SOLUTIONS

WEEKLY TEST-4

RBPA

(JEE MAIN PATTERN)

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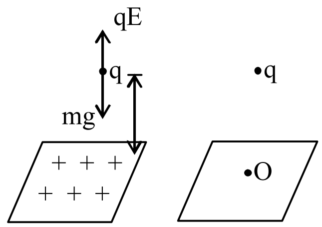
PHYSICS

1. **(B)** **2.** **(A)** **3.** **(A)** **4.** **(C)**

5. **(B)** **6.** **(A)** **7.** **(A)**

8. **(A)**

Equilibrium mg = qE



Now $\sigma \times \pi r^2$ charged disc is removed as r is very less we can treat disc as a point charge

$\therefore$ unbalanced acceleration = $\frac{Q}{4\pi t h^2} \times \frac{q}{m}$

Q = $\sigma \times \pi r^2$

q = $\frac{mg}{E}$

putting in acceleration expression

a = $\frac{g}{2}\left(\frac{r}{h}\right)^2$

9. **(B)** **10.** **(D)** **11.** **(D)** **12.** **(A)**

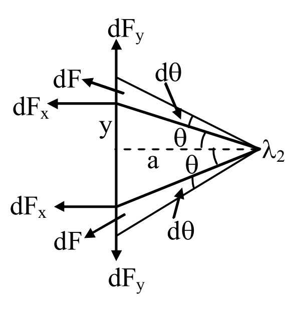
13. **(B)** **14.** **(D)** **15.** **(B)**

16. **(C)**

dF = $\frac{\lambda_2 \times \lambda_1}{2\pi\varepsilon_0\sqrt{a^2+y^2}}.dy$

$dF_x = dF\cos\theta$

and $F_x = \int_{-\infty}^{+\infty} dF_x$



17. **(B)**

The displacement between first stone and aeroplane after *t* second $(h_1) = \frac{1}{2}(g+f)t^2$

After time *t*,

Velocity of aeroplane $= u + ft$

Velocity of first stone $= u - gt$

Where *u* is velocity of aeroplane when first stone is dropped.

The relative speed of second stone with respect to first stone $= (u + ft) - (u - gt)$

$= (g + f)t$

The relative displacement between first and second stone after time $t'(h_2)$

$= (g + f)tt'$

$$h_1 + h_2 = \frac{1}{2}(g + f)t^2 + (g + f)tt' = \frac{1}{2}(g + f)(t + 2t')t$$

18. (A)

$S_1 = \frac{1}{2}a(p-1)^2, S_2 = \frac{1}{2}ap^2$; $S = \frac{1}{2}a\left[2\left(p^2 - p + 1\right) - 1\right] = S_1 + S_2$

19. (C)

Fractional loss in velocity = $\frac{v_1 - v_2}{v_1} = 1 - \frac{v_2}{v_1} = 1 - \sqrt{\frac{2g \times 1.8}{2g \times 5}} = 1 - \frac{3}{5} = \frac{2}{5}$.

20. (C)

$h = ut + \frac{1}{2}gt^2 \Rightarrow \quad 25 = ut - 5t^2$

$5t^2 - ut + 25 = 0 \Rightarrow \quad t_1 + t_2 = \frac{u}{5};\ t_1t_2 = 5$

$(t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1t_2$

$16 = \frac{u^2}{25} - 20 \Rightarrow \frac{u}{5} = 6 \Rightarrow$ *u* = 30 m/s

21. (D)

$h = \frac{1}{2}gt_1^2$; $2h = \frac{1}{2}g(t_1 + t_2)^2$ and $3h = \frac{1}{2}g(t_1 + t_2 + t_3)^2$

i.e., $t_1 : (t_1 + t_2) : (t_1 + t_2 + t_3) = 1 : \sqrt{2} : \sqrt{3}$

or $t_1 : t_2 + t_3 = 1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$

22. (D)

$\frac{d|\vec{v}|}{dt}$ is the tangential acceleration.

