left(-1+(-1)+\frac{1}{2}\right)$$

$U_f = \frac{kq^2}{a}\left(-1+1-\frac{1}{2}\right)$

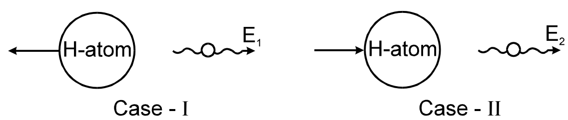
$U_f - U_i = \frac{kq^2}{a}$.

16. (D)

$E_3 + E_2 = E_1 \Rightarrow \frac{1}{\lambda_3} + \frac{1}{\lambda_2} = \frac{1}{\lambda_1}$.

17. (B)

Sol.



In the first case K.E. of H-atom increases due to recoil whereas in the second case K.E. decreases due to recoil.

$\therefore \quad E_2 > E_1$.

18. (A)

Sol. Linear momentum $\Rightarrow$ $mv \,\alpha\, \frac{1}{n}$

angular momentum $\Rightarrow$ $mvr \,\alpha\, n$

$\therefore$ product of linear momentum and angular momentum $\alpha\ n^0$.

19. (B)

Sol. $E = \frac{3}{2}kT$ & $P = \sqrt{2mE} \Rightarrow \lambda_{de-broglie} = \frac{h}{p} = \frac{h}{\sqrt{2m\left(\frac{3}{2}kT\right)}}$ $\lambda_{de-broglie} = \frac{h}{\sqrt{3mkT}}$

Substituting values :

$\lambda_{de-broglie} = 0.63$ Å .

20. (D)

Sol. The electron ejected with maximum speed v_{max} are stopped by electric field E = 4N/C after travelling a distance d = 1m

$\therefore \frac{1}{2} mV_{max}^2 = eE\,d = 4eV$

The energy of incident photon = $\frac{1240}{200} = 6.2$ eV

From equation of photo electric effect $\frac{1}{2}$ $mv_{max}{}^2 = h\upsilon - \phi_0$

$\therefore$ $\phi_0 = 6.2 - 4 = 2.2$ eV.

21. **(A)**

$$\frac{hc}{\lambda} = 5\ eV_0 + \phi$$

$$\frac{hc}{3\lambda} = eV_0 + \phi \Rightarrow \frac{2hc}{3\lambda} = 4eV_0 \Rightarrow \phi = \frac{hc}{6\lambda}.$$

22. **(A)**

$h\nu = eV$

$$\Rightarrow \nu = \frac{eV}{h} = \frac{1.6\times10^{-19}\times10\times10^{3}}{6.6\times10^{-34}} = 2.4 \times 10^{18} \text{ Hz.}$$

23. **(A)**

Conceptual

24. **(C)**

The energy of incident photons is given by $h\nu = eV_s + \phi_0 = 2 + 5 = 7$ eV (V_s is stopping potential and ϕ_0 is work function)

Saturation current = 10^{-5} A = $\frac{\eta P}{h\nu}e = \frac{10^{-5}P}{7\times e}e$ (η is photo emission efficiency)

$\therefore$ P = 7 W.

25. **(B)**

26. **(C)**

Energy released = (80 × 7 + 120 × 8 – 200 × 6.5) = 220 MeV.

27. **(B)** $N = \frac{\lambda E}{hc} \approx 28$

BE of X = 6A

BE of Y = 6A – 2 + 1 = 6A – 1

[Because absorption of energy decreases BE and releas of energy increases BE]

In Y nuclues there are A + 1 nuclues.

$$\therefore \frac{BE}{nucleon} = \frac{6A-1}{A+1}.$$

28. **(C)**

More the BE/A, more stable is the nucleus.

29. (B)

Rate of decay of A keeps on decreasing continuously because concentration of A decreases with time. $\Rightarrow$ A is false

Initial rate of production of B is $\lambda_1 N_0$ and rate of decay is zero. With time, as the number of B atom increase, the rate of its production decrease and its rate of decay increases. Thus the number of nuclei of B will first increase and then decrease. $\Rightarrow$ B is the correct choice

The initial activity of B is zero whereas initial activity of A is $\lambda_1 N_0 \Rightarrow$ C is false.

