# SOLUTIONS

# WEEKLY TEST-5

## RBPA

## (JEE MAIN PATTERN)

## Test Date: 07-10-2017

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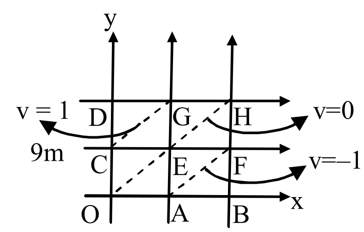
Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd.,
Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

# PHYSICS

**1. (B)**

OEH is an equipotential surface, the uniform E.F. must be perpendicular to it pointing from higher to lower potential as shown.




Hence E = $\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$

E = $\frac{(v_E - v_B)}{EB}$ = $\frac{0-(-2)}{\sqrt{2}} = \sqrt{2}$

$\therefore\ \vec{E} = E.\ \vec{E} = \sqrt{2}\ \frac{(\hat{i}-\hat{j})}{\sqrt{2}} = \hat{i}-\hat{j}$

**2. (C)**

Time to cross river ($t$) = $\frac{AB}{v_{mr}\sin\theta} = \frac{0.4}{5\sin\theta}$

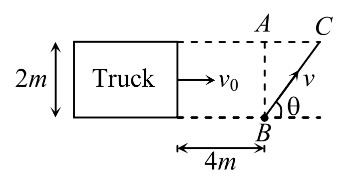
$BC = (v_{mr}\cos\theta + v_r)t$

$\Rightarrow\quad 0.4 = (5\cos\theta + 1)\times\frac{0.4}{5\sin\theta} \quad\Rightarrow\quad 5\sin\theta - 5\cos\theta = 1$

$\Rightarrow\quad 25\sin^2\theta + 25\cos^2\theta - 50\sin\theta\cos\theta = 1 \quad\Rightarrow\quad 25\sin2\theta = 24$

$\Rightarrow\quad \sin2\theta = \frac{24}{25} \quad\Rightarrow\quad \theta = 53°$

**3. (C)**




For safe crossing, the condition is that the man must cross the road by the time the truck covers the distance 4 + $AC$ or 4 + 2cot θ

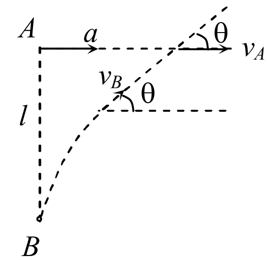
$\therefore\quad \frac{4 + 2\cot\theta}{8} = \frac{2/\sin\theta}{v}$

or $\quad v = \frac{8}{2\sin\theta + \cos\theta}$ …(i)

For minimum $v$, $\frac{dv}{d\theta} = 0 \Rightarrow \tan\theta = 2$

From equation (i), $v_{min} = \frac{8}{\sqrt{5}}$ = 3.57 m/s

**4.** Let after time $t$, the velocity of particle $B$ is directed at an angle $\theta$ with the horizontal, then




$$-\frac{ds}{dt} = bt - at\cos\theta$$

$$\Rightarrow \quad -\int_l^0 ds = b\int_0^t t\,dt - a\int_0^t t\cos\theta\,dt$$

and $\frac{1}{2}at^2 = b\int_0^t t\cos\theta\,dt \quad \therefore \quad l = \frac{bt^2}{2} - \frac{a^2t^2}{2b}$

$$t = \sqrt{\frac{2bl}{b^2 - a^2}}, \qquad S = \frac{1}{2}bt^2 = \frac{1}{2}b\frac{2bl}{b^2-a^2} = \frac{b^2 l}{b^2 - a^2}$$

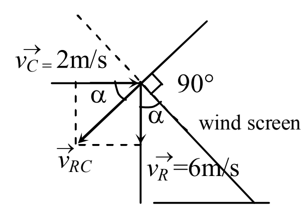
$\therefore$ **(A)**

**5.** The graph will be parabolic and in downward motion velocity will be negative and upward motion velocity will be positive

$\therefore$ **(A)**

**6.** Velocity of rain with respect to car $\vec{v}_{RC} = \vec{v}_R - \vec{v}_C$ should be perpendicular to the wind screen.