23. (A)

The equation of velocity is $v = \frac{1}{10}s + 3$

$a = v\dfrac{dv}{ds} = \left(\dfrac{1}{10}s+3\right)\left(\dfrac{1}{10}\right)$, $a = \dfrac{1}{100}s + \dfrac{3}{10}$

24. **(B)**

At any instant of time, let the length of the string *BP* = l_1 and the length *PA* = l_2. In a further time *t*, let *B* move to the right by *x* and *A* move down by *y*, while *P* moves to the right by *ut*. As the length of the string must remain constant.

$l_1 + l_2 = (l_1 - x + ut) + (l_2 + y)$

or $x = ut + y$

or $\dot{x} = u + \dot{y}$

$\dot{x}$ = speed of *B* to the right $= v_B$, $\dot{y}$ = downward speed of *A* = v_A

$\therefore \quad v_B = u + v_A$.

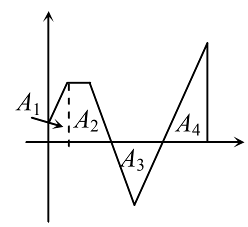
Also $\dot{v}_B = \dot{v}_A$ or $a_B = a_A$

25.: **(D)**

Block *B* again comes to rest when speed of *A* = speed of *C*

$v_A = 6t^2$, $v_c = 3t$, $6t^2 = 3t$, $t = \dfrac{1}{2}s$

26. **(D)**



$A_1 = \dfrac{1}{2}(2+4)\times 1 = 3m$

$A_2 = \dfrac{1}{2}(2+1)\times 4 = 6m$

$A_3 = \dfrac{1}{2}(2\times 4) = 4m$

$A_4 = \dfrac{1}{2}(2\times 6) = 6m$

Distance travelled in 7 s = $A_1 + A_2 + A_3 + A_4$ = 19 m

Average speed = $\dfrac{19}{7}$ m/s

27. **(A)**

This is the situation similar to elastic collision of ball impinging on floor and bouncing back.

28. **(D)**

29. **(A)**

30. $s = t^3 - 6t^2 + 3t + 4$, $v = \dfrac{ds}{dt} = 3t^2 - 12t + 3$,

$a = \dfrac{dv}{dt} = 6t - 12$; a is zero at $t = 2$

$v(t=2) = 3\times 4 - 12\times 2 + 3 = -9$ m/sec

$\therefore$ **(B)**

CHEMISTRY

31. (B)

We have $\frac{P_r V_r}{T_r} = 2$

$$\Rightarrow \frac{\frac{P}{P_c} \cdot \frac{V}{V_c}}{\frac{T}{T_c}} = 2 \Rightarrow \frac{PV}{T} \cdot \frac{T_c}{P_c \cdot V_c} = 2$$

$$\Rightarrow \frac{1 \times V}{300} \times \frac{8}{3R} = 2 \qquad \left(\frac{P_c V_c}{RT_c} = \frac{3}{8}\right)$$

$$\Rightarrow V = \frac{900R}{4} = 18.47 \text{ litre}$$

For 2 mole of gas volume occupied = 2V = $\frac{900R}{2} = 36.95$ litre

32. (B)

P = 1 atm = 1.013×10^5 N/m^2

$\Delta V = (2.5 - 2) = 0.5$ litre = 0.5×10^{-3} m^3

$W = -P\Delta V = -1.01 \times 10^5 \times 0.5 \times 10^{-3}$ J = – 50.5 J

$\Delta U = q + w = 300 - 50.5 = 249.5$ J

33. (D)

$Fe + 2HCl \longrightarrow FeCl_2 + H_2$

Thus mole of H_2 = 1

The work is done against external pressure by H_2 which pushes a pressure of 1 atm

$W = -P\Delta V = -\Delta n_g RT = -1 \times 0.0821 \times 300 = -24.63$ L - atm

34. (B)

$W = -P\Delta V = -3 \times 1.013 \times 10^5 \times 2 \times 10^{-3} = -607.8$ J

this work is used in heating water thus

$-w = q = m s \Delta T$

$607.8 = 10 \times 18 \times 4.184 \times \Delta T$

$\Delta T = 0.81$

$\therefore$ Final Temp. = 290 + 0.81 = 290.81 K

35. (A)

$$\frac{V}{T} = \text{constant}$$

$$P = \text{constant}$$

$$W = -P(V_f - V_i) = -nR\Delta T$$

$$q = nC_P\Delta T$$

$$\frac{q}{|w|} = \frac{nC_P\Delta T}{nR\Delta T} = \frac{C_P}{R} = \frac{5}{2}$$