As time $t \to \infty$: $N_A = 0$, $N_B = 0$ and $N_C = N_0$. $\Rightarrow$ D is false

30. (A)

After first half hrs $N = N_0 \frac{1}{2}$

for $t = \frac{1}{2}$ to $t = 1\frac{1}{2}$ $\quad N = \left(N_0 \frac{1}{2}\right)\left[\frac{1}{2}\right]^4 = N_0 \left(\frac{1}{2}\right)^5$

for $t = 1\frac{1}{2}$ to t = 2 hrs [for both A and B $\frac{1}{t_{1/2}} = \frac{1}{1/2} + \frac{1}{1/4} = 2 + 4 = 6$ $t_{1/2} = 1/6$ hrs.]

$$N = \left[N_0 \left(\frac{1}{2}\right)^5\right] \left(\frac{1}{2}\right)^3 = N_0 \left(\frac{1}{2}\right)^8.$$

CHEMISTRY

31. (A)

$$\frac{(0.5+2)\times 20\times 1000}{40\times 500} = \Delta T_b + \Delta T_f = 2.5$$

$\therefore$ Required difference $= 80 + 2.5 = 82.5^\circ C = 82.5 K$

32. (B)

$y = 1 - x$ ...(1)

$$\Delta T_f = 1.85(2\times x + 3\times y) = (2x + 3(1-x)) = 1.85(3-x)$$

when $x = 0$. $\Delta T_f = 5.55^\circ C$

when $x = 1$. $\Delta T_f = 3.7^\circ C$.

33. (C)

Number of Faraday = number of equivalents

$$\frac{2\times5\times60\times60}{96500} = n\times\frac{22.2}{177}$$

$\therefore$ n = **3**.

34. (C)

Total charge = $\frac{15\times3860}{96500}$ = 0.6 F

Fe^{3+} +	e	$\longrightarrow$	Fe^{2+}
0.3 mol	0.6 F		
0	0.3 F		0.3 mol
Fe^{2+} +	2e	$\longrightarrow$	Fe
0.3 mol	0.3 F		
0.15 mol	0		0.15 mol

35. (C)

36. (B)

$A = 4x \quad e^{-k_1t}$

$B = x \quad e^{-k_2t}$

$4 = e^{(k_2-k_1)t}$

$2 \ln 2 = (k_2 - k_1)t$

$2 \ln 2 = \left[\frac{\ln 2}{5} - \frac{\ln 2}{15}\right]t$

$2 = \frac{15-5}{75}t$

$t = 15$

37. (D)

38. (C)

Number of A atoms = 7

Contribution of each = $\frac{1}{8}$

$\therefore$ Net contribution of A atoms = $\frac{7}{8}$

B atoms at the face centres

$\therefore$ Net contribution = $\frac{6}{2}$ = 3

Formula = $A_{7/8}B_3 = A_7B_{24}$.

39. (B)

Mass of one fcc unit cell = $\frac{4 \times M}{N_A}$.

$\rho = \frac{4 \times M}{N_A\ a^3}$ $\therefore$ $\frac{4M}{N_A} = \rho\ a^3$.

Total number of fcc unit cells = $\frac{100}{\rho\ a^3} = \frac{100}{10 \times (10^{-8})^3} = 10^{25}$

Each fcc unit cell contains 4 atoms. So number of atom is 100 g of fcc unit cell = 4×10^{25}.

