SOLUTIONS

PROGRESS TEST-6

CD-1801(α), CD-1801(β)

CDK-1801 & CDS-1801

(JEE ADVANCED PATTERN)

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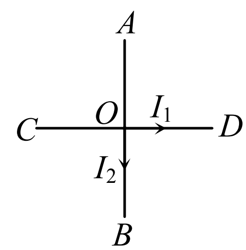
PHYSICS

1. **(C)**

2. $\vec{B}$ due to *AOB* and *COD* will be perpendicular to each other at point *P*

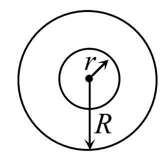


$$B^2 = B_1{}^2 + B_2{}^2$$

$$B = \frac{\mu_0}{2\pi a}\left(I_1^2 + I_2^2\right)^{1/2}$$

$\therefore$ **(C)**

3. No current is enclosed in the circle, so from Ampere's circuital law, the magnetic induction at any point inside the infinitely long straight thin walled tube (cylindrical) is zero.



$\therefore$ **(B)**

4. Since the magnetic field is constant with time and space and exists everywhere, there is no change in magnetic flux when the loop is moved in it. Hence no current is induced.

$\therefore$ **(D)**

5. $T = \frac{2\pi m}{qB}$, $\frac{T_\alpha}{T_p} = \frac{m_\alpha}{m_p}\cdot\frac{q_p}{q_\alpha}$ **=** 2

$\therefore$ **(B)**

6. Magnitude of torque is given by $|\vec{\tau}| = MB\sin\theta$

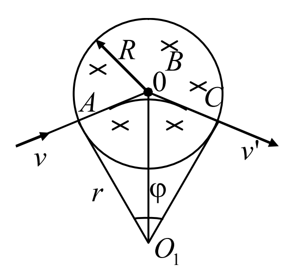
Here, $M = NiA = (1)(1.0)(\pi)(0.2)^2 = (0.04\pi)\text{A}-\text{m}^2$

and θ = angle between $\vec{M}$ and $\vec{B}$ =90^0

$\therefore$ $|\vec{\tau}| = (0.04\pi)(2)\sin 90^0$ = 0.08 π N-m.

$\therefore$ **(B)**

7. The magnetic induction of the solenoid is directed along its axis. Therefore, the Lorentz force acting on the electron at any instant of time will lie in the plane perpendicular to the solenoid axis. Since the electron velocity at the initial moment is directed at right angles to the solenoid axis, the electron trajectory will lie in the plane perpendicular



to the solenoid axis. The Lorentz force can be found from the formula

$F = evB$.

The trajectory of the electron in the solenoid is an arc of the circle whose radius can be deter mined from the relation $evB = mv^2/r,$ whence

$$r = \frac{mv}{eB}$$

The trajectory of the electron in shown in figure, where O_1 is the centre of the arc AC described by the electron, v' is the velocity at which the electron leaves the solenoid. The segments OA and OC are tangents to the electron trajectory at points *A* and *C*. The angle between v and v' is obviously $\varphi = \angle AO_1C$ since $\angle OAO_1 = \angle OCO_1$.

In order to find φ, let us consider the right triangle OAO_1; side $OA = R$ and side $AO_1 = r$.

Therefore, $\tan(\varphi/2) = R/r = eBR/(mv)$.

Therefore, $$\varphi = 2\tan^{-1}\left(\frac{eBR}{mv}\right)$$

Obviously, the magnitude of the velocity remains unchanged over the entire trajectory since the Lorentz force is perpendicular to the velocity at any instant. Therefore, the transit time of electron in the solenoid can be determined from the relation

$$t = \frac{r\varphi}{v} = \frac{m\varphi}{eB} = \frac{2m}{eB}\tan^{-1}\left(\frac{eBR}{mv}\right).$$

