SOLUTIONS

WEEKLY TEST-13

GRA, GRS-1801 & GRKS-1801

[TOP 170 STUDENTS]

(JEE MAIN PATTERN)

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PHYSICS

1. (B)

Conceptual

2. (C)

$$F_{max} = \frac{9\times10^{9}\times6\times2.56\times10^{-38}}{1^2} = 13.8\times10^{-28}\text{N}.$$

3. (C)

The component of velocity of the charge particle parallel to the line of charge will not change. So, at the minimum distance of approach (say *r*) the velocity of the particle will be $v_0 \sin 30^\circ$.

From conservation of mechanical energy,

$$\Delta U + \Delta K = 0$$

$$\Rightarrow \frac{q\lambda}{2\pi\epsilon_0}\ln\frac{r_0}{r} = \frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{v_0}{2}\right)^2$$

$$\Rightarrow r = r_0 e^{-\frac{3\pi\epsilon_0 m v_0^2}{4q\lambda}} = r_0 e^{-3} = \frac{r_0}{e^3}$$

4. (D)

5. (A)

ϕ_{cube} = flux due to single wire from whole cube.

$\phi_{cube} = \frac{\lambda\ell}{8\varepsilon_0}$ similarly four wires out of twelve will have same contribution and eight will have zero.

$$\phi_{face(1)} = \frac{\lambda\ell}{8\varepsilon_0}\times4 = \frac{\lambda\ell}{2\varepsilon_0}$$

6. (D)

$$\vec{E} = \frac{k\lambda}{a}(-\hat{i}+\hat{k}) + \frac{k\lambda}{a}(-\hat{j}+\hat{k}) + \frac{k\lambda}{a}(-\hat{k}) = \frac{k\lambda}{a}(-\hat{i}-\hat{j}+\hat{k}) = -\frac{k\lambda}{a}(\hat{i}+\hat{j}-\hat{k})$$

7. (C)

Before connection charge on 3μF capacitor $= \frac{6}{5}\times5 = 6\mu C$,

After connection charge on 3μF capacitor = 3 × 5 = 15 μC.

Charge flown = 9 μC.

8. (C)

We have

$$\frac{2KQ_A}{3L} - \frac{KQ_B \times 2}{L} = 0$$

$$\Rightarrow \quad \left|\frac{Q_A}{Q_B}\right| = 3:1$$

9. (D)

V = Cr

$$E = -\frac{dV}{dr} = -C$$

By Gauss's law,

$$-C4\pi r^2 = \int_0^r \frac{\rho 4\pi r^2 dr}{\varepsilon_0}$$

Differentiating w.r.t. to r, we get

$$\Rightarrow \quad \rho \propto \frac{1}{r}.$$

10. (A)

For two resistors in series their potential difference are proportional to resistance. For two capacitors in series, their potential differences are inversely proportional to their capacitances. Hence A and B are at the same potential and no charge will flow between them.

11. (D)

For any plane parallel to x-y plane, z = constant.

Hence V = $ax^2 + ay^2 + 2\,az^2$

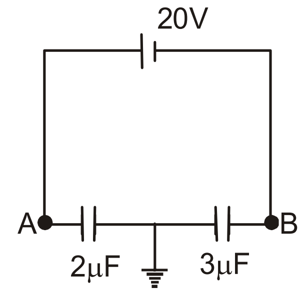
$$6250 = a\left[\left(x^2 + y^2\right) + 2z^2\right]$$

$$\frac{6250}{a} = x^2 + y^2 + 2 \times 2$$

$$\frac{6250}{1250} = x^2 + y^2 + 4$$

$$x^2 + y^2 = 1$$

12. (A)



V_1 = Voltage across 2μF capacitor

$V_1 = \frac{3}{5} \times 20$ = 12 V

$V_A - V_P = 12$

$\therefore$ V_A = 12 volt.

V_2 = voltage across 3μF capacitor

$V_2 = \frac{2}{5} \times 20$ = 8 V

$V_P - V_B = 8$

$\therefore$ V_B = – 8 volt

13. (D)

$$E = \frac{-dv}{dx} = \frac{-d(5+4x^2)}{dx} = -8x$$

$$F = -qE = 8qx = 8 \times 2 \times 10^{-6} \times 0.5\,N = 8 \times 10^{-6}\,N.$$

14. (B)

As the shell is earthed, the potential at any point on the surface is zero. Being a shell, the space inside is field free. So the net potential at any inside point is same as that of a point on the surface.

The distance of point 'P' from q is

$$d = \sqrt{(3R)^2 + \left(\frac{R}{2}\right)^2} = R\sqrt{3^2 + \frac{1}{4}} = R\sqrt{a + \frac{1}{4}} = \frac{\sqrt{37}R}{2}$$

The potential at point P is = 0 = Potential at point P due to q + Potential at point P due to charges accumulated on the surface .

