SOLUTIONS

WEEKLY TEST-6

GZPA-1901 & 1902

(JEE MAIN PATTERN)

Test Date: 09-09-2017

Mentors® Eduserv™

Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd., Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

PHYSICS

1. (C)

Let the vector is

$\vec{a} = 2x\hat{i} + x\hat{j} + z\hat{k}$

$|\vec{a}| = \sqrt{5x^2 + z^2} = 5\sqrt{2}$

Also, $\cos 135° = \frac{Z}{\sqrt{5x^2+z^2}} = \frac{Z}{5\sqrt{2}} = -\frac{1}{\sqrt{2}}$

$\Rightarrow$ z = – 5 then x = $\sqrt{5}$,

The required vector $\vec{a} = 2\sqrt{5}\hat{i} + \sqrt{5}\hat{j} - 5\hat{k}$

2. (C)

Given $(\vec{a} + 3\vec{b}).(7\vec{a} - 5\vec{b}) = 0 \Rightarrow 7a^2 - 15b^2 + 16\,\vec{a}.\vec{b} = 0$ ...(1)

Also, $(\vec{a} - 4\vec{b}).(7\vec{a} - 2\vec{b}) = 0 \Rightarrow 7a^2 + 8b^2 - 30\,\vec{a}.\vec{b} = 0$ ...(2)

Subtracting, $-23b^2 + 46\,\vec{a}.\vec{b} = 0 \Rightarrow \vec{a}.\vec{b} = \frac{b^2}{2}$

Putting this in (1),

$7a^2 - 7b^2 = 0 \Rightarrow |\vec{a}| = |\vec{b}|$. Thus $\vec{a}.\vec{b} = ab\cos\theta$

$\Rightarrow \frac{b^2}{2} = b^2\cos\theta \Rightarrow \cos\theta = \frac{1}{2}$ or $\theta = 60°$.

3. (D)

As we have read,

$\vec{C} = \vec{A} \times \vec{B}$ is a vector perpendicular to both $\vec{A}$ and $\vec{B}$. Hence, a unit vector $\hat{n}$ perpendicular to $\vec{A}$ and $\vec{B}$

can be written as

$$\hat{n} = \frac{\vec{C}}{C} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

Here, $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i}(3+1) + \hat{j}(1-2) + \hat{k}(-2-3) = 4\hat{i} - \hat{j} - 5\hat{k}$

Further, $\left|\vec{A}\times\vec{B}\right| = \sqrt{(4)^2+(-1)^2+(-5)^2} = \sqrt{42}$

$\therefore$ The desired unit vector is $\hat{n} = \dfrac{\vec{A}\times\vec{A}}{|\vec{A}\times\vec{A}|}$ *or*, $\hat{n} = \dfrac{1}{\sqrt{42}}(4\hat{i}-\hat{j}-5\hat{k})$

4. (A)

$\vec{a}\times\vec{b}-\vec{a}\times\vec{c}=0$

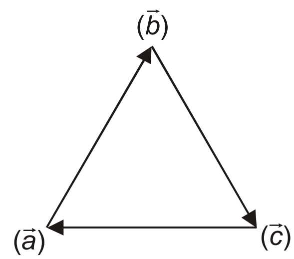
$\vec{a}\times(\vec{b}-\vec{c})=0$

it means $\vec{a}$ is parallel or antiparallel to $\vec{b}-\vec{c}$, so $\vec{b}-\vec{c}=\lambda\vec{a}$ where λ is any constant

$\Rightarrow \vec{b}=\vec{c}+\lambda\vec{a}.$

5. (D)

A $= \dfrac{1}{2}|(\vec{b}-\vec{a})\times(\vec{c}-\vec{b})|$



$= \dfrac{1}{2}|\vec{b}\times\vec{c}-\vec{b}\times\vec{b}-\vec{a}\times\vec{c}+\vec{a}\times\vec{b}|$

$= \dfrac{1}{2}|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}|$

$\therefore \boxed{A=\dfrac{1}{2}|(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})|}$

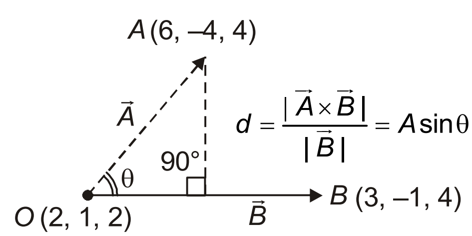
6. (C)

$\vec{A}=4\hat{i}-5\hat{j}+2\hat{k}$

$\vec{B}=\hat{i}-2\hat{j}+2\hat{k}; |\vec{B}|=3$

$|\vec{A}\times\vec{B}| = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -5 & 2\\ 1 & -2 & 2\end{vmatrix} = |6\hat{i}-6\hat{j}-3\hat{k}| = 9$

$d=\dfrac{|\vec{A}\times\vec{B}|}{|\vec{B}|}=\dfrac{9}{3}=3$



7. (D)

Let $\vec{a}$ makes angles α, β & γ with *x*, *y* & *z*-axis.