From figure,




$$\tan\alpha = \frac{v_r}{v_c} = \frac{6}{2}$$

$$\alpha = \tan^{-1}(3)$$

$\therefore$ **(B)**

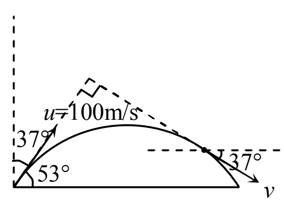
**7.** $y = ax^2$, $\quad \frac{dy}{dt} = a(2x)\frac{dx}{dt} = 2acx$, $\quad \frac{d^2y}{dt^2} = 2ac\frac{dx}{dt} = 2ac^2$

$a_y = 2ac^2$, $\quad a_x = 0$, $\vec{a} = a_x\hat{i} + a_y\hat{j} \quad \therefore \quad \vec{a} = 2ac^2\hat{j}$

$\therefore$ **(B)**

**8.** $u\cos 53° = v\cos 37°$

$$\Rightarrow \quad 100 \times \frac{3}{5} = v \times \frac{4}{5} \Rightarrow v = 75\,m/s$$




$$v_y = -v\sin 37° = -45\,m/s$$

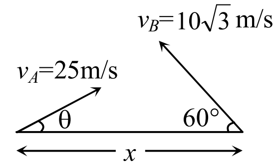
$$u_y = u\sin 53° = 80\,m/s$$

$$v_y = u_y + gt \Rightarrow -45 = 80 - 10t$$

$$t = 12.5\,s$$

$\therefore$ **(B)**

**9.** For collision,




$v_A \sin\theta = v_B \sin 60°$

$25\sin\theta = 10\sqrt{3}\times\frac{\sqrt{3}}{2}$

$\sin\theta = \frac{3}{5}$

or $\theta = 37°$

∴ **(D)**

**10.** $V_{s/g} = 15\text{m/sec}$

$v(t=2) = 15 - 10\times 2 = -5\text{m/sec}$

∴ **(B)**

**11.** $R = \frac{u^2}{g}\sin 2\theta = \frac{u^2}{g}$

Velocity of take off at *P* or $u = \sqrt{Rg} = \sqrt{90\times 10} = 30\text{m/s}$

$v = \sqrt{u^2 + 2g\sin\theta S}$ [$v \rightarrow$ velocity at point *O*]

$= \sqrt{(30)^2 + 2\times 10\times\frac{1}{\sqrt{2}}\times 80\sqrt{2}} = 50\text{m/s}$

∴ **(C)**

**12.** If *h* be the maximum height attained by the projectile then

$h = \frac{u^2\sin^2\theta}{2g}$ and $R = \frac{u^2\sin 2\theta}{g}$, $\frac{R}{h} = \frac{2\sin\theta\cos\theta}{(\sin^2\theta)/2} = 4\cot\theta$

Therefore $\frac{\Delta R}{R} = \frac{\Delta h}{h}$ (if θ is constant)

∴ Percentage increase in *R* = percentage increase in *h* = 5%

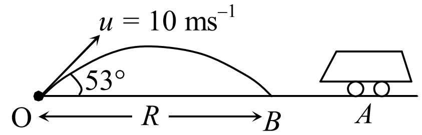
∴ **(A)**

**13.** $\tan\frac{\alpha}{2} = \frac{v_y}{v_x}$, $\tan\frac{\alpha}{2} = \frac{u_y - gt}{4x} = \frac{10\sin\alpha - 10t}{10\cos\alpha}$

$t = \sin\alpha - \cos\alpha\tan\frac{\alpha}{2}$, $t = \sin\frac{\alpha}{2}\left[2\cos\frac{\alpha}{2} - \frac{(2\cos^2\frac{\alpha}{2} - 1)}{\cos\frac{\alpha}{2}}\right]$, $t = \tan\frac{\alpha}{2}$

∴ **(A)**

**14.** $T = \dfrac{2u\sin\theta}{g} = \dfrac{2\times 10\times \frac{4}{5}}{10} = \dfrac{8}{5}\,s$



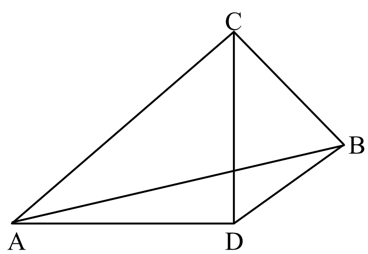

$OB = R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{100\times 2\times \frac{4}{5}\times\frac{3}{5}}{10} = \dfrac{48}{5}\ m$

$AB = \dfrac{8}{5}\times 5 = 8\ m$

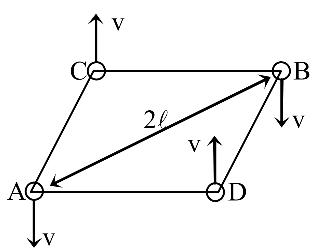
$OA = OB + AB = \dfrac{48}{5} + 8 = 17.6\ m$

$\therefore$ **(D)**

**15.** **(B)**




After the string between A and B is cut. Particle A and B will pivot downward and particle C and D move upward so that momentum is conserved.