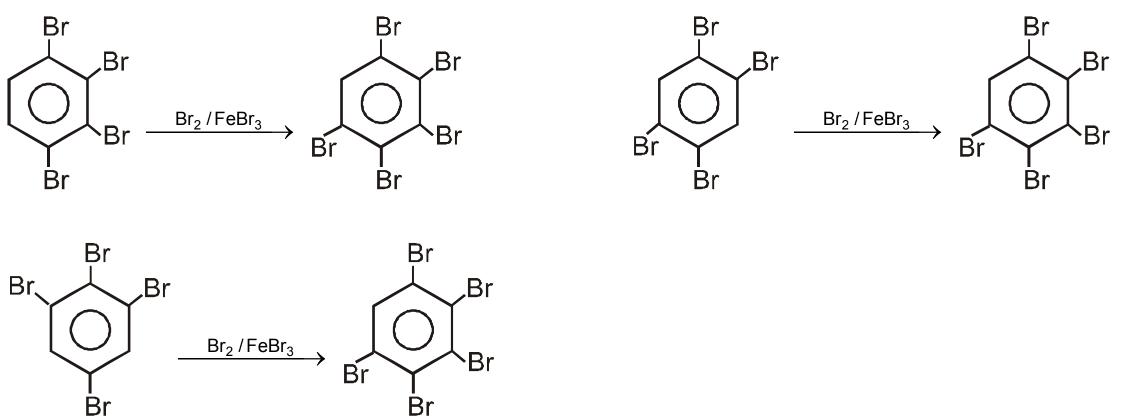
40. (D)

Packing fraction in one layer of square close packing = $\frac{\left(\frac{4}{3}\pi r^3\right)}{(2r)^3} = \frac{\pi}{6}$

Percentage of occupied space in one layer = $\frac{\pi}{6} \times 100$

$\therefore$ Percentage of vacant space in one layer = $100 - \frac{\pi}{6} \times 100$.

41. (C)



42. (A)

Rate of electrophilic substitution $\propto$ Stability of arenium ion.

43. (A)

Electrophile attack on that ring which have more +M effect.

44. (D)

If both +M group are present of benzene ring then electrophilic attack in the influence of more +M group.

45. (A)

46. (D)

47. (B)

$C^1 - C^2$ - is shorter because it is double bond in two of three resonance strucutre ; C_2–C_3 is a single bond in two of three resonance strucutres.

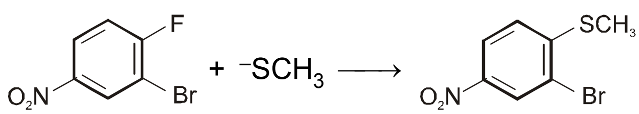
48. (C)

The + M of Cl stabilizes the intermediate carbocation.

49. (A)

– I and – M group increases rate of ArS_{N2} reaction.

50. (B)



51. (D)

$$\left.\begin{array}{l} X = 5 \\ Y = 0 \end{array}\right\}$$

52. (D)

$$KMnO_4 \xrightarrow{\Delta} \underset{(green)}{K_2MnO_4} + \underset{(black)}{MnO_2} + O_2$$

$$K_2Cr_2O_7 \xrightarrow{\Delta} \underset{yellow}{K_2CrO_4} + \underset{green}{Cr_2O_3} + O_2$$

$$KNO_3 + K \xrightarrow{\Delta} K_2O + N_2$$

$$Pb_3O_4 \xrightarrow{\Delta} PbO + O_2$$

53. (D)

54. (A)

55. (C)

56. (A)

X - trivalent ; Y - pentavalent ; Z - monovalent

Hence only A is possible.

57. (C)

$$Cr_2O_7^{--} \xrightarrow[\substack{\downarrow \\ \text{Alkaline med} \\ (pH>7)}]{pH=x} CrO_4^{--} \xrightarrow[\substack{\downarrow \\ \text{acidic med} \\ (pH<7)}]{pH=y}$$

58. (A)

$$FeO + SiO_2 \rightarrow FeSiO_3$$

59. (B)

$$Co + Cl_2 \xrightarrow{h\nu} CoCl_2$$

60. (B)

MATHEMATICS

61. (A)

As $\sum_{i=1}^{6}(\sin^{-1}x_i + \cos^{-1}y_i) = 9\pi$

$= (\sin^{-1}x_1 + \sin^{-1}x_2 + ... + \sin^{-1}x_6) + (\cos^{-1}y_1 + \cos^{-1}y_2 + ... + \cos^{-1}y_6) = 9\pi$