$|\hat{i}\times\vec{a}| = |\hat{i}||\vec{a}|\sin\alpha = (1)(a)\sin\alpha$

$\Rightarrow \boxed{|\hat{i}\times\vec{a}|^2 = a^2\sin^2\alpha}$

Similarly

$|\hat{j}\times\vec{a}|^2 = a^2\sin^2\beta$

$|\hat{k}\times\vec{a}|^2 = a^2 \sin^2\gamma$

$\Rightarrow |\hat{i}\times\vec{a}|^2 + |\hat{j}\times\vec{a}|^2 + |\hat{k}\times\vec{a}|^2 = a^2 (\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$

$= a^2 [3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)] = 2a^2$

8. (B)

$\frac{F}{-4} + \frac{t}{5} = 1$

9. (C)

$f(5) = 28$

$f(f(5)) = 28^2 + 3 = 787$

10. (B)

$= e^{\sin\sqrt{x}} \cdot \frac{d}{dx}\sin\sqrt{x}$

$= e^{\sin\sqrt{x}} \cos\sqrt{x} \cdot \frac{d}{dx}\sqrt{x}$

$= e^{\sin\sqrt{x}} \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}} e^{\sin\sqrt{x}} \cdot \cos\sqrt{x}$

11. (D)

The velocity function is the derivative of the position function :

$s = f(t) = t^3 - 6t^2 + 9t$

$\Rightarrow v(t) = \frac{ds}{dt} = 3t^2 - 12t + 9$

The acceleration is the derivative of the velocity function :

$a(t) = \frac{d^2s}{dt^2} = \frac{dv}{dt} = 6t - 12$

$\Rightarrow a(4) = 6(4) - 12 = 12 \text{ m/s}^2$

12. (A)

In mathematical terms, we want to solve the initial value problem that consists of

The differential condition : $\frac{d\nu}{dt} = 9.8$

The initial condition: $\upsilon = 0$ when $t = 0$ (abbreviated as $v(0) = 0$)

We first solve the differential equation by integrating both sides with respect to t:

$\frac{d\nu}{dt} = 9.8$

The differential equation $\int \frac{d\nu}{dt} dt = \int 9.8 dt$ Integrate with respect to t.

$\upsilon + C_1 = 9.8t + C_2$ Integrals evaluated

$\upsilon = 9.8t + C$. Constants combined as one

This last equation tells us that the body's velocity t seconds into the fall is 9.8t + C m/sec.

For value of C : What value? We find out from the initial condition :

$\upsilon = 9.8t + C$

$0 = 9.8(0) + C$ $\upsilon(0) = 0$

$C = 0$.

Conclusion : The body's velocity t seconds into the fall is

$\upsilon = 9.8t + 0 = 9.8t$ m/sec.

13. (B)

Let $u = 7\theta + 5$, $du = 7d\theta$, $(1/7)\,du = d\theta$.

$$= \int \cos u \cdot \frac{1}{7} du$$

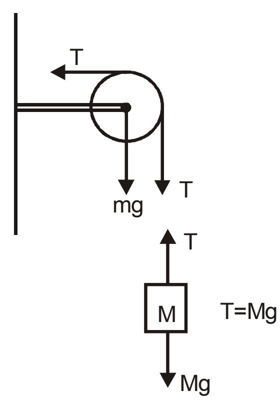
$$= \frac{1}{7}\int \cos u\, du$$ With (1/7) out front, the integral is now in standard form.

$$= \frac{1}{7}\sin u + C$$ Integrate with respect to u.

$$= \frac{1}{7}\sin(7\theta + 5) + C$$ Replace u by $7\theta + 5$.

14. (D)

Force on the pulley by the clamp,

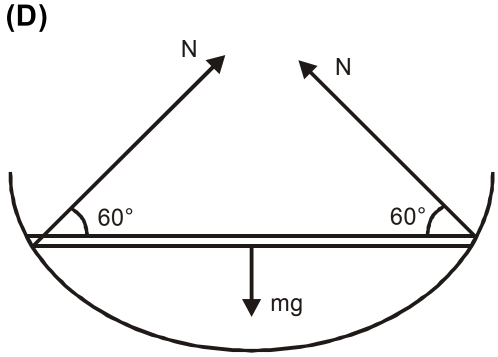


$$F = \sqrt{T^2 + (T + mg)^2}$$

$$= \sqrt{M^2g + (Mg + mg)^2}$$

$$= \left(\sqrt{M^2 + (M + m)^2}\right)g$$

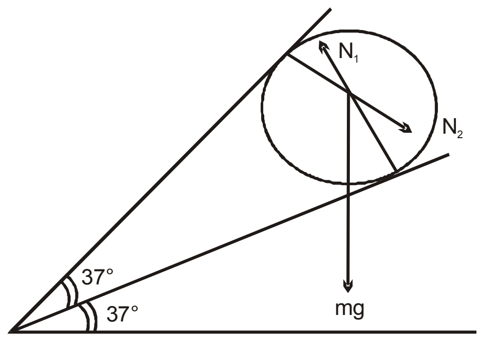
15. (D)

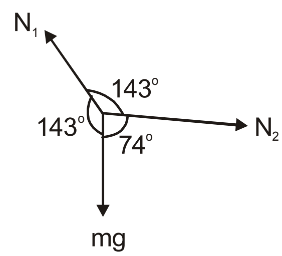


$$2N\sin 60^\circ = mg$$

$$N = \frac{mg}{\sqrt{3}}$$

16. (A)