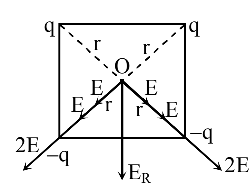

When the particle all lie in the same horizontal plane, then charge in potential energy is greatest so the increment in kinetic energy is greatest. So at this condition particle speed is maximum.
Speed of A and B in downward direction = Speed of C and D in upward direction Charge in Potential energy

$\Delta U = \Delta KE = \left[\frac{1}{2}mv^2\right]\times 4$

where v = speed of each particle

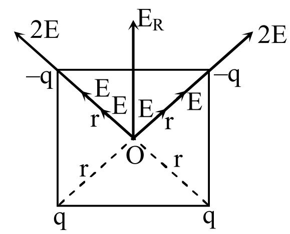
**16.** **(D)**

$V_0 = \frac{kq}{r} + \frac{kq}{r} - \frac{kq}{r} - \frac{kq}{r} = 0$




$E_R = 2\,(2E)\cos 45 = 2\sqrt{2}\ \frac{kq}{r^2}\ -\ \hat{j}$

After interchanging




$V_0 = \frac{kq}{r} + \frac{kq}{r} - \frac{kq}{r} - \frac{kq}{r} = 0$

$E_R = 2\sqrt{2}\,\frac{kq}{r^2}\,\hat{j}$

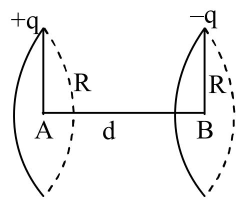
Hence Electric field will change.

**17. (D)**

$E_x = -\frac{\delta V}{\delta x}\,\hat{i} = -\frac{\delta}{\delta x}\left(\frac{20}{x^2-4}\right)\hat{i} = \frac{20\times 2x}{(x^2-4)^2}\,\hat{i}$

$E_x = \frac{40x}{(x^2-4)^2}\,\hat{i} = \frac{40\times 4}{144} = \frac{10}{9}$ in + x direction.

**18. (B)**




$V_A = \frac{kq}{R} - \frac{kq}{\sqrt{R^2+d^2}}$

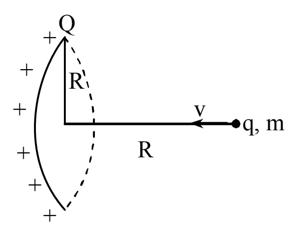
$V_B = \frac{-kq}{R} + \frac{kq}{\sqrt{R^2+d^2}}$

$-\quad + \quad -$

$\therefore \quad V_A - V_B = \frac{2kq}{R} - \frac{2kq}{\sqrt{R^2+d^2}}$

$\therefore \quad V_A - V_B = \frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$

**19. (A)**




$U_i + K_i = U_f + K_f$

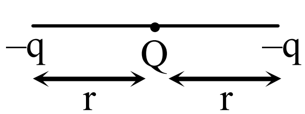
$\frac{kQq}{\sqrt{2}R} + \frac{1}{2}mv^2 = \frac{kQq}{R} + O$