Which is possible only when $\sin^{-1}x_i = \frac{\pi}{2}$ & $\cos^{-1}y_i = \pi \ (\forall i = 1,2,...6)$

$\therefore x_i = 1$ & $y_i = -1 \ \forall i = 1,2,....6.$

$\sum_{i=1}^{6} x_i = 6$ and $\sum_{i=1}^{6} y_i = -6$

Now $\int_{6}^{-6} x\,\ell n(1+x^2)\left(\frac{1}{e^x + e^{-x}}\right) dx = 0$. (Given function is odd function)

62. (C)

$xy^2dx - y(x^2 - y^2)^2\,dx = y^3\,dy - x(x^2 - y^2)^2\,dy$

$y^2(x\,dx - ydy) = (x^2 - y^2)^2(ydx - xdy)$

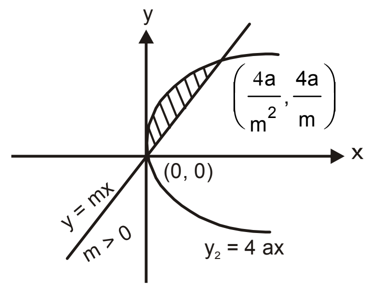
$$\frac{d(x^2-y^2)}{(x^2-y^2)^2}=2d\left(\frac{x}{y}\right)$$

$$\frac{2x}{y}+\frac{1}{x^2-y^2}=c$$

63. (B)

Required area

$$=\int_0^{4a/m^2}\left[y(\text{parabola})dx-y(\text{line})\right]dx$$



$$=\int_0^{4a/m^2}\left(2\sqrt{a}\sqrt{x}-mx\right)dx$$

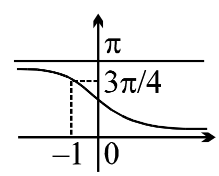
$$=\frac{4}{3}\sqrt{a}\left(\frac{4a}{m^2}\right)^{3/2}-\frac{m}{2}\left(\frac{4a}{m^2}\right)^2$$

$$=\frac{32}{3}\frac{a^2}{m^3}-\frac{8a^2}{m^3}=\frac{8a^2}{3m^3}$$

64. (B)

$y=(x^2-1)^2+2 \Rightarrow \quad y_{min}=2$

$\Rightarrow \ \log_{0.5}(x^4-2x^2+3)\leq -1 \ \Rightarrow$ range $\left[\frac{3\pi}{4},\pi\right)$



65. (B)

$$\lim_{x\to 0^-}\left[\frac{4f(x)-12}{\tan(2f(x)-6)}\right]=\lim_{x\to 0^-}\left[\frac{2(2f(x)-6)}{\tan(2f(x)-6)}\right]=1$$

66. (C)

In a skew symmetric matrix,

diagonal elements are zero.

Also $a_{ij}+a_{ji}=0$

Hence number of matrices = 2 × 2 × 2 = 8

$$\begin{bmatrix}0 & \times & \times\\ \times & 0 & \times\\ \times & \times & 0\end{bmatrix}$$

67. (A)

$$e^{f(x)}f(x)=x$$

$$e^{f(x)}.f'(x)+f(x)e^{f(x)}.f'(x)=1$$

$$f'(x) = \frac{1}{e^{f(x)} + f(x).e^{f(x)}} = \frac{1}{x + e^{f(x)}} > 0 \uparrow$$

68. **(A)**

$3 \le |[x]| + |[y]| \le 6$

$|[x]| = 0 \;\&\; 3 \le |[y]| \le 6 \;\Rightarrow\; 0 \le x < 1 \;\&\; 3 \le y < 7, -6 \le y < -2$

Hence area = 1 × 8 = 8.