The potential at P due to q is $\frac{2Kq}{\sqrt{37}R}$.

15. (D)

Even if $\varepsilon_1 > \varepsilon_2$, due to internal resistance of standard cell $V_1 < \varepsilon_2$.

$\therefore$ **(D)**

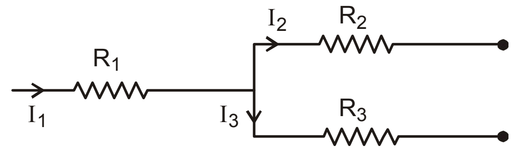
16. (C)

17. (B)

$$\frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_1}$$

$$\frac{1}{2R_3} + \frac{1}{R_3} = \frac{1}{R_1}$$

$$\frac{3}{2R_3} = \frac{1}{R_1}$$



$$R_1 = \frac{2R_3}{3}$$

$$R_2 = 2R_3$$

Now, $I_1 = I_2 + I_3$

$$I_1R_2 = I_3R_3$$

$$2I_1 = I_3$$

$$\therefore \quad I_1 : I_2 : I_3 = 3:1:2$$

$$\therefore \quad P_1 : P_2 : P_3 = 3:1:2$$

$$V_1 : V_2 : V_3 = 1:1:1$$

18. (D)

We have to take two readings one with resistance R and other without R. Option A & B are wrong because connection of E is wrong. Option C is wrong because key should be with R.

19. (A)

$V_E + 4 - 4 - 4 - 4 = V_F$

$\Rightarrow \quad V_E - V_F = 8\,\text{volt}$

$\therefore \quad I = \frac{8}{4} = 2\,A$ from E to F.

20. (D)

21. (B)

22. (A)

23. (D)

$$\frac{R}{S} = \frac{\ell_1}{\ell_2}$$

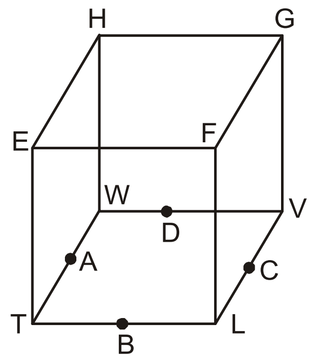
$$\Rightarrow \quad R = 33.33\,\Omega$$

$$\Rightarrow \quad \frac{dR}{R} = \frac{dS}{S} + \frac{d\ell_1}{\ell_1} + \frac{d\ell_2}{\ell_2}$$

$$\Rightarrow \quad dR = 0.2\,\Omega$$

24. (A)

The situation is similar to figure shown.



The maximum resistance across the body diagonal points.

$$R_{max} = \frac{5R}{6} = 10\Omega$$

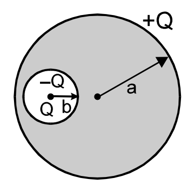
25. (D)

In figure 2, both voltmeters should read 100 volts and in figure 3 both voltmeters should read 25 volts, hence readings in figure 1 and figure 3 are not possible.

26. (A)

–Q & Q will be induced uniformly as shown.

$\therefore$ Total energy $= \Sigma$ Self energy + Σ interaction energy



$$= \frac{K(-Q)^2}{2b} + \frac{K(Q)^2}{2a} + \frac{K(Q)(-Q)}{b} + \frac{K(Q)(+Q)}{a} + \frac{K(Q)(-Q)}{a}$$

$$= KQ^2\left(\frac{1}{2a} - \frac{1}{2b}\right)$$

27. (D)

Potential for each plate remain same over whole area. If potential difference between them is, say V' then V' = Ed

i.e. E is also same inside the plates.

To keep E same, free charge density is changed i.e. charge redistributes itself.

To find new capacitance, two capacitors can be taken as connected in parallel. Then

$$= \frac{K.\epsilon_0 .A/2}{d} + \frac{\epsilon_0 .A/2}{d} = \frac{3\epsilon_0 .A}{2d}$$

By Q = CV, as Q remains unchanged V is changed to $\frac{2}{3}$V.