$\therefore$ **(B)**

8. $$\alpha = \frac{|\vec{\tau}|}{I} = \frac{i\pi r^2 B_0\sqrt{2}}{\frac{1}{2}mr^2}$$

$$\alpha = \frac{2\sqrt{2}\pi B_0 i}{m}$$

Axis of rotation of the loop will be along unit vector $\frac{(\hat{j}-\hat{i})}{\sqrt{2}}$

the moment of inertia of ring about that axis = $\frac{1}{2}mR^2$

$\therefore$ **(B)**

9. $B = \dfrac{\mu_0 I}{4r} + \dfrac{\mu_0 I}{4\pi r} = \dfrac{\mu_0 I}{4\pi r}(1+\pi)$

$\therefore$ **(A)**

10. Potential of centre of sphere = $\dfrac{Kq}{r} + V_i = \dfrac{Kq}{r}$

where V_i = potential due to induced charge at centre = 0 [$\therefore$ Σq_i = 0 and all induced charges are equidistance from centre]

$\therefore$ potential at point P = $\dfrac{Kq}{r} = \dfrac{Kq}{r_1} + V_i$ (For conductor all points are equipotential)

$\therefore$ $V_i = K\left(\dfrac{q}{r} - \dfrac{q}{r_1}\right)$

$\therefore$ **(C)**

11. $Q = CV = 5\times10^{-6}\times0.942 = 4.71\times10^{-6}$ C

$\therefore$ **(A)**

12. $E = \dfrac{Q^2}{2C} = \dfrac{(4.71)^2\times(10^{-6})^2}{2\times10\times10^{-6}} = 11.1\times10^{-7}$ J

$\therefore$ **(B)**

13. **(D)**

14. **(A)**

$\phi = \text{BA}$

$\text{E} = \text{A}\dfrac{\text{dB}}{\text{dt}} = \pi.2\text{t}$

$\text{i} = \dfrac{\text{E}}{\text{R}} = \dfrac{2\pi\text{t}}{2} = \pi\text{t}$

$\tau = \vec{\mu}\times\vec{\text{B}} = \pi^2\text{t}\,\hat{\text{j}}\times\left(\hat{\text{i}} + \text{t}^2\hat{\text{j}}\right)$

$= \pi^2\text{t} = \pi\text{g}$

$\text{t} = \dfrac{\text{g}}{\pi} = \dfrac{10}{\pi}$

15. (B)

$$i^2 = \frac{P}{R_2}$$

$i = 2 \times 10^{-2}$ A

$P_{R_1} = i^2 R_1 = (2.0 \times 10^{-2})^2 \times 4 \times 10^3 = 1.6$ W

16. (C)

$Q_{C_1} = V_{R_1} \times C_1 = 80 \times 3 \times 10^{-6} = 240\ \mu C$

$Q_{C_2} = V_{R_2} \times C_2 = 140 \times 6 \times 10^{-6} = 840\ \mu C$

17. (A)

$$I = \frac{2+3-5+4+6}{2+3+5+4+6} = \frac{1}{2}\text{A}$$

$$V_D = 2 - \frac{1}{2} \times 2 = 1\text{V}$$

$$V_C = V_D + 3 - \frac{1}{2} \times 3 = 2.5\text{V}$$

$$V_B = V_C - 5 - \frac{1}{2} \times 5 = -5\text{V}$$

$$V_A = V_B + 4 - \frac{1}{2} \times 4 = -3\text{V}$$

18. (D)

Let the radius of circle in which particle moves is R. In this magnitude of region electric field is $E = \frac{R}{2}\left(\frac{dB}{dt}\right)$ as $qE = m\frac{dV}{dt}$

$$\Rightarrow \frac{qR}{2}\left(\frac{dB}{dt}\right) = m\frac{dV}{dt}$$

also $R = \frac{mV}{Bq}$

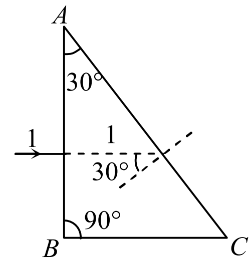
$$\Rightarrow \frac{dV}{V} = \frac{1}{2}\frac{dB}{B}$$

as $R = \frac{mV}{Bq}, \frac{dV}{V} = \frac{dq}{q} + \frac{dB}{B}, \frac{dq}{q} = -\frac{1}{2}\frac{dB}{B}$