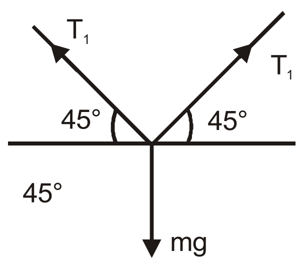


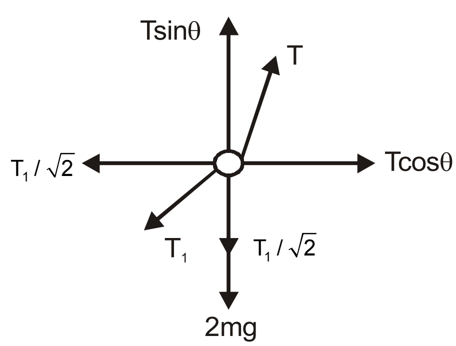
For equilibrium of ball

$$\frac{mg}{\sin 143^\circ} = \frac{N_1}{\sin 74^\circ} = \frac{N_2}{\sin 143^\circ}$$

$\therefore$ $N_2 = mg$

17. (C)





$$\sqrt{2}\,T_1 = mg$$

$$T_1 = \frac{mg}{\sqrt{2}}$$

$$T\sin\theta = \frac{5}{2}mg$$

$$T\cos\theta = \frac{1}{2}mg$$

$$\tan\theta = 5$$

$$\theta = \tan^{-1}(5)$$

18. (A)

$$y = x + \frac{1}{x} + 2$$

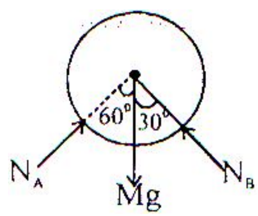
on diff. w.r.t x

$$\frac{dy}{dx} = 1 - \frac{1}{x^2}$$

19. (A)

The FBD of the cylinder is as shown in figure,

As the cylinder is in equilibrium then net force acting on the block is zero.



For Horizontal equilibrium.

$N_A \sin 60 = N_B \sin 30°$ ....(i)

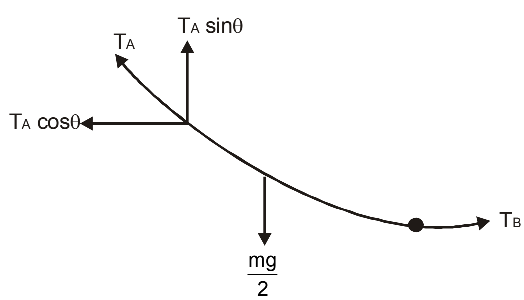
For vertical equilibrium, $N_A \cos 60 + N_B \cos 30° = Mg$ ....(ii)

solving abouve equation we get, $N_B = \sqrt{3}N_A$ & $N_A = \frac{Mg}{2}$

$$N_B = \frac{\sqrt{3}\,Mg}{2}$$

20. (B)

Cut the chain from its lowest point and draw the FBD of one half as shown. Let T' is the tension at end & T is tension of bottom–most point.



For vertical equilibrium

$$T_A \sin\theta = \frac{mg}{2}$$

For Horizontal equilibrium, $T_A \cos\theta = T_B$

$\Rightarrow\ T_B = \dfrac{mg\cot\theta}{2}$

21. (B)

$\because\ \sin x + \cos x = \sqrt{2}$

on squaring $\Rightarrow\ \sin^2 x + \cos^2 x + 2\sin x\cos x = 2$

$\sin 2x = 1$ ....(1)

and $\dfrac{1}{\sin^6 x + \cos^6 x} = \dfrac{1}{1 - 3\sin^2 x\cos^2 x}$

$= \dfrac{1}{1 - \frac{3}{4}\sin^2 2x} = \dfrac{1}{1 - \frac{3}{4}(1)^2}$ using eq^n .... (1)

$= \dfrac{4}{4-3} = 4$

22. (A)

$y = \dfrac{x^3}{3} - \dfrac{5}{2}x^2 + 6x + 4$

$\dfrac{dy}{dx} = x^2 - 5x + 6$ ....(1)

if $x^2 - 5x + 6 = 0$

x = 2, 3

Again diff. (1)

$\dfrac{d^2y}{dx^2} = 2x - 5$

at x= 2 $\dfrac{d^2y}{dx^2} = -1 < 0$

so maximum at x = 2.

23. (D)

$T_1 - 50 - 40 = 0$

$T_1 = 90\text{N}$

24. (C)

25. (D)

$-\left[\dfrac{\cos 2x}{2}\right]_0^{\frac{\pi}{6}} = \dfrac{1}{2}\left(\dfrac{1}{2} - 1\right) = \dfrac{1}{4}$

26. (B)

put $t = 1 + x^2 \Rightarrow dt = 2xdx$

$$\frac{1}{2}\int\frac{dt}{t} = \frac{1}{2}\ell nt + C = \ell n\sqrt{1+x^2} + C$$

27. (C)

Remember $\vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}(\vec{a}\,\vec{c}) - \vec{c}(\vec{a}\,\vec{b})$

Thisis called the vectro triple product.