$|[x]| = 1 \;\&\; 2 \le |[y]| \le 5 \;\Rightarrow\; 1 \le x < 2, -1 \le x < 0 \;\&\; 2 \le y < 6, -5 \le y < -1$

Area = 2 × 8

$|[x]| = 2 \;\&\; 1 \le |[y]| \le 4 \;\Rightarrow\; 2 \le x < 3, -2 \le x < -1 \;\&\; 1 \le y < 5, -4 \le y < 0$

Area = 2 × 8

For $|[x]| = 3,\ 0 \le |[y]| \le 3,\ \text{Area} = 2 \times 7$

For $|[x]| = 4,\ |[y]| \le 2,\ \text{Area} = 2 \times 5$

For $|[x]| = 5,\ |[y]| \le 1,\ \text{Area} = 2 \times 3$

For $|[x]| = 6,\ |[y]| = 0,\ \text{Area} = 2 \times 1$

Hence total area = 8 + 16 + 16 + 14 + 10 + 6 + 2 = 72

$\Rightarrow\ \alpha = 7, \beta = 2$

Now, $I = \int_\beta^\alpha \frac{\sqrt{9-x}}{\sqrt{x} + \sqrt{9-x}} dx = \int_2^7 \frac{\sqrt{9-x}}{\sqrt{x} + \sqrt{9-x}} dx = \int_2^7 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx$

Hence, $2I = \int_2^7 dx = 5 \;\Rightarrow\; I = \frac{5}{2}$

69. **(A)**

$$I_n = 2\int_0^\pi \left(\frac{\pi}{2} - x\right)^{n+1} \cos nx \, dx$$

$$= 2\int_0^\pi \left(\frac{\pi}{2} - (\pi - x)\right)^{n+1} \cos(n(\pi - x)) \, dx$$

$$= 2\int_0^\pi (-1)^{n+1}\left(\frac{\pi}{2} - x\right)^{n+1} (-1)^n \cos nx \, dx = -I_n$$

$\therefore\ I_n = 0\ \forall\, n \in N$

70. (D)

$A^2 = \begin{bmatrix} 3 & 1 \\ -6 & -2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -6 & -2 \end{bmatrix} \Rightarrow A^2 = A$

$\therefore$ A is an idempotent matrix.

Now, $(I + A)^{99} = {}^{99}C_0 I + {}^{99}C_1 A + {}^{99}C_2 A^2 + ... + {}^{99}C_{99} A^{99}$

$= I + A({}^{99}C_1 + {}^{99}C_2 + ... + {}^{99}C_{99})$ $\{\because A = A^2 = A^3 = ... = A^{99}\}$

$= I + (2^{99} - 1)A$]

71. (B)

Using LMVT, $\dfrac{\tan^{-1}\beta - \tan^{-1}\alpha}{\beta - \alpha} = \dfrac{1}{1+c^2}$

where, $0 < \alpha < c < \beta < \sqrt{3}$

So, $\dfrac{1}{4} < \dfrac{1}{1+c^2} < 1$

$\Rightarrow \dfrac{1}{4} < \dfrac{\tan^{-1}\beta - \tan^{-1}\alpha}{\beta - \alpha} < 1 \Rightarrow \dfrac{1}{4} < \dfrac{\tan^{-1}\left(\dfrac{\beta - \alpha}{1+\alpha\beta}\right)}{\beta - \alpha} < 1 \Rightarrow 1 < \dfrac{\beta - \alpha}{\cot^{-1}\left(\dfrac{1+\alpha\beta}{\beta - \alpha}\right)} < 4$

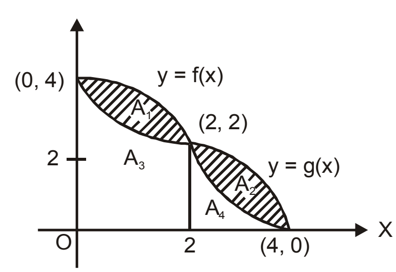
72. (C)

Given $\int_0^4 f(x)\,dx - \int_0^4 g(x)dx = 10$

$(A_1 + A_3 + A_4) - (A_2 + A_3 + A_4) = 10$

$A_1 - A_2 = 10$ ....(i)

