# SOLUTIONS

# PROGRESS TEST-4

## RBA, RB-1806 TO 1809

## RBK-1804

## (JEE MAIN PATTERN)

## Test Date: 07-10-2017

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# PHYSICS

**1.** tan 30° = $\frac{1}{2}\tan\theta$ , tan θ = $\frac{2}{\sqrt{3}}$

∴ **(C)**

**2.** $V = \left(\frac{kq}{x_0}\right) - \frac{kq}{2x_0} + \frac{kq}{3x_0} - \frac{kq}{4x_0} + ..............$ , $V = \frac{q\ln 2}{4\pi\varepsilon_0 x_0}$

∴ **(D)**

**3.** **(A)** **4.** **(C)** **5.** **(A)** **6.** **(D)**

**7.** **(D)**

**8.** **(A)**

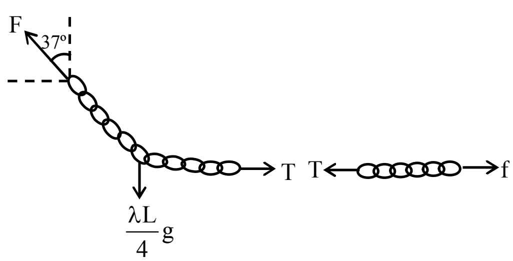
2kx cos 30° = $\left(\frac{4m_1m_2}{m_1+m_2}\right)g$

**9.** **(A)** **10.** **(B)** **11.** **(C)** **12.** **(D)**

**13.** **(A)**

$\mu_k$ depends upon nature of contact surfaces only

**14.** **(B)**




F cos 37° = $\frac{\lambda L}{4}$ g

F sin 37° = T = f

∴ f = $\frac{3\lambda Lg}{16} \le \mu$ N

$\Rightarrow \mu \ge \frac{1}{4}$

**15.** **(D)**

**16.** Time taken for the particle to reach the highest point is $\frac{t_1+t_2}{2}$.

Therefore, initial vertical velocity of the particle is: $u = g\left(\frac{t_1+t_2}{2}\right)$

Therefore, height of *B* from the ground is

$$h = ut_1 - \frac{1}{2}gt_1^2 = g\left(\frac{t_1+t_2}{2}\right)t_1 - \frac{1}{2}gt_1^{\,2} \qquad \text{or} \quad h = \frac{1}{2}gt_1t_2$$

$\therefore$ **(D)**

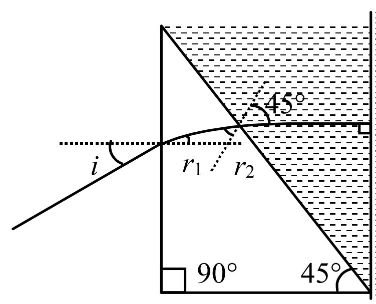
**17.** Acceleration is positive through out the motion.

$\therefore$ **(C)**

**18.** **(A)**

**19.** **(D)**

**20.** Since incident ray retraces its path it must strike the plane mirror perpendicularly.




From Snell's law $\sin i = \mu_1 \sin r_1$

and $\mu_1 \sin r_2 = \mu_2 \sin 45° \Rightarrow \mu_1 \sin r_2 = \frac{\mu_2}{\sqrt{2}}$

$$\Rightarrow r_2 = \sin^{-1}\left(\frac{\mu_2}{\sqrt{2}\mu_1}\right)$$

Also, $r_1 + r_2 = \frac{\pi}{4}$

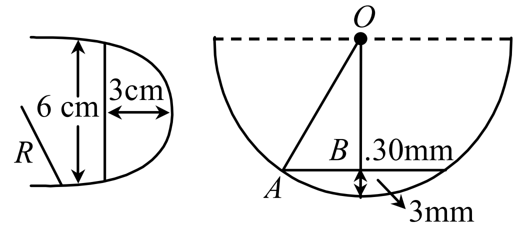
$$\therefore \quad r_1 = \frac{\pi}{4} - \sin^{-1}\left(\frac{\mu_2}{\sqrt{2}\mu_1}\right)$$

$$\therefore \ i = \sin^{-1}\left[\mu_1 \sin\left(\frac{\pi}{4} - \sin^{-1}\frac{\mu_2}{\sqrt{2}\mu_1}\right)\right]$$

$\therefore$ **(B)**

**21.** **(B)**

**22.** In $\Delta$ *OAB*, *OA* = *R* mm




$\therefore$ *OB* = (*R* – 3) mm and *AB* = 30 mm

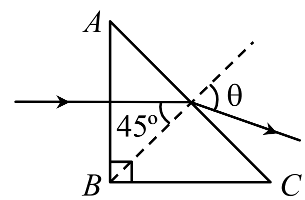
$OA^2 = OB^2 + AB^2$

$R^2 = (R-3)^2 + (30)^2 \Rightarrow R^2 = R^2 + 9 - 6R + 900$

$\Rightarrow R = \frac{909}{6}$ = 151.5 mm ≈ 15 cm.