28. (A)

$i_0, \frac{i_0}{2}, \frac{i_0}{4}, \frac{i_0}{8}, \frac{i_0}{16}, ..., \frac{i_0}{2^n}$

$T_n = t,\ n = t/T$ $\quad i = \frac{i_0}{2^{t/T}}$ $\quad dq = i\,dt$ $\quad \int_0^q dq = \int_0^\infty \frac{i_0}{2^{t/T}}\,dt$

$q = i_0\int_0^\infty \frac{1}{e^{t/T\,\ell n\,2}}\,dt = q = i_0\int_0^\infty e^{-\frac{t}{T}\ell n\,2}\,dt$ $\quad q = i_0\left(\frac{e^{-\frac{t}{T}\ell n\,2}}{-\frac{\ell n\,2}{T}}\right)_0^\infty$

$q = \frac{i_0\,T}{\ell n\,2}\left(\frac{1}{e^\infty} - \frac{1}{e^0}\right)$ $\quad q = -\frac{i_0\,T}{\ell n\,2}.\,(0.1) = \frac{i_0\,T}{\ell n\,2}$

$i_0 = \frac{q}{T}\,ln\,2$

Instantaneous value = $\frac{q}{T}\,ln\,2\ e^{-t/T\,ln\,2}$.

29. (B)

The current density J, electric field E at the cross-section are related by

$J = \frac{E}{\rho}$

multiplying both side by A.

$JA = \frac{EA}{\rho}$ or $\quad I = \frac{\phi}{\rho}$ Where ϕ is electric flux.

or $\quad \phi = \rho I$

30. (C)

The voltmeter is connected across zero resistance (ammeter resistance) hence it shall read zero volts.

CHEMISTRY

31. (C)

32. (C)

Mass of water present in the solution at the freezing pt. = 1000 – 200 = 800 gm.

$\Delta T_f = 0.372 = \frac{1000 \times k_f w}{60 \times 800} = \frac{1000 \times 1.86 \times w}{60 \times 800}$

or w = 9.6 gm

33. (C)

$$\frac{0.009}{20} = \frac{5s \times 18}{1000} \Rightarrow s = 5 \times 10^{-3}$$

$$K_{Sp} = 4 \times 27 \times (5 \times 10^{-3})^5 = 3.375 \times 10^{-10}$$

34. (A)

$$\frac{(0.5+2) \times 20 \times 1000}{40 \times 500} = \Delta T_b + \Delta T_f = 2.5$$

$\therefore$ Required difference $= 80 + 2.5 = 82.5^{o}C = 82.5K$

35. (B)

$$\frac{0.01}{1} = \frac{i \times 1.25 \times 18}{90 \times 50} \Rightarrow i = 2, \quad 2 = 1 - \alpha + 3\alpha \Rightarrow \alpha = 0.5$$

36. (B)

$$8.3 = \frac{(0.02 + 0.03)}{(1-x)} \times 0.083 \times 300$$

or, x = 850 ml

37. (B)

y = 1– x ...(1)

$$\Delta T_f = 1.85(2 \times x + 3 \times y) = (2x + 3(1-x)) = 1.85(3-x)$$

when $x = 0$. $\Delta T_f = 5.55°C$

when $x = 1$. $\Delta T_f = 3.7°C$.

38. (B)

$\because \quad P_A^0 > P_B^0 \Rightarrow Y_A > X_A$

39. (C)

$$kt = \frac{1}{a-x} - \frac{1}{a} \Rightarrow \frac{1}{a-x} = kt + \frac{1}{a}$$

Graph between $(a - x)^{-1}$ and time t is a straight line. Hence $k = \tan\theta = 0.5$

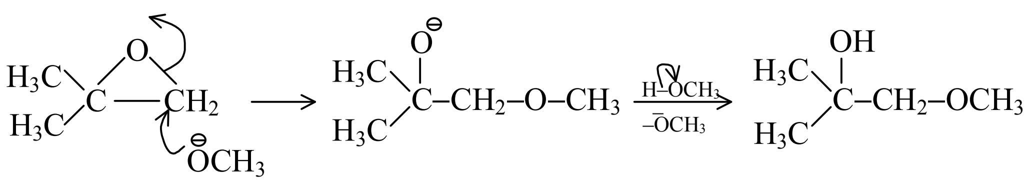
$$OA = \frac{1}{a} = 2$$

Rate at the start of $R = k[A]^2 = 0.5 \times \left(\frac{1}{2}\right)^2 = 0.125 \text{ mol L}^{-1}\text{min}^{-1}$.

40. (D)

$t_{1/2} = \frac{1}{k|A|_0}$ for IInd order reaction.

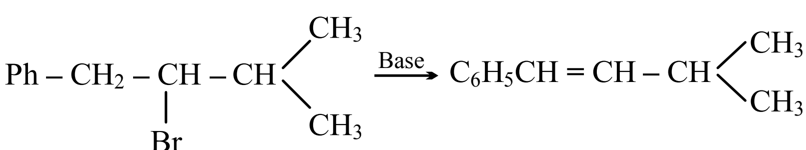
41. (A)



42. (A)

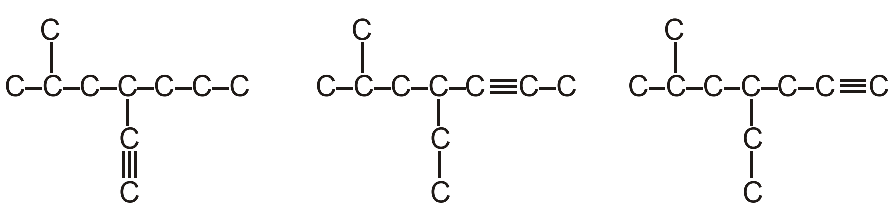
Fluoro alkene gives mainly Hofmann product due to E2 like E1cB mechanism.