$\frac{1}{2}mv^2 = \frac{kQq}{R}\left(1 - \frac{1}{\sqrt{2}}\right)$

$v = \sqrt{\frac{2kQq}{mR}\left(1 - \frac{1}{\sqrt{2}}\right)}$

$v = \sqrt{\frac{kQq}{mR}(2-\sqrt{2})}$

**20. (D)**



U = $\frac{-kQq}{r} \times 2 + \frac{kq^2}{2r}$

$2 \times \frac{kQq^2}{r} = \frac{kq^2}{2r}$

$\therefore \quad \frac{Q}{q} = \frac{1}{4}$

**21. (C)**

$\tau$ = PE sin 30

$10\sqrt{3} = \frac{PE}{2}$

PE = $20\sqrt{3}$

Potential Energy = –PE cos 30

$\therefore$ Potential energy = $-20\sqrt{3} \times \frac{\sqrt{3}}{2}$

= – 10 × 3 = –30 J

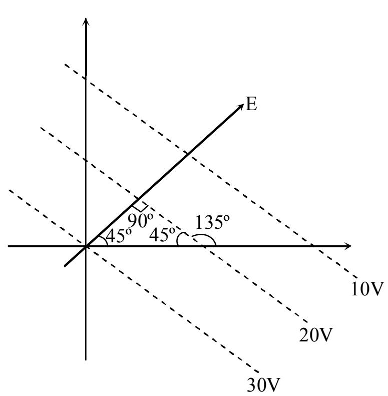
**22. (B)**

$E_x = -\frac{\partial V}{\partial x}\,\hat{i} = -\frac{\partial}{\partial x}(10axy)\,\hat{i} = -10ay\,\hat{i}$

$E_y = \frac{-\partial V}{\partial y}\,\hat{j} = -\frac{\partial}{\partial y}(10axy)\,\hat{i} = -10ax\,\hat{j}$

$E = -10a\,(y\hat{i} + x\hat{j})$

**23. (A)**




Electric field lines are ⊥ to the equipotential surface and are directed from high potential to low potential.

**24. (A)**

**25. (C)**

**26. (B)**

$V_B - V_A = -\int_A^B \vec{E}.d\vec{r} = \int_B^A \vec{E}.d\vec{r}$

Now $V_B = K\frac{q}{2a} - \frac{kq}{a} = -\frac{kq}{2a}$

$\therefore \int_B^A \vec{E}.d\vec{r} = -\frac{kq}{2a} = -\frac{q}{8\pi \in_0 a}$

**27. (C)**

$U_i + K_i = U_f + K_f$

$0 + \frac{1}{2}mv^2 = \frac{-k8q^2}{R} + \frac{1}{2}mv'^2$

$\therefore v' = \sqrt{\frac{16kq^2}{mR} + v^2}$

**28. (D)**

$E_x = -\frac{\partial V}{\partial x} \Rightarrow E_y = -\frac{\partial V}{\partial y} \Rightarrow E_z = -\frac{\partial V}{\partial z}$

$\therefore \vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

**29. (A)**

**30.** Work done to rotate the ring is equal to work done to return the charge at its initial position.

$\therefore$ **(B)**

# CHEMISTRY

**31. (D)**

P decreases and T decreases

So, $\Delta U =$ decreases

**32. (B)**

Work done in isothermal reversible process is

$$W = -nRT\ln\frac{V_2}{V_1} = -nRT\ln\frac{P_1}{P_2}$$

So, to apply the above equation first we calculate 'n' by using PV= nRT

So, $W = -\frac{10^4}{0.0821}\times 8.314\times 2.303\log\frac{10}{1}$

W = **–2332 kJ**

**33. (D)**

**34. (B)**

P = 1 atm = $1.013 \times 10^5$ N/m$^2$

$\Delta V$ = (2.5 –2) = 0.5 litre = $0.5 \times 10^{-3}$ m$^3$

W = $-P\Delta V$ = – $1.01 \times 10^5 \times 0.5 \times 10^{-3}$ J = – 50.5 J

$\Delta U$ = q + w = 300 – 50.5 = 249. 5 J

**35. (D)**

$Fe + 2HCl \longrightarrow FeCl_2 + H_2$

Thus mole of $H_2$ = 1

The work is done against external pressure by $H_2$ which pushes a pressure of 1 atm

W = $-P\Delta V$ = – $\Delta n_g$ RT = – 1×0.0821 × 300 = – 24.63 L - atm

**36. (B)**

W = $-P\Delta V$ = $-3 \times 1.013 \times 10^5 \times 2 \times 10^{-3}$ = –607.8 J

this work is used in heating water thus

–w = q = $m\,s\Delta T$

607.8 = 10 ×18 × 4.184 × $\Delta T$

$\Delta T$ = 0.81

∴ Final Temp. = 290 + 0.81 = 290.81 K

**37. (A)**

$\frac{V}{T} = \text{constant}$

$P = \text{constant}$

$W = -P(V_f - V_i) = -nR\Delta T$

$q = nC_P\Delta T$

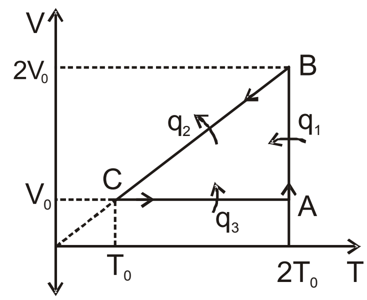
$\frac{q}{|w|} = \frac{nC_P\Delta T}{nR\Delta T} = \frac{C_P}{R} = \frac{5}{2}$