$\therefore$ **(A)**

**23.** $\mu \sin 45 = 1 \sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{\mu}{\sqrt{2}}\right)$




$$\frac{d\theta}{dt} = \frac{1}{\sqrt{1-\frac{\mu^2}{2}}} \times \frac{1}{\sqrt{2}}\left(\frac{d\mu}{dt}\right)$$ = 2 rad/sec.

$\therefore$ **(B)**

**24.** $m = \frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right) = m_0\left(1+\frac{D}{f_e}\right)$

$\Rightarrow \; 30 = m_0\left(1+\frac{25}{5}\right) = m_0 \times 6$

$\Rightarrow \; m_0 = 5$

$\therefore$ **(B)**

**25.** **(A)**

Applying Snell's law between the points *O* and *P*, we have

$2\times\sin 60^0 = (\sin 90^0)\times\frac{2}{(1+H^2)}$, $\qquad 2\times\frac{\sqrt{3}}{2} = 1\times\frac{2}{(1+H^2)}$

$(1+H^2) = \frac{2}{\sqrt{3}}$, $\qquad H = \sqrt{\left(\frac{2}{\sqrt{3}}-1\right)}$

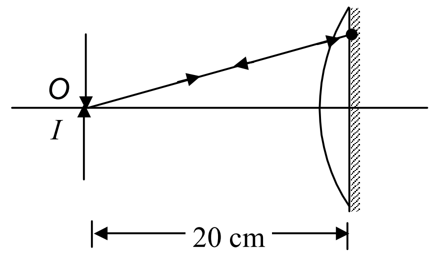
**26.** $\Delta x = (\mu_1 - \mu_2)\, t$

$I_P = 2I_0\left[1+\cos\left(\frac{2\pi}{\lambda}\right)(\mu_1-\mu_2)\, t\right]$

$= 4I_0 \cos^2 \frac{\pi}{\lambda}(\mu_1 - \mu_2)\, t$

$\therefore$ **(A)**

**27.** When the object is placed at the focus of the lens, the refracted rays will be incident normally on the silvered surface. So, they will retrace their path.




Hence, the image will be formed at the location of the object. In this way, the combination behaves as a concave mirror of radius of curvature (*R*) = 20 cm

$\therefore$ $f = \frac{R}{2}$ = 10 cm

$\therefore$ **(D)**

**28.** $\delta = \left(\frac{\mu_2}{\mu_1} - 1\right)A = \left(\frac{1.5}{5/4} - 1\right) \times 4^0 = 0.8^0$ downward

$\therefore$ **(B)**

**29.** At maximum depth the ray graze the surface (i.e. the angle made by the ray with normal will become $90^0$)

Applying Snell's law $1 \times \sin 45^0 = \left(\sqrt{2} - \frac{1}{\sqrt{2}}x\right)\sin 90^0$

$\Rightarrow \sqrt{2} - \frac{1}{\sqrt{2}}x = \frac{1}{\sqrt{2}}$ or $x$ = 1 m

$\therefore$ **(D)**

**30.** Since consecutive prisms are kept in inverted position w.r.t. each other, they would (a pair of prisms) cancel each others deviations. So if even number of prisms are there deviation would be zero and if odd number of prisms are there it would be $\delta$.

$\therefore$ **(B)**

# CHEMISTRY

**31. (B)**

$$\upsilon_{av} = \sqrt{\frac{8RT}{\pi m}}$$

Average speed depends only on temperature and molecular mass of the gas, hence it will be same in both vessels A and C.

**32. (D)**

$$n_f = n_i$$

$$\frac{p_f v}{RT_1} + \frac{p_f \cdot 4v}{RT_2} = \frac{p_1 v}{RT_1} + \frac{p_2 \cdot 4v}{RT_2}$$

$$p_f\left(\frac{1}{300} + \frac{4}{400}\right) = 10^5\left(\frac{5}{300} + \frac{2\times 4}{400}\right)$$

$$p_f = \frac{11}{4}\times 10^5 = 2\cdot 8\times 10^5\ N/m^2$$

**33. (A)**

PV = Constant hence its differential will be zero, i.e.,

$$\lim_{P\to 0}\frac{d(PV)}{dP} = 0$$

Boyle's temperature $T_B = \frac{a}{Rb}$

**34. (D)**

**35. (C)**

The temperature at which real gas behaves like ideal gas is called Boyle temperature at this temperature Z = 1, Thus, graph (II) represents oxygen and graph (I) represents $H_2$(Z>1).