43. (A)

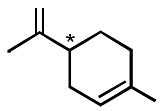


Stable due to conjugation

44. (C)

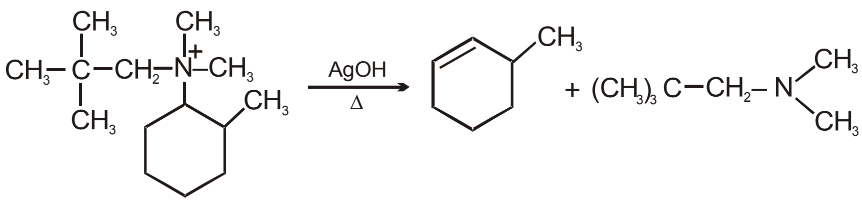


45. (C)



Limonene has one Asymmetric C.

46. (D)

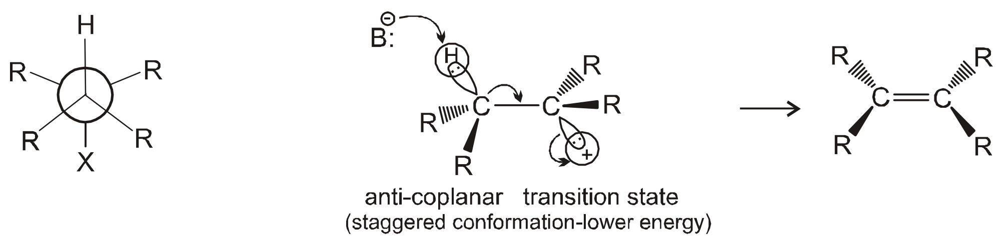


more acidic hydrogen
is removed

47. (C)

Elimination occurs through to anti-periplanar transition state

48. (C)



But Syn-coplanar transition state (eclipsed conformation-higher energy)

49. (D)

Rate of both E1 and E2 follows the order $3^0 > 2^0 > 1^0$.

50. (B)

Reactivity for combination of 1º Radical > 2º Radical > 3º Radical

51. (A)

3 mole Br_3 ionized

So, Molar conductivity is highest.

52. (A)

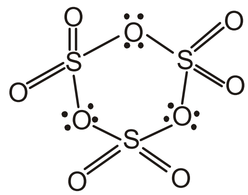
53. (C)

$KMnO_4$ and $K_2Cr_2O_7$ both has Charge transfer ligand to metal.

54. (A)

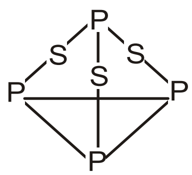
$(SO_3)_3 = S_3O_9$

6S = O –bonds ; 3S–O–S –bonds

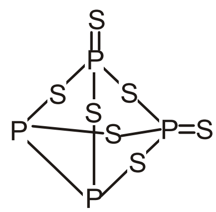


55. (C)

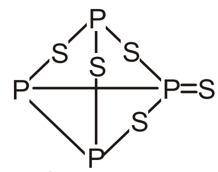
$P_4S_3(x)$ $P_4S_7(y)$ $P_4S_5(z)$



P-S Bond = 6



P-S Bond = 12



P-S Bond = 9

$| x + y - z |$

x = 6, y = 12, and z = 9

56. (A)

Due to ligand to metal charge transfer.

57. (C)

Cyclic Silicate Structure : $[SiO_3]_n^{-2n}$

58. (A)

59. (A)

$$Na_2CO_3 + Fe_2O_3 \xrightarrow{\Delta} NaFeO_2 + CO_2$$

60. (C)

$$\underset{(X)}{2MnO_2} + 2K_2CO_3 + \underset{(air)}{O_2} \xrightarrow{\Delta} \underset{green(Y)}{2K_2MnO_4} + 2CO_2\uparrow$$

$$2K_2MnO_4 + Cl_2 \longrightarrow 2KMnO_4 + 2KCl$$

MATHEMATICS

61. (B)