19. **(D)**

20. **(A)**

(A) Refractive index of the prism is the minimum value required for ray (1) to undergo total internal reflection at face *AC*. Ray (1) falls on face *AC* at an angle of incidence 30°



$\therefore$ 30° > i_C

sin 30° > sin i_C

$\therefore$ $\mu > 2$

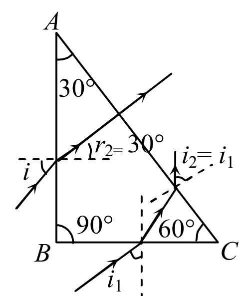
Minimum value of μ can be taken as 2.

(B) For ray 2, refractive angle of prism is 30°. Apply Snell's law for refraction at face *AB*.

1 sin $i = \mu$ sin r

$i = 90°$

(C) Using the relation $i_1 + i_2 = A + \delta$ for ray 2.



90° + 0° = 30° +δ

δ = 60°

(D) $\mu = \dfrac{\sin\left(\dfrac{A+\delta m}{2}\right)}{\sin\dfrac{A}{2}} \Rightarrow \delta m = 120°$

CHEMISTRY

21. (A)

$$\frac{P_o - P_s}{P_s} = \frac{n}{N}$$

or, $$\frac{P_o - P_s}{P_s} = \frac{w_B}{M_B} \times \frac{M_A}{w_A} = \frac{185 - 183}{183} = \frac{1.2}{M_B} \times \frac{58}{100}$$

$$\frac{185 - 183}{183} = \frac{1.2 / M_B}{100 / 58}$$

M_B = 64 g/mol

22. (A)

$$P_T = P_A^0 + \left(P_B^0 - P_A^0\right) x_B$$

$$120 = 150 + (50 - 150) x_B$$

$$-30 = -100\ x_B$$

$$x_B = \frac{3}{10}$$

$$x_A = \frac{7}{10}$$

$$y_A = \frac{P_A^0\, x_A}{P_T} = \frac{150x}{120x}\frac{7}{10}$$

$$y_B = \frac{P_B^0 x_B}{P_T} = \frac{50}{120} \times \frac{3}{10}$$

$$\frac{y_A}{y_B} = \frac{150 \times 7}{50 \times 3} = \frac{7}{1}$$

23. (B)

$$kt_{½} = 2.303\ \log_{10} \frac{100}{50}\ \text{.........(i)}$$

$$kt_{99\%} = 2.303\ \log_{10} \frac{100}{1}\ \text{..........(ii)}$$

$(ii) \div (i)$

$$\frac{t_{99\%}}{t_{½}} = \frac{\log_{10}^{10^2}}{\log_{10}^{2}} = \frac{2}{\log_{10}^{2}}$$

or, $t_{99\%} = \frac{2}{0.301} \times t_{½} = \frac{2}{0.301} \times 6.93 = 46.06$ min.

or, $k = \frac{0.693}{t_{y_2}} = \frac{0.693}{6.93} = 0.1$

also, $kt_{99\%} = 2.303 \log_{10} \frac{100}{10}$

or 0.10 $t_{99\%}$ =2.303 × 2

or, $t_{99\%} = \frac{4.606}{0.1} = 46.06$ min.

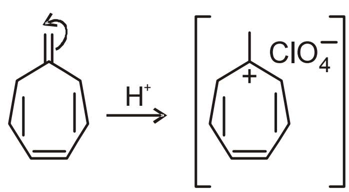
24. (C)

25. (B)

Its conjugate base is very much stablised due to locking of phenyl ring.

26. (B)

$HClO_4 = H^+ + ClO_4^{\ominus}$



Aromatic ion

27 . (D)

Half chair is less stable due to high torsional strain in the molecule.