**36. (D)**

Equivalent mass of $KMnO_4$ in acidic, basic and neutral medium are 31.6, 158 and 52.6. The ratio will be 31.6 : 158 : 52.6.

| Acid medium | : | Basic medium | : | Neutral medium |
|---|---|---|---|---|
| $E_w = \frac{M_w}{5} = \frac{158}{5}$ | : | $E_w = \frac{M_w}{1} = \frac{158}{1} = 158$ | : | $E_w = \frac{M_w}{3} = \frac{158}{3}$ |
| 3 | : | 15 | : | 5 |

**37. (A)**

$$X^{n+} \quad + \quad Xr_2O_7^{2-} \longrightarrow \overset{+5}{X}O_3^- + Cr^{3+}$$

$9\times 10^{-3}$ mol $\quad 6\times 10^{-3}$ mol

v.f. $= (5-n)$ $\quad$ v.f. $= 6$

$$9 \times 10^{-3}(5-n) = 6 \times 10^{-3} \times 6$$

$$5 - n = 4 \Rightarrow n = 1$$

**38. (B)**

Volume of $H_2$ = Volume of Cylinder increased

$$= 0.025 \times 1 = 0.025\ m^3 = 25\ lit$$

$$PV = \frac{w}{m}RT$$

$$1 \times 25 = \frac{w}{2} \times 0.082 \times 300$$

$$w_{H_2} = 2.032\ g$$

Reaction : $\underset{56g}{Fe(s)} + 2HCl(aq) \longrightarrow FeCl_2(aq.) + \underset{2g}{H_2}$

Mass of pure iron = $\frac{56}{2} \times 2.032 = 56.896\ g$

% Purity = $\frac{\text{Mass of pure iron}}{\text{Mass of Impure iron}} \times 100$

$$= \frac{56.896}{75} \times 100 = 75.86$$

**39. (C)**

milli–equivalent of $NH_3$ reacted with $HNO_3$

= 45 × 0.4 – 20 × 0.1 = 16

$\therefore\ \frac{W}{17} \times 1000 = 16;\qquad W_{NH_3} = 0.272\,g;$

$$\Rightarrow\ wt_N = \frac{0.272}{17} \times 14 = 0.224\,g$$

% N in the sample = $\frac{0.224}{1.12} \times 100 = 20\%$

**40. (D)**

$$C_nH_{2n+2}(g) + \left(\frac{3n+1}{2}\right)O_2(g) \longrightarrow nCO_2(g) + (n+1)H_2O(g)$$

Let initial pressure of $C_nH_{2n+2}$ is P then increase in pressure

$$= P\left[(2n+1)-1-\left(\frac{3n+1}{2}\right)\right] = \left(\frac{n-1}{2}\right)P$$

546 K and 4.6 atm $\longrightarrow$ 273 K and 2.3 atm; Increase in pressure = 0.3 atm

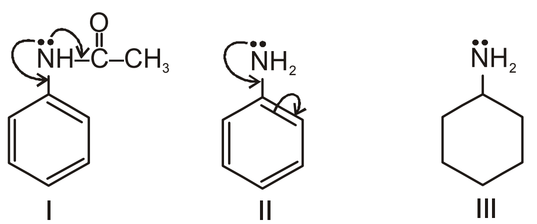
$$P = \frac{nRT}{V} = \frac{11.6}{M} \times \left(\frac{0.0821 \times 273}{22.41}\right)$$

$$\left(\frac{n-1}{2}\right) \times \frac{11.6}{(14n+2)} = 0.3$$

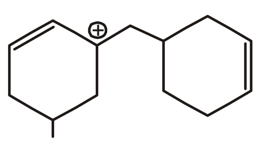
$$\left(\frac{n-1}{14n+2}\right) = \frac{0.6}{11.6} = n = 4$$

$\therefore$ Compound is $C_4H_{10}$

**41. (B)**



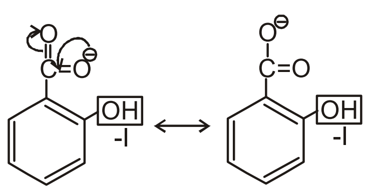
**42. (C)**


; 3º carbocation and it is resonance stabilized.