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[\frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + \ldots + \frac{1}{n+\sqrt{n^2}}\right]$$

$$= \lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{\frac{1}{n}+\frac{1}{\sqrt{n}}} + \frac{1}{\frac{2}{n}+\sqrt{\frac{2}{n}}} + \ldots + \frac{1}{\frac{n}{n}+\sqrt{\frac{n}{n}}}\right]$$

$$\int_0^1 \frac{dx}{\sqrt{x}\left(\sqrt{x}+1\right)}$$

Put $\sqrt{x} = z$ or $\frac{1}{2\sqrt{x}} dx = dz$

$$\therefore \lim_{n\to\infty} S_n = \int_0^1 \frac{2dz}{z+1} = 2|\log(z+1))|_0^1$$

= 2 (log 2 – log 1)

= 2 log 2 = log 4

62. (C)

We have $\int_0^1 e^{x^2}(x-\alpha)dx = 0,$

or $\int_0^1 e^{x^2} x dx = \int_0^1 e^{x^2} \alpha dx$

or $\frac{1}{2}\int_0^1 e^t dt = \alpha\int_0^1 e^{x^2} dx,$ where $t = x^2$

or $\frac{1}{2}(e-1) = \alpha\int_0^1 e^{x^2} dx$

Since, e^{x^2} is an increasing function for $0 \le x \le 1$

$1 \le e^{x^2} \le e$ when $0 \le x \le 1$

or or $1(1-0) \le \int_0^1 e^{x^2} dx \le e(1-0)$

or $1 \le \int_0^1 e^{x^2} dx \le e$

63. (C)

$$I_{11} = \int_0^1 \underbrace{(1-x^5)^{11}}_{I} \cdot \underset{II}{1} dx$$

$$= 0 - 55\int_0^1 (1-x^5)^{10}(1-x^5-1)dx$$

$$= -55\int_0^1 (1-x^5)^{11}dx + 55\,I_{10}$$

or $56I_{11} = 55I_{10}$

or $\frac{I_{10}}{I_{11}} = \frac{56}{55}$

64. (A)

Let $x = n + f,\ n \in I^+,\ 0 < f < 1$

$$\int_0^x [x]dx = \int_0^n [x]dx + \int_n^{n+f} [x]\,dx$$

$$= \frac{n(n-1)}{2} + \int_n^{n+f} n\,dx \ = \frac{n(n-1)}{2} + n(x)_n^{n+f} \ = \frac{n(n-1)}{2} + nf \ = [x]\left(\frac{[x]-1}{2} + x - [x]\right)$$

65. (A)

Substituting $x = p^6, dx = 6p^5dp,$ we have

$$I = \int \frac{6p^5(p^6+p^4+p)}{p^6(1+p^2)}dp = \int \frac{6(p^5+p^3+1)}{(p^2+1)}dp = \int 6p^3dp + \int\left(\frac{6}{p^2+1}\right)dp$$

$$= \frac{6p^4}{4} + 6\tan^{-1}p + c = \frac{3}{2}x^{2/3} + 6\tan^{-1}(x^{1/6}) + c$$

66. (C)

$$\int_0^\infty \left[\frac{6}{x^2+2}\right]dx = \int_0^1 2\,dx + \int_1^2 1dx + \int_2^\infty 0\,dx = 3$$

67. (B)

$f(x) < g(x)\ \forall\, x \in (0,\infty)$ and $d(x\,g(x)) = f(x)dx$

$$\Rightarrow \quad x\,g(x) = \int_0^x f(x)\,dx\ \ \forall\, x \in (0,\infty) \ \Rightarrow\ x\,f(x) < \int_0^x f(x)\,dx\ \forall\, x \in (0,\infty)$$

68. (B)

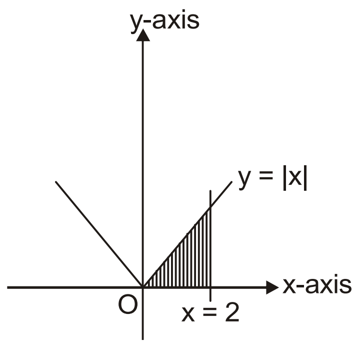
$$\int_{-3}^{3} f(|x|)\,dx \ + \ \int_3^5 f(x)\,dx \ = \ 2\int_0^3 f(x)\,dx \ + \ \int_3^5 f(x)\,dx$$

$$= 2\left[\int_0^1 f(x)dx + \int_1^2 f(x)dx + \int_2^3 f(x)dx\right] + \int_3^4 f(x)dx + \int_4^5 f(x)dx$$

$$= 2\left[0 + \frac{1}{2} + \frac{4}{2}\right] + \frac{9}{2} + \frac{16}{2} = 5 + \frac{25}{2} = \frac{35}{2}$$

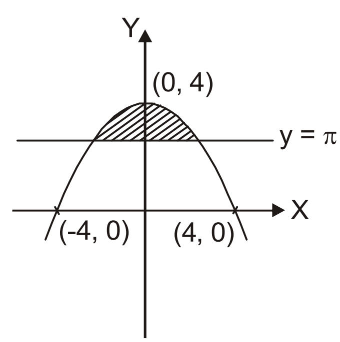
69. (C)

Area = 2 sq. units



70. (B)

$$x\,dx + 2dy = 0 \Rightarrow \frac{x^2}{2} = -2y + C \Rightarrow x^2 = -4(y-4)$$



$[-\sin^2 x] = 0$ or -1

but $\sec^{-1}(0)$ is not defined.