Again $\int_2^4 g(x)dx - \int_2^4 f(x)dx = 5$



$(A_2 + A_4) - A_4 = 5$ ....(ii)

$A_2 = 5$

$\therefore$ (i) + (ii)

$A_1 = 15$

73. (A)

$$I = \int_{\pi/4}^{\pi/2} 2\sin t\cos t\,dt + \underbrace{\int_{\pi/2}^{\pi} (-\sin t\cos t)+(\sin t\cos t)dt}_{\text{zero}} + \int_{\pi}^{5\pi/4} -2\sin t\cos t\,dt$$

$$= \int_{\pi/4}^{\pi/2} \sin 2t\,dt - \int_{\pi}^{5\pi/4} \sin 2t\,dt = 0$$

74. (A)

$$u = \int_0^{\pi/2} \cos\left(\frac{2\pi}{3}\sin^2 x\right)dx \Rightarrow u = \int_0^{\pi/2} \cos\left(\frac{2\pi}{3}\cos^2 x\right)dx$$

$$\Rightarrow 2u = \int_0^{\pi/2}\left(\cos\left(\frac{2\pi}{3}\sin^2 x\right)+\cos\left(\frac{2\pi}{3}\cdot\cos^2 x\right)\right)dx$$

$$\Rightarrow 2u = \int_0^{\pi/2} 2\cos\frac{\pi}{3}\cdot\cos\left(\frac{\pi}{3}\cos 2x\right)dx = \frac{1}{2}\int_0^{\pi}\cos\left(\frac{\pi}{3}\cos t\right)dt \qquad \text{[Put } 2x = t\text{]}$$

$$= \int_0^{\pi/2}\cos\left(\frac{\pi}{3}\cos t\right)dt = \int_0^{\pi/2}\cos\left(\frac{\pi}{3}\sin t\right)dt = v$$

75. (C)

$$A^2 = \begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\ -\sin 2\alpha & \cos 2\alpha\end{bmatrix},\ A^3 = \begin{bmatrix}\cos 3\alpha & \sin 3\alpha\\ -\sin 3\alpha & \cos 3\alpha\end{bmatrix},\ A^4 = \begin{bmatrix}\cos 4\alpha & \sin 4\alpha\\ -\sin 4\alpha & \cos 4\alpha\end{bmatrix}$$

Now, $\cos\alpha + \cos 2\alpha + \cos 3\alpha + \cos 4\alpha = \cos\alpha + \cos 2\alpha + \cos(\pi - 2\alpha) + \cos(\pi - \alpha) = 0$

$\sin\alpha + \sin 2\alpha + \sin 3\alpha + \sin 4\alpha$

$= \sin\alpha + \sin 2\alpha + \sin(\pi - 2\alpha) + \sin(\pi - \alpha) = 2(\sin\alpha + \sin 2\alpha)$

$$= 4\sin\frac{3\alpha}{2}\cos\frac{\alpha}{2} = 4\sin\frac{3\pi}{10}\cos\frac{\pi}{10} = a$$

$$\therefore B = \begin{bmatrix}0 & a\\ -a & 0\end{bmatrix}$$

76. (B)

$g(x) = \lambda\sin x$

where $\lambda = 1 - 2\int_0^{\pi/2}(\cos t)g(t)dt$

$$\Rightarrow \lambda = 1 - 2\int_0^{\pi/2} \cos t.\lambda \sin t\,dt \quad \Rightarrow \lambda = 1 - \lambda\int_0^{\pi/2} \sin 2t\,dt \quad \Rightarrow \lambda = 1 - 2\lambda\int_0^{\pi/4} \sin 2t\,dt$$

$$\Rightarrow \lambda = 1 + 2\lambda\left[\frac{\cos 2t}{2}\right]_0^{\pi/4} \Rightarrow \lambda = 1 + 2\lambda\cdot\left(-\frac{1}{2}\right) \quad \Rightarrow \quad 2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$g'(x) = \lambda\cos x \Rightarrow g'(\pi/4) = \frac{\lambda}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