Using above,

$$\hat{i}\times(\hat{i}\times\vec{a}) = \hat{i}(\hat{i}.\vec{a}) - \vec{a}(\hat{i}.\hat{i}) = \hat{i}(\hat{i}.\vec{a}) - \vec{a}$$

$$\hat{j}\times(\hat{j}\times\vec{a}) = \hat{j}(\hat{j}.\vec{a}) - \vec{a}(\hat{j}.\hat{j}) = \hat{j}(\hat{j}.\vec{a}) - \vec{a}$$

$$\hat{k}\times(\hat{k}\times\vec{a}) = \hat{k}(\hat{k}.\vec{a}) - \vec{a}(\hat{k}.\hat{k}) = \hat{k}(\hat{k}.\vec{a}) - \vec{a}$$

$$\therefore\quad \hat{i}\times(\hat{i}\times\vec{a}) + \hat{j}(\hat{j}\times\vec{a}) + \hat{k}\times(\hat{k}\times\vec{a})$$

$$= \hat{i}.(\hat{i}.\vec{a}) + \hat{j}(\hat{j}.\vec{a}) + \hat{k}(\hat{k}.\vec{a}) - 3\vec{a} \qquad \text{...(i)}$$

Since, $\vec{a} = \hat{i}\,a_x + \hat{j}\,a_y + \hat{k}\,a_z$

and $\quad a_x = \hat{i}.\vec{a}; a_y = \hat{j}.\vec{a}; a_z = \hat{k}.\vec{a}$

$$\therefore\quad \hat{i}(\hat{i}\times\vec{a}) + \hat{j}(\hat{j}.\vec{a}) + \hat{k}(\hat{k}.\vec{a}) = \hat{i}a_x + \hat{j}a_y + \hat{k}a_z = \vec{a}$$

on putting in eq. (i) we get

$$\hat{i}\times(\hat{i}\times\vec{a}) + \hat{j}\times(\hat{j}\times\vec{a}) + \hat{k}\times(\hat{k}\times\vec{a}) = \vec{a} - 3\vec{a} = -2\vec{a}$$

28. (C)

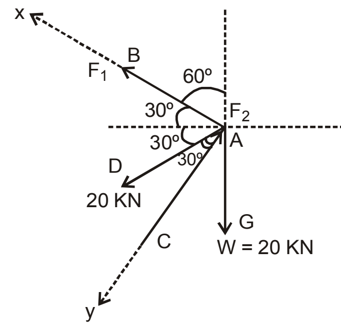
$mg - B = mf$

$B - (m - m')g = (m - m')f$

$\Rightarrow\quad m'g = (2m - m')f \quad \Rightarrow m' = \dfrac{2mf}{g+f} \qquad \Rightarrow \quad w' = \dfrac{2wf}{g+f}$

29. (D)

Since pulley is frictionless, same force exists throughout in the flexible cable. Hence force in AD is also 20 KN as shown in figure. Also we have $AC \perp AB$ (see fig.). Selecting AB and AC as cartesian X- and Y-axis.



$\Sigma F_x = 0 \quad \Rightarrow F_1 + 20\sin 30^\circ - 20\sin 30^\circ = 0$

$\therefore \quad F_1 = 0$

$\Sigma F_y = 0 \quad \Rightarrow -F_2 + 20\cos 30^\circ + 20\cos 30^\circ = 0$

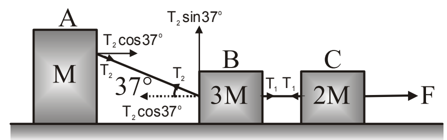
$\Rightarrow F_2 = 40\cos 30^\circ \approx 20\sqrt{3}$ kN.

$\therefore \quad F_2 = (20\sqrt{3})\text{kN} \approx (34.6)\text{kN}$

30. (C)

For 3 M :- for loosing contact N = 0

$T_2 \sin 37^\circ = 3Mg$



$T_2 = 5Mg$ ....(i)

$F - T_1 = 2Ma$ ....(ii)

$T_1 - T_2\cos 37^\circ = 3Ma$ ....(iii)

$T_2\cos 37^\circ = Ma$ ....(iv)

From (i) & (iv)

$a = 4g$

From (ii), (iii) & (iv)

$F = 24Mg$

CHEMISTRY

31. (A)

$$\frac{(e/m)_p}{(e/m)_a} = \frac{e/m_p}{2e/4m_p} = \frac{2}{1}$$

32. (D)

Charge/mass for n = 0, for $\alpha = \frac{2}{4}$,

for $p = \frac{1}{1}$, for $e^- = \frac{1}{1/1837}$

33. (C)

$$\frac{m}{e} = 1.5 \times 10^{-8} \times 10^{+3} = \frac{m}{1.6 \times 10^{-19}}$$

m = 2.4×10^{-24} g

34. (C)

35. (B)

$r_{atom} \approx 10^{-10}\,m$

$r_{nucleus} \approx 10^{-15}\,m$

$$\frac{V_{Atom}}{V_{Nucleus}} = \frac{4/3\pi\left(10^{-10}\right)^3}{4/3\pi\left(10^{-15}\right)^3} = 10^{15}$$

36. (C)

$h\nu = h\nu_0 + KE$

37. (C)

Nature of canal rays does not depend on electrode material.