$\therefore\quad y = \sec^{-1}(-1) = \pi$

$$\therefore\quad \text{Area} = 2\int_\pi^4 x\,dy = 2\int_\pi^4 \sqrt{16-4y}\,dy = 4\int_\pi^4 \sqrt{4-y}\,dy$$

$$= -4\left[\frac{(4-y)^{3/2}}{3/2}\right]_\pi^4 = \frac{8}{3}(4-\pi)^{3/2}$$

71. (D)

$$\int_0^x f(t)dt = \frac{1}{2}\cdot\left(\frac{1}{2}x\cdot f(x)\right)$$

$$\therefore\quad f(x) = \frac{1}{4}(x f'(x) + f(x)) \Rightarrow f(x) = cx^3$$

$$f(2)=2 \Rightarrow c=\frac{1}{4} \quad \therefore \; f(x)=\frac{x^3}{4}.$$

72. (C)

$xy-4x+6y=0$

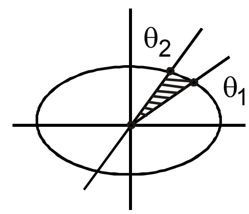
$\Rightarrow \; (x+6)(y-4)=-24$ is a rectangular hyperbola and x = – 6 and y = 4 are it's asymptotes.

Area of the region enclosed by a tangent to the rectangular hyperbola xy = c^2 and it's asymptotes is = $2c^2$

Here, c^2 = 24

Hence, Area = 2 × 24 = 48 sq units.

73. (B)



Area = $\frac{ab}{2}(\theta_2-\theta_1)$ ...(i)

$$\frac{b\sin\theta}{a\cos\theta}=m \Rightarrow \tan\theta=\frac{am}{b} \Rightarrow \theta=\tan^{-1}\left(\frac{am}{b}\right)$$

$\Rightarrow \; \theta=\tan^{-1}(2m)$

$\Rightarrow \; \theta_1=\tan^{-1}2, \; \theta_2=\tan^{-1}4$

Putting the values of $\theta_1, \theta_2,$ a and b in equation (i), we will get

Area $=\frac{2\times4}{2}(\tan^{-1}4-\tan^{-1}2)=4(\tan^{-1}4-\tan^{-1}2)$

74. (A)

$f(x+4)=f(x+2)-f(x)$

$\Rightarrow f(x+6)=f(x+4)-f(x+2)$

$=f(x+2)-f(x)-f(x+2)$

$f(x+6)=-f(x)$

hence T = 12

$\int_{\lambda}^{\lambda+12} f(x)=\int_0^{12} f(x)\,dx$

75. (B)

$$S_n = \int_1^2 \{x\}\,dx + \int_2^3 \{x\}^2 dx + \int_3^4 \{x\}^3 dx + ... + \int_{n-1}^{n} \{x\}^{n-1} dx$$

$$\Rightarrow S_n = \int_0^1 x\,dx + \int_0^1 x^2 dx + \int_0^1 x^3 dx + ... + \int_0^1 x^{n-1} dx \Rightarrow S_n = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + .. + \frac{1}{n}$$

$$\Rightarrow S_{16} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{16} \Rightarrow S_{16} > \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right) + \left(\frac{1}{16} + \frac{1}{16} + ...8\text{times}\right)$$

$$\Rightarrow S_{16} > 2$$

Similarly,

$$S_{32} > \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right) + \left(\frac{1}{16} + \frac{1}{16} + ...8\text{times}\right) + \left(\frac{1}{32} + \frac{1}{32} + ...16\text{times}\right) \Rightarrow S_{32} > \frac{5}{2}$$

76. (D)

$$\because \quad f(x) = \max\left\{\sin x, \cos x, \frac{1}{2}\right\}$$

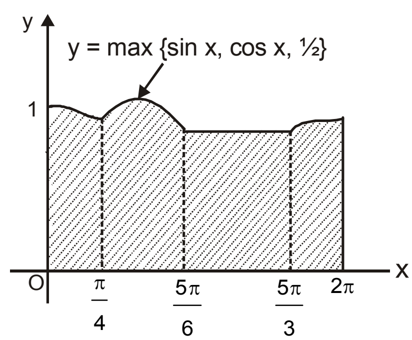
Interval value of f(x)