77. (B)

$$I = \int \frac{dx}{x^{29}\left(1 - \frac{6}{x^7}\right)}$$

Let $\left(1 - \frac{6}{x^7}\right) = p \Rightarrow \frac{42}{x^8}dx = dp$ and $x^7 = \left(\frac{6}{1-p}\right)$

$$I = \frac{1}{42}\int \frac{(1-p)^3}{(6)^3 p}dp = \frac{1}{(42)(216)}\int \frac{1 - p^3 - 3p + 3p^2}{p}$$

$$= \frac{1}{54432}\ [\ln p^6 + 9p^2 - 2p^3 - 18p] + c$$

78. (A)

Equation of tangent at $P(x_1, f(x_1))$

$$y - f(x_1) = f'(x_1)(x - x_1) \quad \Rightarrow B \equiv (x_1, 0),\ A \equiv \left(x_1 - \frac{f(x_1)}{f'(x_1)}, 0\right)$$

According to equation, $2x_1 - \frac{f(x_1)}{f'(x_1)} = 4$

on generalising, $f'(x) = \frac{f(x)}{2(x-2)}$

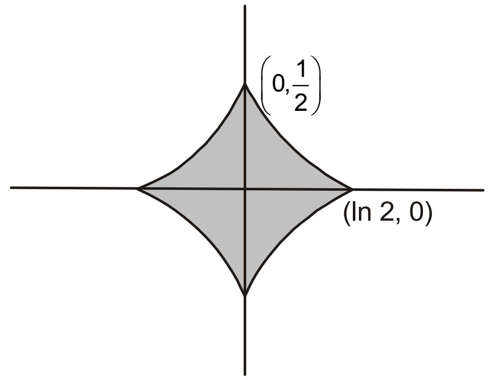
$$\frac{dy}{dx} = \frac{y}{2(x-2)} \quad \Rightarrow \int 2\frac{dy}{y} = \int \frac{dx}{x-2} \Rightarrow 2\ln y = \ln(x-2) + k \Rightarrow y^2 = c(x-2)$$

79. (D)

$|y| \le e^{-|x|} - \frac{1}{2}$

Graph is symmetric about x-axis and y-axis,

and $e^{-|x|} - \frac{1}{2} \ge 0$



$$\text{Area} = 4\int_0^{\ln 2}\left(e^{-x} - \frac{1}{2}\right)dx$$

$= 2 - 2\ln 2$

80. (A)

$$\left(\frac{dy}{dx} - e^{-x}\right)\left(\frac{dy}{dx} - e^{x}\right) = 0$$

$\Rightarrow \frac{dy}{dx} = e^{-x}$ or $\frac{dy}{dx} = e^{x}$

$\Rightarrow y = -e^{-x} + c$ or $y = e^{x} + c$

$\Rightarrow y + e^{-x} = c$ or $y = e^{x} + c$

81. (A)

Case –1 : Let $\frac{1}{2} \le x < 1$

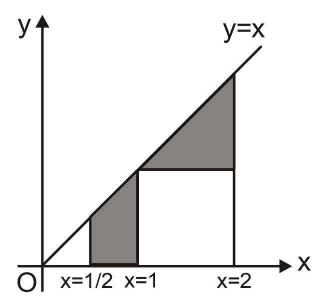
$\Rightarrow 0 < \log_y x \le 1$

$\Rightarrow 1 < x \le y$, if y > 1(not possible) and $1 > x \ge y$, if 0<y<1

Case –2 : Let $1 < x \le 2$

$\Rightarrow \log_y x \ge 1 \Rightarrow x \ge y$, if y > 1

and $x \le y$, if y < 1(not possible)



So, the possibilities are $x \ge y$, if $\frac{1}{2} \le x < 1$ and 0<y<1 and $x \ge y$, if $1 < x \le 2$ and y > 1

So, required area is 7/8.

82. (B)

As the matrix is skew-symmetric, hence |A| = 0, if n is odd

$\Rightarrow$ A is not an invertible matrix when n is odd.