38. (C)

39. (C)

As per Einstein's equation of photoelectric effect, $h\nu = h\nu_0 + K.E.$

$$\therefore \quad \frac{1}{2}mv^2 = h\nu - h\nu_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

$$v^2 = \frac{2hc}{m}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right);$$

$$\Rightarrow \quad v = \left[\frac{2hc}{m}\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)\right]^{1/2}$$

40. (C)

$$\frac{hc}{\lambda} = 1 + \phi$$

$$3 \times \frac{hc}{\lambda} = 4 + \phi$$

from eq. (1) and (2) $\phi = 0.5\,eV$

41. (B)

$$E = E_1 + E_2; \frac{hc}{\lambda} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\Rightarrow \lambda = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$$

42. (A)

$$4Ca_5\underset{0.1}{(PO_4)_3F} + 18\underset{0.36}{\overset{L.R.}{SiO_2}} + 30\underset{0.9}{C} \longrightarrow 3P_4 + 2CaF_2 + 18CaSiO_3 + 30CO$$

18 mole SiO_2 gives 3 mole P_4

0.36 mole SiO_2 will give $= \frac{3}{18} \times 0.36 = 3.33g$

43. (D)

Absorbed Energy = Emitted Energy

$\frac{hc}{\lambda} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$ or, $\frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

$\frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda_2}$ $\lambda_2 = 743$ nm

44. (A)

Apply POAC for Zn

1×moles of ZnS = 1×mol Zn

Moles of ZnS $= \frac{2}{0.8 \times 0.8 \times 1} = 3.125$ mole

45. (B)

$$CO_2 + 2NaOH \longrightarrow Na_2CO_3 + H_2O$$

$n_{NaOH} = 1$;

$\therefore$ **CO_2** present in mixture = 0.5 and CO present = 0.3 mole

Moles of CO_2 obtained from CO = 0.3, extra moles of NaOH required = 0.3×2 = 0.6 moles

46. (D)

Mass of solution = 100 g

Mass of solute = 56 g

$\Rightarrow$ moles of solute $\frac{56}{60} = 0.93$

Mass of solvent = 44 g

$\Rightarrow$ moles of solvent $= \frac{44}{18} = 2.44$

$$X_{solute} = \frac{0.93}{0.93 + 2.44} = \frac{0.93}{3.37} = 0.276$$

47. (A)

Millimoles of Pb $(NO_3)_2$ = 25×0.15 =3.75 m. moles

Millimoles of $Al_2(SO_4)_3 = \frac{1}{3} \times 3.75 = M \times 20$

M = 0.0625 = 6.25×10^{-2} M

48. (B)

m-moles of HNO_3 required = 250×1.2= 300

100 g solution contains 63g HNO_3 or

$\frac{100}{1.4}$ mL solution contain 1 mole HNO_3

$\therefore$ molarity of HNO_3 solution

$$= \frac{1000}{100} \times 1.40 = 14\,M$$

$\therefore$ 14×V=300 or V=21.42 mL

49. (B)

50 mL $BaCl_2$ (aq) solution contain 10.4 g $BaCl_2$

$$\therefore\ n_{BaCl_2} = \frac{10.4}{137+71} = 0.05$$

moles of H_2SO_4 in 100 mL H_2SO_4 (aq)

$$\text{solution } = \frac{9.8}{98} = 0.1$$

$$BaCl_2 + H_2SO_4 \longrightarrow BaSO_4 + 2HCl;$$

moles of HCl formed = 0.1

$$\left[Cl^-\right] = \frac{0.1 \times 1000}{50+100} = 0.666\ M$$

50. (C)

$$H_2O + SO_3 \rightarrow H_2SO_4$$

100 g

H_2O required is $\frac{100}{80} \times 18 = 22.5$

Maximum limiting labelling is 122.5 %

51. (B)

$$E_n = \frac{-1E}{n^2}$$

$$\Delta E = E_2 - E_1 = \frac{-13.6}{2^2} - \left[\frac{-13.6}{12}\right]$$

$$= 10.2\,eV$$

52. (D)

$$M = \frac{55 \times 0.07 + 20 \times 0.12 + 25 \times 0.15}{55+20+25} = 0.1$$

53. (C)

$$\text{Molarity} = \frac{28}{11.2} = 2.5\text{M}$$

1000 mL of solution contains 2.5 moles H_2O_2

$$\text{molality, } m = \frac{2.5}{1250-85} \times 1000 = 2.15\,m$$

54. (B)

(i) Mass of NaOH $= 50 \times \frac{40}{100} = 20\text{g}$

(ii) $\frac{w}{v}\text{NaOH} \rightarrow 50\text{g NaOH in } 100\,\text{mL}$

So, in 50 mL, mass of NaOH will be = 25 g

(iii) 50g of 15 M NaOH (d = 1g/mL)

15 M NaOH $\rightarrow$ 15 moles NaOH in 1000 mL solution

Mass of solution = 1000×1=1000 g

Mass of NaOH =15×40 = 600 g

in 1000 g solution mass of NaOH = 600 g

So, in 50g solution, mass of NaOH $= \frac{600}{1000} \times 50 = 30\text{g}$

Order (iii) > (ii) > (i)

55. (B)

$$H_2PtCl_6 \longrightarrow Pt; \frac{12}{M_B + 410} = \frac{5}{195}$$

56. (D)

Velocity of (+)vely charge particle is set according to requirement of experiment it is not a part of conclusion of Rutherford.

57. (A)

Let the number of moles of BaO be *x*.