For $0 \le x < \frac{\pi}{4}, \cos x$

For $\frac{\pi}{4} \le x < \frac{5\pi}{6}, \sin x$

For $\frac{5\pi}{6} \le x < \frac{5\pi}{3}, \frac{1}{2}$

For $\frac{5\pi}{3} \le x < 2\pi, \cos x$



Hence, required area

$$= \int_0^{\pi/4} \cos x dx + \int_{\pi/4}^{5\pi/6} \sin x\,dx + \int_{5\pi/6}^{5\pi/3} (1/2)\,dx + \int_{5\pi/3}^{2\pi} \cos x\,dx$$

$$= [\sin x]_0^{\pi/4} - [\cos x]_{\pi/4}^{5\pi/6} + \frac{1}{2}[x]_{5\pi/6}^{5\pi/3} + [\sin x]_{5\pi/3}^{2\pi}$$

$$= \left(\frac{1}{\sqrt{2}} - 0\right) - \left(-\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right) + \frac{1}{2}\left(\frac{5\pi}{3} - \frac{5\pi}{6}\right) + \left(0 + \frac{\sqrt{3}}{2}\right)$$

$=\left(\frac{5\pi}{12}+\sqrt{2}+\sqrt{3}\right)$ sq unit.

77. (C)

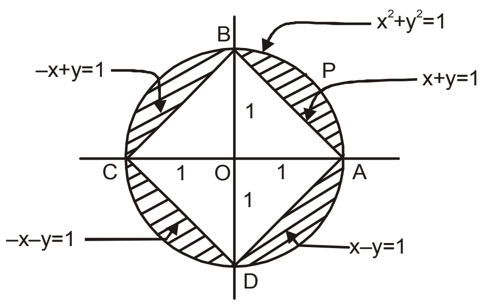
$f'(x)=e^{x^2}+4\ge 5 \qquad \forall x\in R$

Hence, $\frac{f(x+1)-f(x-2)}{3}=f'(c_1)\ge 5$ and

$\frac{f(x+2)-f(x-2)}{4}=f'(c_2)\ge 5$

78. (C)

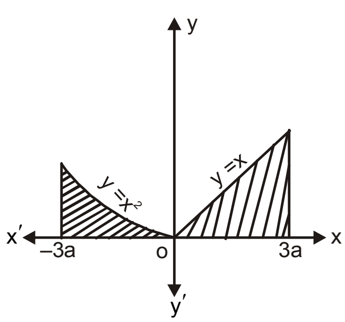
$\therefore$ Required area



= area of circle – area of square

79. (D)

If a > 0, then



$\therefore$ Required area $=\int_{-3a}^{0}x^2dx+\int_0^{3a}x\,dx=\int_0^{3a}x^2\,dx+\int_0^{3a}x\,dx=\int_0^{3a}(x^2+x)dx$

$$=\left.\frac{x^3}{3}+\frac{x^2}{2}\right|_0^{3a}=\frac{27a^3}{3}+\frac{9a^2}{2}=\frac{9a}{2}$$

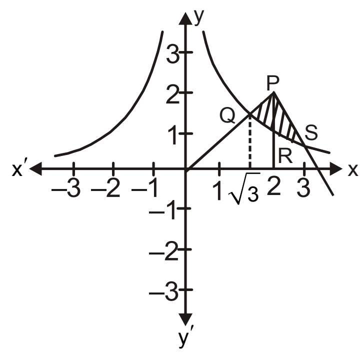
$\Rightarrow\ a^2+\frac{a}{2}=\frac{1}{2}\ \Rightarrow\ 2a^2+a-1=0$

We get, $a=\frac{1}{2}, a=-1$

$\therefore\ a=\frac{1}{2}, a\neq -1$

80. (C)

$y=2-|2-x|, y=\frac{3}{|x|}$



$y=\begin{cases} x, & x\le 2 \\ 4-x, & x\ge 2\end{cases}; y=\begin{cases}\frac{3}{x}, & x>0 \\ -\frac{3}{x} & x<0\end{cases}$

Hence, required area

PQRSP = area PQRP + area PRSP

$=\left|\int_{\sqrt{3}}^{2}\left(x-\frac{3}{x}\right)dx\right|+\left|\int_{2}^{3}(4-x)-\frac{3}{x}dx\right|$

$=\frac{4-3\ln 3}{2}$ sq unit.