28. (B)

29. (C)

30. (A)

31. (C)

$$Y_A P_T = P_A^0 \times x_A$$

$$Y_A = \frac{P_A^0 x_A}{P_B^0 + (P_A^0 - P_B^0) x_A}$$

$$Y_A - x_A = \frac{P_A^0 x_A}{P_B^0 + (P_A^0 - P_B^0) x_A} - x_A$$

$$\frac{d(Y_A - x_A)}{dx_A} = \frac{(P_B^0 + (P_A^0 - P_B^0)x_A)P_A^0 - P_A^0 x_A [P_A^0 - P_B^0)}{[P_B^0 + (P_A^0 - P_B^0)x_A]^2} - 1$$

$$0 = \frac{P_B^0 \cdot P_A^0 \cdot}{[P_B^0 + (P_A^0 - P_B^0)x_A]^2} - 1$$

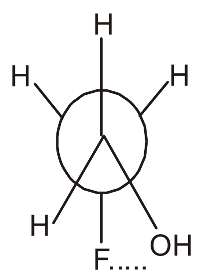
$$P_B^0 + (P_A^0 - P_B^0)x_A = \sqrt{P_A^0 \cdot P_B^0}$$

$$x_A = \frac{\sqrt{P_A^0 \cdot P_B^0} - P_B^0}{P_A^0 - P_B^0}$$

32. (D)

$$P_T = P_B^0 + (P_A^0 - P_B^0)x_A = \sqrt{P_A^0 P_B^0}$$

33. (C)



more stable due to intramolecular H–bonding

Intramolecular H–bonding

34 . (D)

Two larger substituent CH_3 group ups are presental 60° angle so this is ganch form

35. (B)

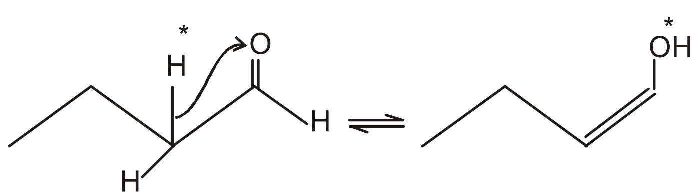
36. (B)

37. (A)

(A)–(Q); (B)–(P); (C)–(S); (D)–(R)

38. (B)

A $\rightarrow$ (R), B $\rightarrow$ (Q), C $\rightarrow$ (S), D $\rightarrow$ (P)

(A) 

both are fantomers

(B) One methyl is above the plane other is below the plane

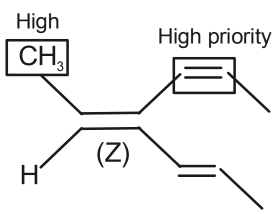
(C) Due to unequal distribution of alkyl group around N

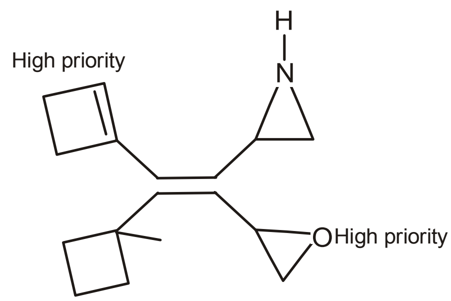
39. (C)

(A)$\rightarrow$(S); (B)$\rightarrow$(P); (C)$\rightarrow$(Q); (D)$\rightarrow$(R)

(A) Both Cl atoms are present at 180° angle so it is trans staggered

(B) Two methyl groups are present at 60° so gauch

(C) 

(D) 

40. (B)

(A) — (P); (B) — (Q); (C) — (P); (D) — (S)

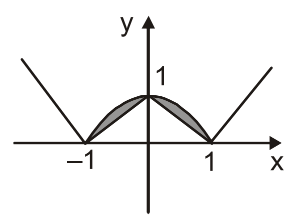
MATHEMATICS

41. (D)

Required area

$$= 2\left\{\frac{1}{4}\times\pi\times 1^2 - \frac{1}{2}\times 1\times 1\right\}$$

$$= \frac{\pi}{2} - 1$$



42. (B)