$\therefore$ Moles of CaO = (0.125 – *x*) mol

Mass of BaO = *x* × 153 g

Mass of CaO = (0.125 – *x*)56 g

153*x* + (0.125 – *x*)56 = 10

$x = \frac{3}{97} = 0.0309.$

$\therefore$ mass of BaO = 0.0309 × 153 = 4.73 g

% BaO = **47.3%**.

58. (B)

59. (A)

$$CaCO_3 \xrightarrow{\Delta} CaO + CO_2(X)$$

So, X is CO_2

$Na_2CO_3 + H_2O + CO_2 \rightarrow 2NaHCO_3$

moles of $CaCO_3 = \frac{10}{100} =$ moles of CO_2

mole of Na_2CO_3 = 1 mole

So, moles of $NaHCO_3$ formed according to the moles of CO_2, which is limiting reagent so moles of $NaHCO_3$ formed will be (0.2) mole.

60. (B)

$N_2O_5 \quad + \quad H_2O \rightarrow 2HNO_3$

27 gm

0.25 mole 0.25 mole

4.5 gm

27 + 23 = 50 g mixture gives 54.5 g HNO_3

100 g mixture gives 109 g HNO_3

MATHEMATICS

61. (B)

$|x^2 - 2x| + |x - 2| = |x^2 - x - 2|$

$(x^2 - 2x)(x - 2) \geq 0$

$x(x - 2)^2 \geq 0$

$x \in [0, \infty)$

62. (A)

$3 < |x - 1| < 5 \quad \Rightarrow \quad x \in (-4, -2) \cup (4, 6)$

63. (A)

It is obvious

64. (A)

We have $(\log_9 x)^2 - \frac{9}{2} \log_9 x + 5 = \frac{3}{2}$

Put $\log_9 x = y$

$\Rightarrow y^2 - \frac{9}{2} y + 5 = \frac{3}{2}$

$\therefore 2y^2 - 9y + 10 = 3$

$\Rightarrow 2y^2 - 9y + 7 = 0$

$\therefore (2y - 7)(y - 1) = 0$

$\Rightarrow y = \frac{7}{2}, 1$

$\therefore$ Either $\log_9 x = 1$ or $\log_9 x = \frac{7}{2}$

$\Rightarrow$ Either x = 9 or $x = 9^{7/2} = 3^7$

$\therefore$ Sum of all solutions = 9 + 2187 = 2196 **Ans.**]

65. (D)

$b = a^{3/2}$ and $d = c^{5/4}$

let $a = x^2$ and $c = y^4$, $x, y \in N$

$b = x^3$; $d = y^5$

given $a - c = 9$

$x^2 - y^4 = 9$

$(x - y^2)(x + y^2) = 9$; Hence $x - y^2 = 1$ and $x + y^2 = 9$

(no other combination in the set of + ve integers will be possible)

x = 5 and y = 2

$\therefore$ $b - d = x^3 - y^5 = 125 - 32 = 93$ **Ans.**]

66. (C)

Let $\log_{10}x = a$; $\log_{10}y = b$ and $\log_{10}z = c$

Here $xyz = 10^{81}$

$\Rightarrow$ $\log_{10}x + \log_{10}y + \log_{10}z = 81$

i.e. a + b + c = 81 ....(1)

Also a(b + c) + bc = 468

ab + bc + ca = 468....(2)

Now $a^2 + b^2 + c^2 = (a + b + c)^2 - 2\sum ab = (81)^2 - (2)(468) = 6561 - 936 = 5625$ **Ans.**]

67. (A)

$x = \log_c ab$ or $x + 1 = \log_c ab + \log_c c = \log_c abc$

$\therefore$ $x + 1 = \log_c abc$

|||ly $y + 1 = \log_a abc$ and $z + 1 = \log_b abc$

$\therefore$ $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = \log_{abc}c + \log_{abc}a + \log_{abc}b = \log_{abc}abc = 1$

or $(x + 1)(y + 1) + (y + 1)(z + 1) + (z + 1)(x + 1) = (x + 1)(y + 1)(z + 1)$

$\therefore$ on simplifying, we get

$(xyz - x - y - z) = 2$ **Ans.**]

68. (B)

$\log_{3x}45 = \log_{4x} 40\sqrt{3} = t$ (let)

$\Rightarrow$ $45 = (3x)^t$ ......(1)

and $40\sqrt{3} = (4x)^t$ ......(2)

From (1) and (2), we get

$$\frac{45}{40\sqrt{3}} = \left(\frac{3}{4}\right)^t \Rightarrow t = \frac{3}{2}\left(\frac{45}{40\sqrt{3}} = \frac{9}{8\sqrt{3}} = \frac{3\sqrt{3}}{8} = \left(\frac{3}{4}\right)^{3/2}\right)$$

$\therefore$ From (1) , we get

$45 = (3x)^{3/2} \Rightarrow x^3 = 75$

Hence characteristic of x^3 on base 7 is 2.