**38. (C)**

$$W = q_2\left(\frac{T_2 - T_1}{T_2}\right) = 1897.8\left(\frac{373-273}{373}\right) = 508.7\text{ kJ}$$

$\therefore$ Work done by the engine = 508.7 kJ

**39. (A)**




AB process

$0 = q_1 + w_1 = q_1 - nR(2T_0)\ln 2$

$\therefore q_1 > 0$

BC process $\Delta U_2 = q_2 + w_2$

$$\frac{nR}{(\gamma-1)}(T_0 - 2T_0) = q_2 - \left(\frac{nRT_0}{V_0}\right).(V_0 - 2V_0)$$

$\therefore q_2 < 0$

CA process

$\Delta U_3 = q_3 + w_3$

$$\frac{nR}{(\gamma-1)}(2T_0 - T_0) = q_3 + 0$$

$\therefore q_3 > 0$

$$\text{Efficiency} = \frac{\text{Total work done}}{\text{Total heat absorbed}} = \frac{w_1 + w_2}{q_1 + q_3} = \frac{(2nRT_0\ \ln 2) - (nRT_0)}{(2nRT_0\ \ln 2) + \left(\frac{nRT_0}{\gamma - 1}\right)} = \frac{(\gamma-1)(2\ln 2 - 1)}{1 + (\gamma-1)2\ \ln 2}$$

**40. (D)**

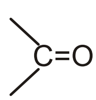
$\Delta H = \Delta E + (P_2V_2 - P_1V_1)$ = 60 + ( 20 – 6 ) = 60 + 14 = 74

**41. (B)**

Degree of unsaturation = no. of $\pi$ bond + no. of ring

$6\pi$ bond + 2 ring = 8

**42. (C)**

Z–H is conjugated with  and form aromatic enol.

**43. (A)**

If aromatic enol is form from non aromatic than % of enol is high.

**44. (C)**

Enolisable –H is replaced by dueterium.

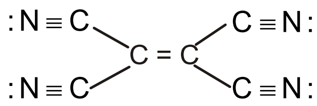
**45. (A)**

Phenol and Alcohol are functional group isomer of each other

**46. (A)** **47. (C)** **48. (A)** **49. (D)**

**50. (A)**

**51. (C)**



$\langle Sp, Sp^2 \rangle$

**52. (B)**

Repulsion order: l.p – l.p > l.p – B.P > B.p –B.p

**53. (B)**

| | $O_2$, | $O_2^+$, | $O_2^-$, | $O_2^{-2}$ |
|---|---|---|---|---|
| Total V.E = | 12, | 11, | 13, | 14 |
| B.O = | 2, | 2.5 | 1.5, | 1 |

so bond strength = $= O_2^+ > O_2 > O_2^- > O_2^{-2}$

**54. (B)**

$CaC_2 \rightarrow Ca^{+2} + C_2^{-2}$

$:\bar{C}\equiv\bar{C}:$

$2\pi$, $1\sigma$

**55. (A)**

X - trivalent ; Y - pentavalent ; Z - monovalent

Hence only A is possible.

**56. (B)**

**57. (D)**

| **Compound** | **B.O.** |
|---|---|
| $XeO_3$ | 2 |
| $ClO_3^-$ | 1.67 |
| $PO_4^{3-}$ | 1.25 |
| $SO_4^{2-}$ | 1.5 |

**58. (C)**

**59. (B)**

**60. (A)**

$$I_2 + \underset{(Solution)}{I^-} \longrightarrow \underset{(Linear)}{I_3^-}$$

# MATHEMATICS

**61. (A)**

Centre of circle S = 0 lies on radical axis of two given circle

radical axis is x = 0

For smallest radius of S = 0

Centre should be intersection point of radical axis and line joining their centres

So, it is (0, 0).

radius = length of tangent drawn from (0, 0) to each circle $= \sqrt{16} = 4$

**62. (C)**

Tangents are perpendicular so, their point of intersection lies on directrix P (1, 1) lies on directrix