$n(A) = n - 1$

$n(A\times A) = (n-1)^2$

Number of relation on A $= 2^{(n-1)^2}$

43. (C)

$$\int_0^x f(t)\,dt = x + \int_x^1 t^2\cdot f(t)\,dt$$

differentiating, $f(x) = 1 - x^2 \cdot f(x)$

$$(1 + x^2) f(x) = 1 \Rightarrow \quad f(x) = \frac{1}{1+x^2}$$

$$\therefore \int_{-1}^{1} \frac{1}{1+x^2} dx = 2\int_0^1 \frac{dx}{1+x^2} = 2\tan^{-1}x\Big]_0^1 = \frac{\pi}{2} \text{ Ans}$$

44. (B)

$$I = \int \frac{2x+1}{(x^2+4x+1)^{3/2}} dx = \int \frac{2x+1}{x^3\left(1+\frac{4}{x}+\frac{1}{x^2}\right)^{3/2}} dx = \int \frac{2x^{-2}+x^{-3}}{\left(1+\frac{4}{x}+\frac{1}{x^2}\right)^{3/2}} dx$$

now put $\frac{1}{x^2} + \frac{4}{x} + 1 = t^2$ then $I = -\int \frac{dt}{t^2} = \frac{1}{t} + c$

45. (A)

$$\left.\frac{t^3}{3}\right|_0^{f(x)} = x\cos\pi x \quad \Rightarrow [f(x)]^3 = 3x\cos\pi x \quad ....(1)$$

$[f(9)]^3 = -27 \Rightarrow f(9) = -3$

also differentiating $\int_0^{f(x)} t^2\, dt = x\cos\pi x$

$$[f(x)]^2 \cdot f'(x) = \cos\pi x - x\pi\sin\pi x$$

$\therefore [f(9)]^2 \cdot f'(9) = -1$

$\Rightarrow f'(9) = -\frac{1}{(f(9))^2} = -\frac{1}{9}$ $\qquad f'(9) = -\frac{1}{9} \Rightarrow$ (A)

46. (D)

$$2y.\frac{dy}{dx} = 4a; \frac{dy}{dx} = \frac{2a}{y}$$

$$\frac{dy}{dx} = -\frac{1}{2a}e^{-\frac{x}{2a}} = -\frac{y}{2a}$$

47. (A)

$\left[\frac{x}{99}\right] = \left[\frac{x}{101}\right] = 0$ $\qquad$ iff $x \in \{1, 2, ..., 98\}$. 98 such numbers

$\left[\frac{x}{99}\right]=\left[\frac{x}{101}\right]=1$ iff $x\in\{101,102,...,197\}$. 97 such numbers

In general; If

$\left[\frac{x}{99}\right]=\left[\frac{x}{101}\right]=k$ where $k\geq 1$, then $x\in\{101k,101k+1,...,99(k+1)-1\}$

(99 – 2k) such numbers.

$\therefore \quad 99(k+1)-1\geq 101k$

$98\geq 2k$, $k\leq 49$

so, $98+\sum_{k=1}^{49}(99-2k)$

$=98+2401=2499$

48. (D)

$4=t^2$; $2=t^3-3t$

$t=-2,2\Rightarrow t=2$

$\frac{dy}{dx}=\frac{3t^2-3}{2t}=\frac{9}{4}$

49. (A)

$\because \sqrt{1+\cos 2x}=\sqrt{2}\sin^{-1}(\sin x)$

or $|\cos x|=\sin^{-1}(\sin x)$

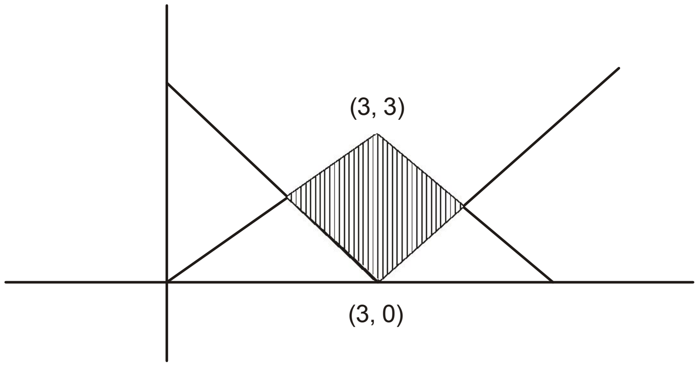
If $-\frac{\pi}{2}\leq x\leq\frac{\pi}{2}$, then $\cos x=x$ $\quad\therefore$ one solution