69. (D)

On adding and subtracting

$$x = \frac{3-\cos 4\theta + 4\sin 2\theta}{2} \quad ;y = \frac{3-\cos 4\theta - 4\sin 2\theta}{2}$$

$$x = \frac{4(1+\sin 2\theta)-(1+\cos 4\theta)}{2} \quad ;y = \frac{4(1-\sin 2\theta)-(1+\cos 4\theta)}{2}$$

$x = 2(1 + \sin 2\theta) - \cos^2 2\theta$; $y = 2(1 - \sin 2\theta) - \cos^2 2\theta$

$x = 1 + 2\sin 2\theta + \sin^2 2\theta$; $y = 1 - 2\sin 2\theta + \sin^2 2\theta$

$x = (1 + \sin 2\theta)^2$; $y = (1 - \sin 2\theta)^2 \Rightarrow \sqrt{x} + \sqrt{y} = 2$]

[Alternate : Or put $\theta = \frac{\pi}{4}$ and verify]

70. (C)

$C = 4\log_{10}b = 2\pi r$

$\therefore\ 4\log_{10}b = 2\pi \cdot 2\log_{10}a$ (as $r = 2\log_{10}a$)

$$\frac{\log_{10} b}{\log_{10} a} = \pi$$

$\therefore\ \log_a b = \pi$ **Ans.**]

71. (A)

$f(x) = 9\sin^2 x - 16\cos^2 x - 10(3\sin x - 4\cos x) - 10(3\sin x + 4\cos x) + 100$

$= 25\sin^2 x - 60\sin x + 84$

$= (5\sin x - 6)^2 + 48$

$\therefore\ f(x)_{min}$ occurs when $\sin x = 1$

minimum value = 49]

72. (B)

$2^{\sin\theta} > 1 \Rightarrow \sin\theta > 0 \Rightarrow \theta \in 1^{st}$ or 2^{nd} quadrant

$3^{\cos\theta} < 1 \Rightarrow \cos\theta < 0 \Rightarrow \theta \in 2^{nd}$ or 3^{rd} quadrant

hence $\theta \in 2^{nd} \Rightarrow$ possible answer is (B)

73. (D)

Given $\log_{10}\left(\frac{\sin 2x}{2}\right) = -1 \Rightarrow \frac{\sin 2x}{2} = \frac{1}{10} \Rightarrow \sin 2x = \frac{1}{5}$ ....(1)

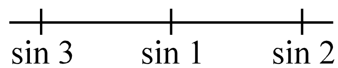
Also $\log_{10}(\sin x + \cos x) = \dfrac{\log_{10}\left(\frac{n}{10}\right)}{2} \Rightarrow \log_{10}(\sin x + \cos x)^2 = \log_{10}\left(\frac{n}{10}\right)$

$\Rightarrow$ 1 + sin 2x = $\frac{n}{10}$ $\Rightarrow$ 1 + $\frac{1}{5}$ = $\frac{n}{10}$ $\Rightarrow$ $\frac{6}{5}$ = $\frac{n}{10}$ $\Rightarrow$ n = 12 $\Rightarrow$ (D)

74. (D)

sin 1 – sin 2

= – 2 cos$\frac{3}{2}$ · sin$\frac{1}{2}$ < 0



$\therefore$ sin 1 < sin 2

sin 1 – sin 3

= – 2 cos 2 sin 1 > 0 $\Rightarrow$ sin 1 > sin 3]

75. (A)

$2^{\log_5 16\cdot\log_4 x+\log_{x\sqrt{2}} 5} + 5^x + x^{\log_3 4+5} + x^5 = 0$

$2^{2\log_5 4\cdot\log_4 x+x\log_2 5} + 5^x + x^{\log_5 4}\cdot x^5 + x^5 = 0$

$2^{2\log_5 x}\,2^{x\log_2 5} + 5^x + x^{2\log_5 2}\cdot x^5 + x^5 = 0$

$\left(2^{\log_5 x}\right)^2\cdot 5^x + 5^x + \left(2^{\log_5 x}\right)^2\cdot x^5 + x^5 = 0$

$5^x\left[\left(2^{\log_5 x}\right)^2+1\right] + x^5\left[\left(2^{\log_5 x}\right)^2+1\right] = 0$

$\left(5^x+x^5\right)\left[\left(2^{\log_5 x}\right)^2+1\right] = 0$

$5^x + x^5 = 0$ $\qquad$ $\left(2^{\log_5 x}\right)^2 + 1 = 0$

This possible only when x will be –ve $\qquad$ No solution

while according to question $x \ge 2$

$\therefore$ number of values of x = zero]

76. (B)

tan(A + B + C + D) = $\frac{S_1 - S_3}{1 - S_2 + S_4}$ ($\therefore$ A + B + C + D = 360°)

$S_1 = S_3$

tanA + tanB + tanC + tanD = $\prod \tan A\left(\sum \cot A\right)$ $\Rightarrow$ B]

77. (B)

$\log_{abc}\sqrt{bc} + \log_{abc}\sqrt{ca} + \log_{abc}\sqrt{ab} \Rightarrow \log_{abc}abc = 1$]

78. (B)

$4\cos^2\theta - 2\sqrt{2}\cos\theta - 1 = 0$

$\cos\theta = \dfrac{2\sqrt{2} \pm \sqrt{8+16}}{8} = \dfrac{\sqrt{2} \pm \sqrt{6}}{4}$

$\cos\theta = \dfrac{\sqrt{6}+\sqrt{2}}{4} \quad \Rightarrow \theta = \dfrac{\pi}{12}; 2\pi - \dfrac{\pi}{12} = \dfrac{23\pi}{12}$