**43. (B)**

$H_2C — CH = \overset{\ominus}{C}H — CH = \overset{\oplus}{O} — CH_3$ : Octet is complete and opposite charge at minimum distance.

**44. (D)**



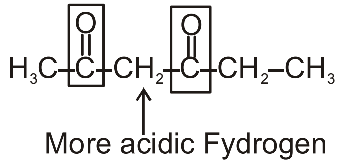
**45. (A)**

$CH_3–CH_2–CH_3$ } No lone pair least basic

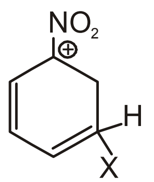
$CH_3–CH_2–SH$ } size↑ B.S. ↓

$CH_3–CH_2–OH$
$CH_3–CH_2–NH_2$ } E.N. ↑ B.S. ↓ Size same

**46. (B)**

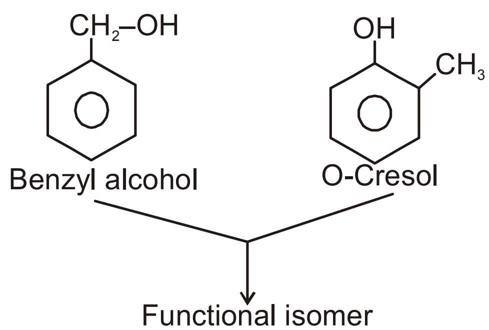



**47. (A)**




**48. (D)**

**49. (A)**




**50. (D)**

**51. (A)**

$La^{3+} > Eu^{3+} > Gd^{3+} > Lu^{3+}$

**52. (D)**

Configuration based.

**53. (B)**

Configuration based.

**54. (D)**

A = H, B = He, C = Li

**55. (B)**

Due to large size of S.

**56. (C)**

HF(s) has zig-zag structure.

**57. (D)**

For dipole moment of $CH_3Cl$, bond length dominates over charge.

**58. (D)**

Due to presence of 3 lone pairs on each small size O-atom.

**59. (D)**

Bond angle $OCl_2$ = 110.9º and $O(SiH_3)_2$ = 144.1º, hence hybridization different.

**60. (D)**

Due to 1 unpaired electron, $N_2^+$ is paramagnetic.

# MATHEMATICS

**61. (B)**

$\tan\theta = \cot\theta - 2\cot 2\theta$

$$\therefore \quad \frac{1}{2}\tan\frac{\theta}{2} = \frac{1}{2}\cot\frac{\theta}{2} - \cot\theta$$

$$\frac{1}{2^n}\tan\frac{\theta}{2^n} = \frac{1}{2^n}\cot\frac{\theta}{2^n} - \frac{1}{2^{n-1}}\cot\frac{\theta}{2^{n-1}}$$

$$\therefore \text{ Required limit} = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{1}{\dfrac{2^n\tan\theta/2^n}{\theta/2^n}\cdot\dfrac{\theta}{2^n}} - 2\cot 2\theta\right) = \frac{1}{\theta} - 2\cot 2\theta$$

**62. (B)**

Clearly AC is parallel to y-axis. Its midpoint is (2, 2). Thus B ≡ (1, 2).

Parabola will be in the form of $(x-2)^2 = \lambda(y-3)$.

It passes through (1, 2)

$\Rightarrow 1 = -\lambda$. Thus parabola is $(x-2)^2 = -1(y-3)$.

It focus is $x - 2 = 0$. $y - 3 = -\frac{1}{4}$, i.e., $\left(2, \frac{11}{4}\right)$.

**63. (C)**

Let $g(x) = \sin^{-1}|\sin x| + \cos^{-1}(\cos x)$

$$= \begin{cases} 2x & ; \quad 0 \le x \le \dfrac{\pi}{2} \\ \pi & ; \quad \dfrac{\pi}{2} < x \le \dfrac{3\pi}{2} \\ 4\pi - 2x & ; \quad \dfrac{3\pi}{2} < x \le 2\pi \end{cases}$$

g (x) is periodic with period $2\pi$ and is constant in the interval $\left[2n\pi+\frac{\pi}{2}, 2n\pi+\frac{3\pi}{2}\right]$, $n \in I$

now, $f(x) = g(\lambda x)$

$\therefore$ f(x) is constant in the interval $\left[\frac{2n\pi}{\lambda}+\frac{\pi}{2\lambda}, \frac{2n\pi}{\lambda}+\frac{3\pi}{2\lambda}\right]$