If $\frac{\pi}{2}<x\leq\pi$, then $-\cos x=\pi-x$ $\quad\therefore$ one solution

If $-\pi\leq x<-\frac{\pi}{2}$, then $-\cos x=-x-\pi$ therefore no solution

$\therefore$ Total no. of solutions = 2

50. (C)



Requred area = $\frac{3}{\sqrt{2}} \times \frac{3}{\sqrt{2}} = \frac{9}{2}$

51. (C)

$f(x) + f'''(3) = x^3 + x^2 f'(1) + xf''(2)$

differetiating w.r.t x

$f'(x) = 3x^2 + 2xf'(1) + f''(2)$ ......(i)

$f''(x) = 6x + 2f'(1)$ ......(ii)

$f'''(x) = 6$ ......(iii)

Putting x = 1 in (i) and x = 2 in (ii) on solving we get,

$f'(1) = -5$ & $f''(2) = 2$, $f(x) = x^3 - 5x^2 + 2x - 6$

52. (B)

$f(x) = x^3 - 5x^2 + 2x - 6$

$f'(x) = 3x^2 - 10x + 2$ two real and distinct root

53. (D)

For continous at x = 0 $\Rightarrow$ r = 0 and at x = –2 $\Rightarrow$ 2p – q = –4

Straight line L is y = 4x – 1 touches f(x) at x = –3, –1 & 1

54. (C)

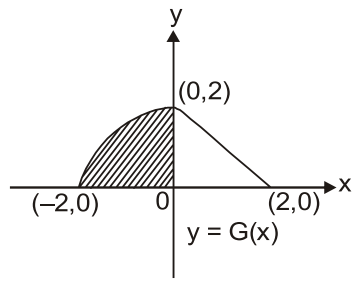
f(x) is differentiable if r = 0, 4p – 2q + r = –8

q = 2 and 4p – q + 6 = 0

$\Rightarrow$ r = 0, q = 2, p = –1

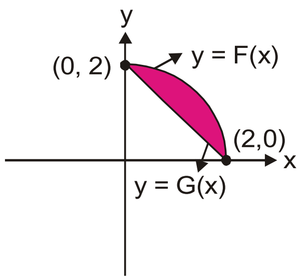
55. (A)

$$G(x)=\begin{cases}\sqrt{4-x^2}, & -2\le x<0\\ |x-2|, & 0\le x\le 2\end{cases}$$



$$\text{Area}=\frac{\pi r^2}{4}=\frac{\pi\times 2^2}{4}=\pi \text{ sq. units}$$

56. (C)

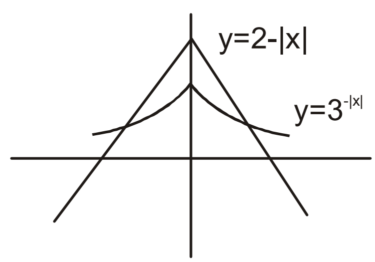


$$\text{Area}=\frac{\pi\times 2^2}{4}-\frac{1}{2}\times 2\times 2=\pi-2 \text{ sq. units}$$

57. (C)

(P) From graph the number of roots of the equation

$2-|x|=3^{-|x|}$ is two



(Q) $f(x)=x^2|x|=\begin{cases}x^3, & x\ge 0\\ -x^3, & x<0\end{cases}$

$f'''(0^+)=6$ & $f'''(0^-)=-6$

(R) For f(x) to have local minima at x = 2

$$a^2-9a-9\ge 1$$

$(a-10)(a+1)\ge 0$

$a\in(-\infty,-1]\cup[10,\infty)$

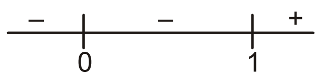
(S) $6\int_0^3|(x-1)(x-2)|\,dx=6\{\int_0^1(x-1)(x-2)\,dx-\int_1^2(x-1)(x-2)\,dx+\int_2^3(x-1)(x-2)\,dx\}$