$\cos\theta = -\dfrac{\sqrt{6}-\sqrt{2}}{4}$

$\cos\theta = \cos(\pi - 5\pi/12)$; $\cos(\pi + 5\pi/12)$

$\theta = 7\pi/12$; $17\pi/12 \Rightarrow$ (B)]

79. (C)

It is obvious

80. (D)

$x^{\log_2 3} = 3^{\log_2 x} = y \Rightarrow 6y = 162 \Rightarrow y = 27 = 3^{\log_2 x} = 3^3$

$\log_2 x = 3 \Rightarrow x = 8$ then $\log_4 8 = 3/2$ Ans]

81. (A)

$4 \cos 10°[\cos 120° + \cos 20°]$

$= -2 \cos 10° + 2 \cdot 2 \cos 10° \cos 20°$

$= -2 \cos 10° + 2[\cos 30° + \cos 60°] = \sqrt{3}$ **Ans.**

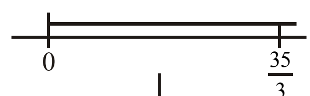
Aliter: $8\cos\theta \cos(60° - \theta) \cos(60° + \theta) \qquad \theta = 10°$

$\dfrac{8}{4} \cos 3\theta = 2\cos 30° = 2 \times \dfrac{\sqrt{3}}{2} = \sqrt{3}$. **Ans.**]

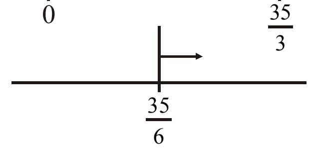
82. (C)

Let $35 - 3x \geq 0 \Rightarrow 0 \leq x \leq \dfrac{35}{3}$ ($x \geq 0$, think!)

$\Rightarrow |3x - 35 + 3x| = x$ or $|(6x - 35)| = x$



Now if $x \geq \dfrac{35}{6} \Rightarrow 6x - 35 = x \Rightarrow \boxed{x = 7}$



if $0 \leq x < \dfrac{35}{6} \Rightarrow 35 - 6x = x \Rightarrow 7x = 35 \Rightarrow \boxed{x = 5}$

If $x \geq \dfrac{35}{3} \Rightarrow |3x - (3x - 35)| = x \Rightarrow x = |35| = 35$ **Ans.**]

Hence $x \in \{5, 7, 35\} \Rightarrow$ sum = 47 **Ans.**]

83. (C)

$$S = \sum_{n=1}^{10} (n+10)^2 - n^2 = \sum_{n=1}^{10} (100+20n) = 1000 + 20\left(\frac{10\cdot 11}{2}\right) = 100\ (10+11) = 2100 \textbf{ Ans.}$$

Aliter: 10[(11 + 1) + (12 + 2) + (13 + 3) + ... + (20 + 10)] = 10 (1 + 2 + 3 + ... + 20)

$= 10 \times \frac{20}{2} \times (1 + 20) = 10 \times 10 \times 21 = 2100$ **Ans.**]

84. (D)

$S_n = \frac{n}{2}[2b + (n-1)d]$ $\quad b = 1^{st}$ term $= 1 - ad$

$20 = 10\ [2(1 - ad) + 19d]$

$2 = 2(1 - ad) + 19d$

$2ad = 19d \quad \Rightarrow \quad a = \frac{19}{2}$ **Ans.**]

85. (B)

If d is the common difference of the arithmetic progression and r is the common ratio of geometric progression, then

$2 + 9d = 3$ and $2r^9 = 3 \Rightarrow d = \frac{1}{9}$ and $r = \left(\frac{3}{2}\right)^{1/9}$

So, $a_7 g_{19} = \left(2 + 6 \times \frac{1}{9}\right)(2r^{18}) = \frac{8}{3} \cdot 2 \cdot \frac{9}{4} = 12$

Now$a_{19}\ g_{28} = \left(2 + 18 \times \frac{1}{9}\right)(2r^{27}) = 27$

Hence,$(a_7\ a_{19} + a_{19}\ g_{28}) = 12 + 27 = 39.$ **Ans.**]

86. (A)

It is obvious

87. (D)

Let d_1 be the common difference,

so, $S_1 = a = T_1$

$S_2 = 4a + \frac{d}{2} \Rightarrow T_1 + T_2 = 4a + \frac{d}{2}$

$\Rightarrow T_2 = 3a + \frac{d}{2} = a + d_1$

$\Rightarrow d_1 = T_2 - T_1 = 2a + \frac{d}{2}$ **Ans.**]

88. (C)

89. (B)

Let the roots of the cubic in A.P. be a – d, a, a + d.

Now, sum of the roots = 3a = $\frac{144}{64} = \frac{9}{4} \Rightarrow a = \frac{3}{4}$

Also, product of the roots = $a(a^2 - d^2) = \frac{15}{64} \Rightarrow a(a^2 - d^2) = \frac{3}{4}\left(\frac{9}{16} - d^2\right)$

$\Rightarrow d^2 = \frac{9}{16} - \frac{5}{16} \Rightarrow d^2 = \frac{4}{16} \Rightarrow d = \pm\frac{1}{2}$

Hence difference between largest and smallest roots = 2d = 1 **Ans.**]

90. (B)

It is obvious