$\therefore 2\pi = \frac{3\pi}{2\lambda} - \frac{\pi}{2\lambda} \Rightarrow \lambda = \frac{1}{2}$

**64. (C)**

Let P be $(-2+r\cos\theta, 0+r\sin\theta) \Rightarrow r^2\sin^2\theta - 4(-2+r\cos\theta) = 0$

$\Rightarrow r^2\sin^2\theta - 4r\cos\theta + 8 = 0$

$\Rightarrow r_1 + r_2 = \frac{4\cos\theta}{\sin^2\theta}$ and $r_1r_2 = \frac{8}{\sin^2\theta}$

$\frac{1}{AP} + \frac{1}{AQ} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{\cos\theta}{2} = \frac{1}{4} \Rightarrow \tan\theta = \sqrt{3}$

Hence equation of incident ray is $y - 0 = -\sqrt{3}(x+2) \Rightarrow y + x\sqrt{3} + 2\sqrt{3} = 0$

**65. (B)**

$\cot\left\{\cot^{-1}3 + \cot^{-1}7 + \cot^{-1}13 + \cot^{-1}21\right\}$

$= \cot\left\{\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \tan^{-1}\frac{1}{21}\right\}$

$= \cot\left\{\tan^{-1}2 - \tan^{-1}1 + \tan^{-1}3 - \tan^{-1}2 + ... + \tan^{-1}5 - \tan^{-1}4\right\}$

$= \cot\left[\tan^{-1}5 - \tan^{-1}1\right] = \frac{3}{2}$

**66. (B)**

Focus is $\left(\frac{7}{2}, \frac{7}{2}\right)$ and it's axis is the line y = x. Corresponding value of 'a' is $\frac{1}{4}\sqrt{(1+1)} = \frac{\sqrt{2}}{4}$.

Let the equation of it's directrix be y + x + $\lambda$ = 0. $\Rightarrow \frac{\left|\frac{7}{2}+\frac{7}{2}+\lambda\right|}{\sqrt{2}} = 2.\ \frac{\sqrt{2}}{4} \Rightarrow \lambda = -6, -8$

Thus equation of parabola is

$$\left(x-\frac{7}{2}\right)^2 + \left(y-\frac{7}{2}\right)^2 = \frac{(x+y-6)^2}{2}$$

or $$\left(x-\frac{7}{2}\right)^2 + \left(y-\frac{7}{2}\right)^2 = \frac{(x+y-8)^2}{2}.$$

**67. (D)**

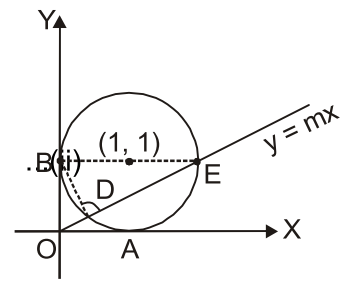
[Given limit = 0 + ($2^2$ – 1) + ($3^2$ – 1) + ...... + ($10^2$ – 1) = $\sum_{n=2}^{10}\left(n^2-1\right)$

$= \frac{10\times 11\times 21}{6}-1-9$ = 385 – 10 = 375 ]

**68. (B)**

The equation of the circle is $(x-1)^2+(y-1)^2=1$

i.e. $x^2+y^2-2x-2y+1=0$ ...(i)




Let the equation of the variable straight line be

y = mx

Solving (i) and (ii), we get

$(1+m^2)x^2-2x(1+m)+1=0$

$\therefore$ Length $DE=\sqrt{\frac{8m}{1+m^2}}$

A = Area of $\Delta DEB=\frac{1}{2}DE\cdot$ (distance of B from DE)

$$A^2=\frac{1}{4}\cdot\frac{8m}{1+m^2}\cdot\frac{1}{1+m^2}$$

$$A=\frac{\sqrt{2m}}{1+m^2}\Rightarrow\frac{dA}{dm}=\frac{1-3m^2}{\sqrt{m}(1+m^2)^2}=0 \qquad \Rightarrow \qquad m=\pm\frac{1}{\sqrt{3}}.$$

Also $\frac{d^2A}{dm^2}<0$ at $m=\frac{1}{\sqrt{3}}$.