Mid point of AB is $Q\left(\frac{7}{2},\frac{3}{2}\right)$

Slope of $PQ = \frac{3/2 - 1}{7/2 - 1} = \frac{1/2}{5/2} = 1/5$

Slopes of directrix = –5

**63. (D)**

The ellipse is passing through O (0,0) and has foci P(3,3) and Q(–4,4). Then,

$$e = \frac{PQ}{OP + OQ} = \frac{\sqrt{50}}{3\sqrt{2}+4\sqrt{2}} = \frac{5}{7}$$

**64. (C)**

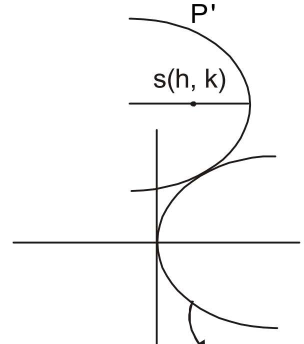
We have PQ = BP

or , $2ae = \sqrt{a^2e^2 + b^2} = \sqrt{a^2} = a \qquad \Rightarrow e = \frac{1}{2}$

**65. (D)**

Equation of parabola P' is $(y-k)^2 = -8(x-h-2)$

Solving it with $y^2 - 8x = 0$, we have




$$(y-k)^2 = -8\left(\frac{y^2}{8} - h - 2\right)$$

$\Rightarrow \quad y^2 - 2ky + k^2 = -y^2 + 8h + 16$

$\Rightarrow \quad 2y^2 - 2ky + k^2 - 8h - 16 = 0 \qquad$ ...(i)

Since two parabolas touches each other,
we have D = 0 of equation (i)

$\Rightarrow \quad 4k^2 - 8(k^2 - 8h - 16) = 0 \Rightarrow -4k^2 + 64h + 128 = 0$

$\Rightarrow \quad k^2 = 16(h+2) \Rightarrow y^2 = 16(x+2)$

**66. (D)**

f(x) = f(2 – x)

$x \to x+1;\ f(x+1) = f(1-x)$

Now, $g(-x) = f(-x+1) = f(x+1) = g(x)$

**67. (C)**

$$f(\alpha) = \frac{\alpha(\alpha-1)}{2}$$

$$\therefore \quad f(|x|) = 0 \Rightarrow \frac{|x|(|x|-1)}{2} = 0$$

**68. (D)**

Radical axis is perpendicular to line joining the centre of circle.

**69. (A)**

$$\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}\times\frac{-h}{h\left(h^2+3\right)}=\lim_{h\to 0}-f'(1).\frac{1}{h^2+3}=-\frac{1}{3}f'(1)$$

**70. (B)**

$$f(x) = \frac{x}{e^x-1} + \frac{x}{2} + 1 = \frac{2x+xe^x-x}{2(e^x-1)} + 1 = \frac{x+xe^x}{2(e^x-1)} + 1$$

$$f(-x) = \frac{-x-xe^{-x}}{2(e^{-x}-1)} + 1 = \frac{x+xe^x}{2(e^x-1)}+1$$

$\therefore$ f(–x) = f(x) for all x

$\therefore$ f(x) is an even function.

**71. (D)**

$$\lim_{x\to\frac{1}{2}^+} f(x)=0+0=0$$

$$\lim_{x\to\frac{1}{2}^-} f(x)=0+1=1$$

**72. (B)**

put $x=\frac{\pi}{4}+h$

$$\therefore \lim_{h\to 0}\frac{2\sqrt{2}(1-\cos^3 h)}{(1-\cos 2h)}=\lim_{h\to 0}\frac{2\sqrt{2}(1-\cos h)(1+\cos h+\cos^2 h)}{2\sin^2 h}=\frac{3}{\sqrt{2}}$$

**73. (C)**

For $n\in I$,

$$\lim_{x\to n^+} f(x) = \lim_{x\to n^+}[x]\cos\frac{2x-1}{2}\pi = n\cos\frac{2n-1}{2}\pi=0$$

and $\lim_{x\to n^-} f(x)=\lim_{x\to n^-}[x]\cos\frac{2x-1}{2}\pi=0$

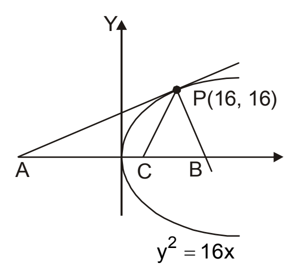
Hence, f is continuous for $x=n\in I$. Since, the functions g(x) = [x] and $h(x)=\cos\frac{2x-1}{2}\pi$ are continuous on $x\in R-I$, so f is continuous everywhere.