81. (A)

$I=\int(\sin(100x+x)\cdot(\sin x)^{99}dx = \int\left((\sin(100x)\cos x+\cos 100x\cdot\sin x)(\sin x)^{99}\right)dx$

$=\int \underbrace{\sin(100x)}_{I}\underbrace{\cos x\cdot(\sin x)^{99}}_{II}dx + \int\cos(100x)\cdot(\sin x)^{100}dx$

$=\frac{\sin(100x)(\sin x)^{100}}{100}-\frac{100}{100}\int\cos(100x)(\sin x)^{100}dx + \int\cos(100x)(\sin x)^{100}dx$

$=\frac{\sin(100x)(\sin x)^{100}}{100}+C$

82. (C)

we know $\sec^{-1}\sqrt{1+x^2} = \tan^{-1}x$; $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$ for x > 0

$$I = \int \frac{e^{\tan^{-1}x}}{1+x^2}\left((\tan^{-1}x)^2 + 2\tan^{-1}x\right) dx \quad \text{put } \tan^{-1}x = t$$

$$= \int e^t(t^2+2t)dt = e^t \cdot t^2 = e^{\tan^{-1}x}\left(\tan^{-1}x\right)^2 + C$$

83. (B)

$$nx = t \Rightarrow I = \frac{1}{n}\int_0^{\pi/2} \frac{dt}{1+(\tan t)^n} = \frac{1}{n}\int_0^{\pi/2} \frac{(\cos t)^n}{(\sin t)^n + (\cos t)^n} dt = \frac{\pi}{4n}$$

84. (B)

For $f\left(x+\frac{1}{2}\right) + f\left(x-\frac{1}{2}\right) = f(x)$

$\Rightarrow$ Period of f(x) is 3

Since $\int_0^6 f(x)dx = 4 \Rightarrow \int_0^3 f(x)dx = 2$

$$\Rightarrow \int_{-3}^{30} f(x)dx = 22$$

85. (C)

0 < sinx < 1

$-\alpha < -\sin x.\alpha < 0$

$\sin(-\alpha) < \sin(-\alpha\sin x) < 0$

$1-\sin\alpha < 1+\sin(-\alpha\sin x) < 1$

$$\Rightarrow \frac{\pi}{2}(1-\sin\alpha) < \int_0^{\pi/2}\left(1+\sin(-\alpha\sin x)\right)dx < \frac{\pi}{2}$$

Hence $I > (1-\cos\alpha).\frac{\pi}{2}$

since $\cos\alpha > \sin\alpha$

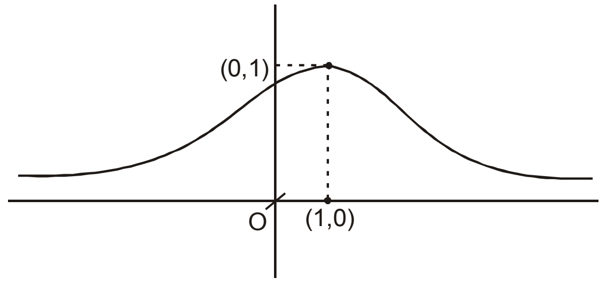
for $\alpha \in (0, \pi/4)$

86. (B)

$$y = \frac{1}{(x-1)^2 + 1}$$

$y_{max} = 1$ at $x = 1$

The graph is symmetrical about x = 1



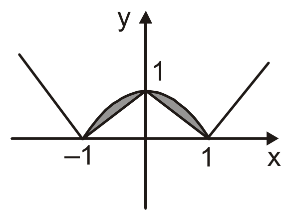
$$A = 2\int_1^{\infty} \frac{1}{(x-1)^2 + 1} dx = \left[2\tan^{-1}(x-1)\right]_1^{\infty} = \pi.$$

87. (D)

Required area

$$= 2\left\{\frac{1}{4} \times \pi \times 1^2 - \frac{1}{2} \times 1 \times 1\right\}$$

$$= \frac{\pi}{2} - 1$$

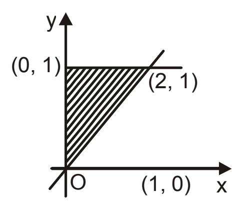


88. (A)

y= sgn (–3x + 10)

$= 1, x \in \left(-\infty, \frac{10}{3}\right)$

$\therefore$ Required area = $\frac{1}{2}.1.2 = 1$ sq. unit



89. (D)

$$1 \le x \le \frac{4}{3},\ y = \sqrt{\sin[x] + [\sin x]} = \sqrt{\sin 1}$$

$$\therefore \text{ Required area} = \int_1^{4/3} \sqrt{\sin 1}\, dx = \frac{1}{3}\sqrt{\sin 1} \text{ sq. unit}$$

90. (D)

$-4 < x < 4 \Rightarrow y = 2$

$\therefore$ Required area $= \frac{1}{2}(1 + 3) \times 2$

= 4 sq. units.