**69. (D)**

$$\lim_{x\to 2}\frac{\sec^x\theta-\tan^x\theta-1}{x-2} = \lim_{x\to 2}\frac{(\sec^x\theta-\sec^2\theta)-(\tan^x\theta-\tan^2\theta)}{x-2}$$

$= \lim_{x\to 2} \dfrac{\sec^2\theta\cdot(\sec^{x-2}\theta-1)-\tan^2\theta(\tan^{x-2}\theta-1)}{x-2}$ Let $(x-2)=y$ then as $x\to 2,\ y\to 0$

$\therefore$ Given limit $=\sec^2\theta\left(\lim_{y\to 0}\dfrac{(\sec\theta)^y-1}{y}\right)-\tan^2\theta\left(\lim_{y\to 0}\dfrac{(\tan\theta)^y-1}{y}\right)$

$= \sec^2\theta\cdot \ln\sec\theta - \tan^2\theta\cdot\ln\tan\theta$. ]

**70. (D)**

Here A, B, C lie on a circle having centre at origin. So, A, B, C form a right angled triangle, if any side is a diameter.

Now, $\gamma-\alpha=\pi \Rightarrow \angle AOC=\pi$

$\Rightarrow$ AC is a diameter

$\Rightarrow$ $\Delta ABC$ is right angled.

**71. (D)**

Here $R=\{(x,y):\left|x^2-y^2\right|<16\}$ and given $A=\{1,2,3,4,5\}$

$\therefore R=\{(1,1);(1,2)(1,3)(1,4);(2,1)(2,2)(2,3)\ (2,4);(3,1)(3,2)\ (3,3)(3,4);(4,1)(4,2)(4,3);$

$(4,4)(4,5),(5,4)(5,5)\}$

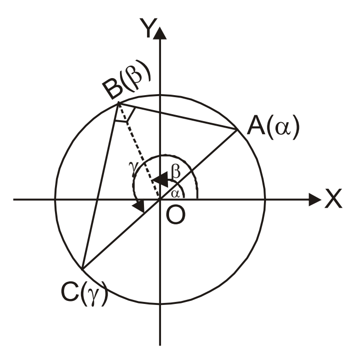
**72. (A)**

$\because\quad 4\cos^2\theta-3=\dfrac{4\cos^3\theta-3\cos\theta}{\cos\theta}=\dfrac{\cos3\theta}{\cos\theta}$

$\therefore\left(4\cos^2 9^\circ-3\right)\left(4\cos^2 27^\circ-3\right)\left(4\cos^2 81^\circ-3\right)\left(4\cos^2 243^\circ-3\right)$

$=\dfrac{\cos27^\circ}{\cos9^\circ}\cdot\dfrac{\cos81^\circ}{\cos27^\circ}\cdot\dfrac{\cos243^\circ}{\cos81^\circ}\cdot\dfrac{\cos729^\circ}{\cos243^\circ}$

$=\dfrac{\cos729^\circ}{\cos9^\circ}=\dfrac{\cos(2\times360^\circ+9^\circ)}{\cos9^\circ}=1$




**73. (A)**

$\sin^2x\cos^2x-\cos^2x\sin^4x=1$

$\Rightarrow\sin^2x\cos^2x\left(1-\sin^2x\right)=1$

$\Rightarrow\sin^2x\cos^4x=1$, No value of 'x'

**74. (C)**

Given $A=\{1,2,3,4\}$

$R=\{(1,3),(4,2),(2,4),(2,3),(3,1)\}$

$(2,3) \in R$ but $(3,2) \notin R$. Hence R is not symmetric.

R is not reflexive as $(1,1) \notin R$.

R is not a function as $(2,4) \in R$ and $(2,3) \in R$.

R is not transitive as $(1,3) \in R$ and $(3,1) \in R$ but $(1,1) \notin R$.

**75.** **(C)**

Both circles cut each other orthogonally. 'C' and 'D' will be centres of two circles also CD will be diameter of circumcircle of quadrilateral ACBD.

Diameter $= \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

**76.** **(C)**

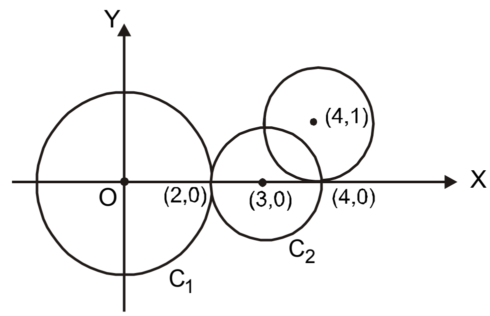
Circles $C_1$ and $C_2$ touch each other externally, so they have three common tangents.

Circles $C_2$ and $C_3$ cut each other at two points, so they have two common tangents.

Circles $C_1$ and $C_3$ are external to each other, so they have four common tangents.