**74. (D)**

By property centre of circle coincides with focus of parabola.

$\Rightarrow\ C\equiv(4,0)$

$\tan\alpha=\text{slope of PC}=\frac{16}{12}\ \Rightarrow\ \alpha=\tan^{-1}\left(\frac{4}{3}\right)$



**75. (D)**

$$\lim_{x\to 3-}\frac{[x]^2-9}{x^2-9}=\infty$$

$$\lim_{x\to 3+}\frac{[x]^2-9}{x^2-9}=0$$

**76. (C)**

For differentiability at x = 1

$$f'(1^+)=f'(1^-)\ \Rightarrow\ a=6x-\frac{4}{2\sqrt{x}}\Rightarrow a=4$$

For continuity at x = 1

$$f(1^+)=f\left(1^-\right)=f(1)\ \Rightarrow a+b=3-4+1=0\ \Rightarrow b=-4$$

**77. (B)**

Since f(x) is continuous function, therefore, $\lim\limits_{x\to 1} f(x)=f(1)$

From the given equation $(f(1))^2 = 2f(1)$

$\Rightarrow$ f(1) = 0 or 2 but f(1) > 0 $\Rightarrow$ f(1) = 2.

**78. (A)**

$$x=1\Rightarrow f(y)=\frac{f(1)}{y}$$

$$y=30\Rightarrow f(30)=\frac{f(1)}{30}\Rightarrow f(1)=600$$

$$f(40)=\frac{f(1)}{40}=\frac{600}{40}=15$$

**79. (C)**

$$\lim_{x\to 0}\log_{\cos\left(\frac{x}{2}\right)}\cos x=\lim_{x\to 0}\frac{\ln(\cos x)}{\ln\left(\cos\frac{x}{2}\right)}=4.$$

**80. (D)**

$$\lim_{x\to 0}\frac{x-e^x+1-(1-\cos 2x)}{x^2}=-\frac{1}{2}-2=-\frac{5}{2}.$$

$\therefore$ For continuity, f(0) = $-\frac{5}{2}$.

**81. (D)**

$$g'(0^+)=\lim_{h\to 0}\frac{\cosh-1}{h}=0$$

$g'(0^-)=\lim\limits_{h\to 0}\dfrac{-h+b-1}{-h}$ and for existence of limit, b = 1 $\Rightarrow$ $g'(0^-)=1$.

**82. (B)**

$$f(x)=\begin{cases} x-2k\pi; & 2k\pi-\frac{\pi}{2}\le x\le 2k\pi+\frac{\pi}{2} \\ (2k+1)\pi-x; & 2k\pi+\frac{\pi}{2}< x\le 2k\pi+\frac{3\pi}{2} \end{cases}$$

**83. (B)**

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{f(h)}{h}$$

and $\because\ f(0)=0 \quad \Rightarrow\ f'(x)=\lim_{h\to 0}\left(\frac{f(h)-f(0)}{h}\right)$

$\Rightarrow\ f'(x)=f'(0) \quad \Rightarrow\ f'(x)=2x$

**84. (D)**

Let $P\equiv(OP\cos\theta, OP\sin\theta)$ and $Q\equiv\left(OQ\cos\left(\theta\pm\frac{\pi}{2}\right), OQ\text{Sin}\left(\theta\pm\frac{\pi}{2}\right)\right)$

We have,

$$\frac{1}{|OP|^2}=\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}$$

$$\frac{1}{|OQ|^2}=\frac{\sin^2\theta}{a^2}+\frac{\cos^2\theta}{b^2} \quad \text{or} \quad \frac{1}{|OP|^2}+\frac{1}{|OQ|^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

Therefore the minimum value of $|OP|.|OQ|=\frac{2a^2b^2}{a^2+b^2}$

**85. (D)**

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

$$e=\sqrt{\frac{9-4}{9}}=\frac{\sqrt{5}}{3}$$

$F_1$, $F_2$ are $(\pm\sqrt{5},0)$

Area $=\sqrt{10}\ =\frac{1}{2}\times(2\sqrt{5})\times|y|=\sqrt{10}$

$|y|=\sqrt{2}$, $y=\pm\sqrt{2}$ hence possible points are $\left(\pm\frac{3}{\sqrt{2}},\pm\sqrt{2}\right)$