$=6\left\{\frac{5}{6}+\left(\frac{1}{6}\right)+\frac{5}{6}\right\}=11$

58. (D)

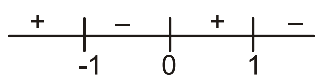
(P) $f(x) = x^{4/3} - 4x^{1/3}$

$$f'(x) = \frac{4}{3}\left(\frac{x-1}{x^{2/3}}\right)$$



(Q) $f(x) = 5x^{2/5} - x^2$

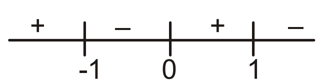
$$f'(x) = 2\left(\frac{1-x^{8/5}}{x^{3/5}}\right) = 2\left(\frac{(1-x^{1/5})(1+x^{1/5})(1+x^{2/5})(1+x^{4/5})}{x^{3/5}}\right)$$



(R) $f(0) = \lim_{x\to 0}\frac{1}{x}\log\left(\frac{e^x-1}{x}\right) = \lim_{x\to 0}\frac{1}{\frac{e^x-1}{x}}\cdot\frac{xe^x-(e^x-1)}{x^2} = \frac{1}{2}$

(S) $f(x) = 3x^{2/3} - x^2$

$$f'(x) = 2\left(\frac{1-x^{4/3}}{x^{1/3}}\right) = 2\frac{(1-x^{1/3})(1+x^{1/3})(1+x^{2/3})}{x^{1/3}}$$



59. (A)

(P) f(x) is continuous $\forall x \in R$ but not differentiable at x = 2

(Q) g(x) is discontinuous and non-differentiable at x = I

(R) h(x) is continuous and differentiable for $\forall x \in R$

(S) $\Psi(x)$ is continuous everywhere but not differentiable at x = 1

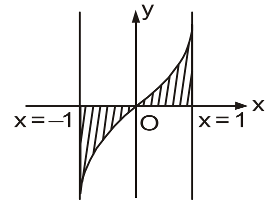
60. (A)

(P) Required area = 4s

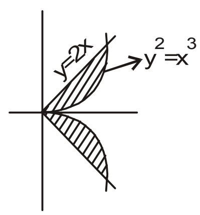
$$s = \int_0^{\pi}(x+\sin x)\ dx - \int_0^{\pi} x\ dx)$$

$$= \frac{\pi^2}{2} - \cos p + \cos 0 - \frac{\pi^2}{2} = 2.\ \text{sq. units.}$$

(Q) Required area = $2\int_0^1 xe^x\ dx = 2\ [xex - ex]_0^1 = 1$



(R) $y^2 = x^3$ and $|\ y\ | = 2x$ both the curve are symmetric about y-axis.



$4x^2 = x^3 \quad \Rightarrow \quad x = 0, 4$

required area = $2\int_0^4 (2x - x^{3/2})\,dx = \frac{16}{5}$

(S) $\sqrt{x} + \sqrt{|\,y\,|} = 1$

Above curve is symmetric about x-axis

$\sqrt{|\,y\,|} = 1 - \sqrt{x}$ and $\sqrt{x} = 1 - \sqrt{|\,y\,|} \ \Rightarrow$ for $x > 0$, $y > 0$ $\sqrt{y} = 1 - \sqrt{x}$

$$\frac{1}{2\sqrt{y}}\ \frac{dy}{dx} = -\frac{1}{2\sqrt{x}}\ ,\ \frac{dy}{dx} = -\sqrt{\frac{x}{y}}$$

$\frac{dy}{dx} < 0$, function is decreasing required area = $\int_0^1 \left(2\sqrt{x} - 2x\right) = \frac{1}{3}$