No common tangent can be drawn to touch all the three circles.

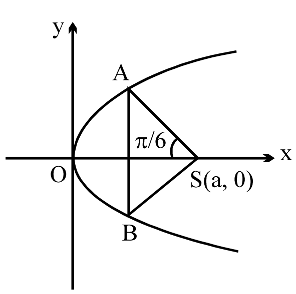
So, total no. of common tangents = 3 + 2 + 4 = 9.




**77.** **(B)**

Let A ≡ $(at_1^2, 2at_1)$, B ≡ $(at_1^2, -2at_1)$. We have $m_{AS} = \tan\left(\frac{\pi}{6}\right) \Rightarrow \frac{2at_1}{at_1^2 - a} = -\frac{1}{\sqrt{3}}$

$\Rightarrow t_1^2 + 2\sqrt{3}\,t_1 - 1 = 0 \Rightarrow t_1 = -\sqrt{3} \pm 2$




Clearly $t_1 = -\sqrt{3} - 2$ is rejected. Thus $t_1 = (2 - \sqrt{3})$. Hence AB = $4at_1 = 4a(2 - \sqrt{3})$.

**78.** **(D)**

$$[f(x) = \lim_{n\to\infty} \frac{2}{n^2}\frac{n(n+1)}{2}\cdot x\left(\frac{3^{nx}-1}{3^{nx}+1}\right) = \lim_{n\to\infty}\frac{(n+1)}{n}\left(\frac{(3^x)^n - 1}{(3^x)^n + 1}\right)\cdot x$$

$$= \begin{bmatrix} x & x>0 \\ 0 & x=0 \\ -x & x<0 \end{bmatrix} \left( \because \lim_{n\to\infty} \frac{(3^x)^n - 1}{(3^x)^n + 1} = \begin{bmatrix} 1 & x>0 \\ 0 & x=0 \\ -1 & x<0 \end{bmatrix} \right) \therefore \quad f(x) = |x|$$

Now, $|x| = |x^2 - 2| \Rightarrow (x^2-2)^2 - x^2 = 0$

$\Rightarrow (x^2 - 2 - x)(x^2 - 2 + x) = 0$

$\Rightarrow (x-2)(x+1)(x+2)(x-1) = 0 \Rightarrow x = -2, -1, 1, 2$ $\therefore$ Sum of all the solutions is zero.

**79. (A)**

$3\sin^2 A + 2\sin^2 B = 1 \Rightarrow 3\cos 2A + 2\cos 2B = 3$

$\Rightarrow \cos 2A + \frac{2}{3}\cos 2B = 1$ .............(i)

Also 3sin2A = 2sin2B $\Rightarrow \frac{2}{3} = \frac{\sin 2A}{\sin 2B}$

putting this value in (i), we get

$\cos 2A + \frac{\sin 2A}{\sin 2B}\cdot\cos 2B = 1$ $\Rightarrow \sin 2A\cdot\cos 2B + \sin 2B\cdot\cos 2A = \sin 2B$

$\Rightarrow 2\sin A\cdot\cos(A+2B) = 0$

$\because 0 < \sin A < 1$, as $0 < A < \pi/2 \therefore \cos(A+2B) = 0 \Rightarrow A + 2B = \pi/2$

From $3\sin^2 A + 2\sin^2 B = 1$, A & B both lie between 0 to $\frac{\pi}{4}$. $\therefore \lambda = 2$

**80. (C)**

Put $\theta = 0$, then $2^7 = 1 + a + b + c + d$

$\Rightarrow a + b + c + d = 127$

$\therefore (a+b+c+d-2)^{\frac{1}{3}} = (125)^{\frac{1}{3}} = 5$

**81. (A)**

By ratio and proportion

$\frac{a}{b} = \frac{c}{d} = \frac{xa+yc}{xb+yd}$

$$\frac{\sin(\alpha+\beta+\gamma)(\sin\alpha+\sin\beta+\sin\gamma)+\cos(\alpha+\beta+\gamma)(\cos\alpha+\cos\beta+\cos\gamma)}{\sin^2(\alpha+\beta+\gamma)+\cos^2(\alpha+\beta+\gamma)}$$

$$\frac{\cos(\alpha+\beta)+\cos(\beta+\gamma)+\cos(\gamma+\alpha)}{1} = 2$$

**82. (D)**

We have $f(x) = \begin{cases} 3, & x \le 0 \\ 3^{-x} - 3^{x} + 3, & x > 0 \end{cases}$ (As $sgn(e^{-x}) = 1 \ \forall x \in R$) Clearly f (x) is many one.