**86.** **(B)**

Equation of Tangent to ellipse $y = 2x \pm \sqrt{4a^2 + b^2}$ is normal to circle whoe centre (–2, 0)

So, $0 = -4 \pm \sqrt{4a^2 + b^2} \Rightarrow 4a^2 + b^2 = 16$

using $A.M. \geq G.M$

$$\frac{4a^2 + b^2}{2} \geq \sqrt{4a^2b^2} \Rightarrow 8 \geq 2ab \Rightarrow 4 \geq ab$$

**87.** **(D)**

Normals are perpendicular means tangents are also perpendicular.
Let tangents at P and Q intersect at $(x_1, y_1)$.

then $x_1^2 + y_1^2 = a^2 + b^2$ .............(i)

Suppose mid point of P and Q be R (h, k).
Equation of chord with respect to mid point is

$$\frac{xh}{a^2} + \frac{yk}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2} \quad \text{............(ii)}$$

Also equation of chord of contact with respect to $(x_1, y_1)$ be

$$\frac{x x_1}{a^2} + \frac{y y_1}{b^2} = 1 \quad \text{............(iii)}$$

Compairing (ii) and (iii)

$$\frac{h}{x_1} = \frac{k}{y_1} = \frac{h^2}{a^2} + \frac{k^2}{b^2} = \frac{\sqrt{h^2 + k^2}}{\sqrt{x_1^2 + y_1^2}} = \frac{\sqrt{h^2 + k^2}}{\sqrt{a^2 + b^2}} \quad \Rightarrow \left(\frac{h^2}{a^2} + \frac{k^2}{b^2}\right)^2 = \frac{h^2 + k^2}{a^2 + b^2}$$

$$\therefore \text{ locus is } \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = \frac{x^2 + y^2}{a^2 + b^2}$$

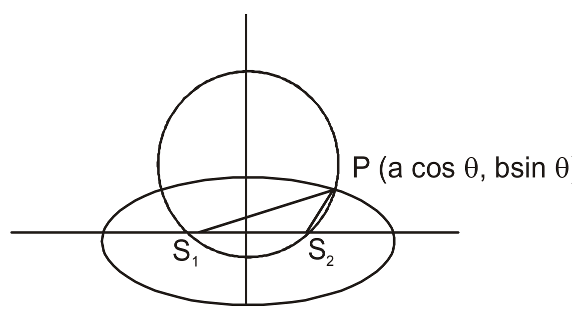
**88.** **(C)**

$\because$ We have $R = \dfrac{abc}{4\ Ar(\Delta)}$

so, $R = \dfrac{2ae \times (a - ae\cos\theta)(a + ae\cos\theta)}{4 \times \left(\frac{1}{2} \times S_1S_2 \times b\sin\theta\right)}$

$$= \frac{2ae(a^2)(1 - e^2\cos^2\theta)}{2(2ae)(b\sin\theta)} = \frac{a^2(1 - e^2\cos^2\theta)}{2b\sin\theta}$$

$$= \frac{a^2\left(1 - e^2 + e^2\sin^2\theta\right)}{2b\sin\theta}$$



$$=\frac{a^2}{2b}\left[e^2\sin\theta+\frac{b^2}{a^2}\operatorname{cosec}\theta\right]\geq\frac{a^2}{2b}\times 2\sqrt{\frac{e^2b^2}{a^2}}$$

$$=\frac{a^2}{b}\times\frac{eb}{a}=ae$$

**89. (B)**

$\sqrt{6}x\sec\theta - \sqrt{3}y\ \operatorname{cosec}\theta = 3$

slope = $\sqrt{2}\tan\theta=1$

$\tan\theta=\frac{1}{\sqrt{2}}$

**90. (A)**

Let $x=r\cos\theta$ and $y=r\sin\theta$, then $x^2+y^2=r^2$

Now $3x^2-4xy+2y^2=12$

$\Rightarrow\ r^2(3\cos^2\theta-4\cos\theta\sin\theta+2\sin^2\theta)=12$

$\Rightarrow\ r^2=\frac{24}{5+\cos 2\theta-4\sin 2\theta}$

$\therefore\ m=\frac{24}{5+\sqrt{17}}=3(5-\sqrt{17})$ and $n=\frac{24}{5-\sqrt{17}}=3(5+\sqrt{17})$.