83. (B)

(i) AB is symmetric $(AB)^T = B^T A^T = AB \Rightarrow BA = AB$

(II) $(B^T AB)^T = B^T A^T (B^T)^T = B^T A^T B = B^T AB$

(iii) and (iv)

Let A be skew symmetric, then $A^T = -A$

and $(A^n)^T = (A^T)^n, \forall n \in N$

$$\Rightarrow (A^n)^T = \begin{cases} A^n & \text{If n is even} \\ -A^n & \text{If n is odd} \end{cases}$$

Hence A^n is symmetric if n is even

Hence Answer is B.

84. (D)

Rearranging the given differential eqn.

$$xdx + \frac{ydx - xdy}{y^4} = 0 \Rightarrow x^3 dx + \frac{x^2}{y^2}.\frac{ydx - xdy}{y^2} = 0 \Rightarrow x^3 dx + \left(\frac{x}{y}\right)^2 .d\left(\frac{x}{y}\right) = 0$$

on integration, $\frac{x^4}{4} + \frac{1}{3}\left(\frac{x}{y}\right)^3 = c$

85. (C)

$$x^2(4ydx + xdy) = \frac{xdy - 2ydx}{x^4 + y^2}$$

$$4x^2 ydx + x^3 dy = \frac{xdy - 2ydx}{x^4 + y^2}$$

$$4x^3 ydx + x^4 dy = \frac{x^2 dy - 2xydx}{x^4 + y^2}$$

$$d(x^4 y) = d\left(\tan^{-1}\frac{y}{x^2}\right)$$

$$\Rightarrow x^4 y = \tan^{-1}\frac{y}{x^2} + c$$

86. (C)

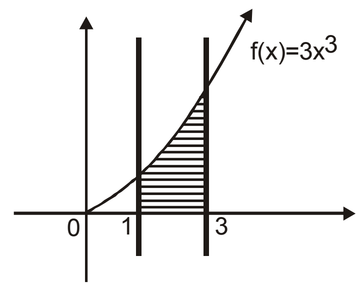
$$y^2 = kx \qquad 2y.\frac{dy}{dx} = k = \frac{y^2}{x} \qquad \Rightarrow \frac{dy}{dx} = \frac{y}{2x}$$

$$\text{now, } \tan\frac{\pi}{4} = \left|\frac{\frac{dy}{dx} - \frac{y}{2x}}{1 + \frac{y}{2x}\cdot\frac{dy}{dx}}\right| \Rightarrow \frac{dy}{dx} = \frac{2x+y}{2x-y} \text{ or } \frac{y-2x}{y+2x}$$

87. (A)

$f(x) = ax^3 + bx$

$$\text{Now } \lim_{x\to 0} \frac{f(x)}{x} = 0 \Rightarrow b = 0$$



$$\lim_{x\to 0}\left(\frac{f(x)}{x} + 1\right)^{1/x^2} = e^3 \Rightarrow e^a = e^3 \Rightarrow a = 3$$

$$\therefore f(x) = 3x^3$$

$$A = \int_1^3 3x^3 dx = 60$$

88. (C)

$$x\,dy = y\,dx + \sqrt{x^2 + y^2}dx \Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sqrt{1 + (y/x)^2}$$

Put y/x = v $\qquad \frac{dy}{dx} = x\frac{dv}{dx} + v$

$$\therefore \int \frac{1}{\sqrt{1+v^2}} dv = \int \frac{dx}{x} \Rightarrow \ell n(v + \sqrt{1+v^2}) = \log x + \log c \Rightarrow y + \sqrt{x^2 + y^2} = cx^2$$

89. (D)

$$xy(x) = x^2 y'(x) + y(x).2x$$

$$xy(x) + x^2 y'(x) = 0$$

$$x\frac{dy}{dx} + y = 0$$

x y = 6

90. (B)

$$\frac{dx}{dy} = \frac{x}{y} + 2y^2$$

$$\frac{dx}{dy} - \frac{x}{y} = 2y^2$$

$$\frac{x}{y} = \int 2y dy$$

$$\frac{x}{y} = y^2 + c$$