For $x > 0$, f (x) is decreasing, hence range of f (x) for $x > 0$ is $(-\infty, 3)$

$\therefore$ Range of the function f (x) is $(-\infty, 3]$, which is subset of R.

Hence f is neither injective nor surjective. ]

**83. (C)**

It is obvious that a, b and c are the roots of the equation $mt^3 + (l - p)t - kq = 0$, where (p, q) is the point of concurrency.

Obviously sum of roots = a + b + c = 0

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

**84. (C)**

$\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$ ; on subtraction , $2\sin^{-1} x = \frac{\pi}{2} - \cos^{-1}(x\sqrt{3}) = \sin^{-1}(x\sqrt{3})$

$\Rightarrow 2\alpha = \beta \Rightarrow 2\sin\alpha\cos\alpha = \sin\beta \Rightarrow x = 0, \pm 1/2$ ]

**85. (D)**

We have $f(x) - g(x) = x^2 + 5x + 7$ ....(1)

Replace $x \to -x$ in equation (1), we get

$f(-x) - g(-x) = x^2 - 5x + 7 \qquad \Rightarrow -f(x) - g(x) = x^2 - 5x + 7$

$\Rightarrow f(x) + g(x) = -x^2 + 5x - 7$....(2)

$\therefore$On solving (1) and (2), we get $f(x) = 5x$ and $g(x) = -x^2 - 7$

Henceg (2) = – 11 **Ans.**]

**86. (A)**

$$2\sin^2 x + \frac{1}{2}\sin 2x = n$$

$$\sin 2x - 2\cos 2x = 2n - 2$$

$$-\sqrt{5} \le 2n - 2 \le \sqrt{5}$$

$$1 - \frac{\sqrt{5}}{2} \le n \le 1 + \frac{\sqrt{5}}{2}$$

**87. (B)**

$m_1 m_2 m_3 m_4 = 1$

So, product of slopes of each pair is (–1)

Let a pair of straight line $ax^2 + 2hxy - ay^2 = 0$

Equation of bisector $\frac{x^2 - y^2}{2a} = \frac{xy}{h}$

$\Rightarrow hx^2 - 2axy - hy^2 = 0$

So, $(ax^2 + 2hxy - ay^2)(hx^2 - 2xay - hy^2) = 0$

$\Rightarrow ah(x^4 + y^4) + 2(h^2 - a^2)(x^3y - xy^3) - 6ahx^2y^2 = 0$ ....... (i)

On compassing given equation and (i) then

$ah = 1, c = -6ah$

$\Rightarrow c = -6 \Rightarrow c + 10 = 4$

**88. (B)**

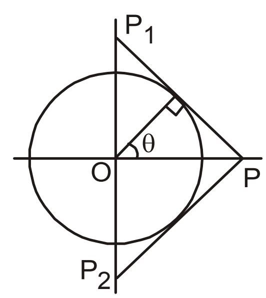
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{f(h) + |x|h + x \cdot h^2}{h}$$

Also x = y = 0 $\Rightarrow f(0) = 0$

$$\therefore f'(x) = \lim_{h\to 0}\left(\frac{f(h) - f(0)}{h} + |x| + xh\right) \Rightarrow f'(x) = f'(0) + |x|$$

**89. (B)**

$OP = 5\sqrt{2}\sec\theta,\ OP_1 = 5\sqrt{2}\ \text{cosec}\theta$




Area = $\dfrac{100}{\sin 2\theta}$

$\Rightarrow$ min. Area $= 100$, when $\theta = \dfrac{\pi}{4}$

$\therefore$ OP = 10

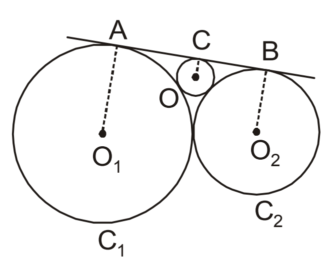
$\therefore$ $P \equiv (10, 0)$

**90. (B)**

Suppose radius of C is r.

If common tangent touches $C_1$, $C_2$ and C at points A, B and C, then

AC + BC = AB




$$2\sqrt{8r} + 2\sqrt{2r} = 2\sqrt{8\times 2} \Rightarrow 3\sqrt{2}\sqrt{r} = 4 \Rightarrow \sqrt{r} = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow r = \frac{8}{9}$$

Now, $O_1O_2^2 > OO_1^2 + OO_2^2$

$\Rightarrow \Delta$ is obtuse