36. (D)

$\Delta H = \Delta E + (P_2V_2 - P_1V_1)$ = 60 + (20 – 6) = 60 + 14 = 74

37. (B)

For monoatomic gas $C_v = \frac{3}{2}R$

$$\therefore C_p = \frac{5}{2}R$$

$$\because \Delta H = nC_p\Delta T$$

$$= 5 \times \frac{5}{2}R.200$$

$$= 2500R = 20.8\,KJ$$

38. (D)

$$P \;\alpha\; D \Rightarrow P = KD$$

$$V = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{1}{6}\pi D^3$$

$$dV = \frac{1}{2}\pi D^2 dD$$

$$\therefore W = \int_{D_1}^{D_2} P\,dv = \int KD.\frac{1}{2}\pi D^2 dD$$

$$W = \frac{1}{2}\pi K\int_{D_1}^{D_2} D^3.dD = \frac{1}{8}\pi K\left[D_2^4 - D_1^4\right]$$

i.e. $w \propto D^4$

39. (A)

W = – 15 (3 – 6)

= 45 atm L = ΔU

$$\gamma = \frac{Cp_m}{Cv_m} = \frac{(20.91+8.314)}{20.91} = 1.4$$

$$\Delta H = \gamma \Delta U$$
$$= 1.4 \times 45 = 63\,L\,atm$$
$$= 6.4 KJ$$

40. (B)

$$T \alpha \frac{1}{\sqrt{V}} \Rightarrow TV^{\frac{1}{2}} = constant$$

$$TV^{\frac{1}{2}} = TV^{\gamma-1} \Rightarrow \gamma - 1 = \frac{1}{2}$$

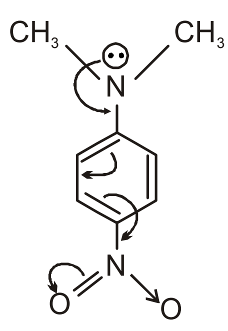
$$\gamma = \frac{3}{2} = 1.5$$

41. (C)

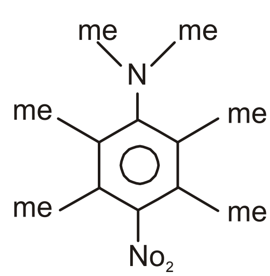
42. (D)

43. (A)

Resonance increases dipolemoment.



Resonance



due to SIR
no resonance

44. (A)

Due to strong –M effect of NO_2 group.

45. (C)

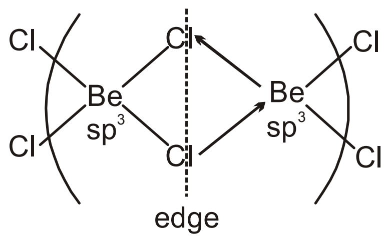
Due to more resonance positive charge spread over 'C' atom more.

46. (A)

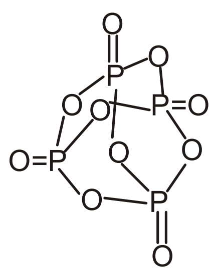
Because both the oxygen make $p\pi - p\pi$ backbonding with carbon.

47. (C) **48. (B)** **49. (D)** **50. (D)**

51. (B)



52. (C)



53. (B)

54. (C)

HF_2^- contains hydrogen bond and covalent bond.

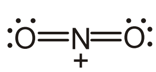
55. (D)

56. (B)

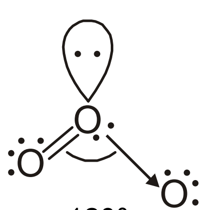
NH_2 group has sp^3 hybridisation.

57. (A) **58. (A)** **59. (D)**

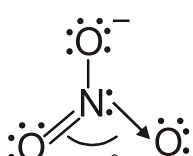
60. (C)



$\theta = 180^\circ$



$\alpha < 120^\circ$
sp^2



sp^2
$\theta = 120^\circ$

MATHEMATICS

61. (A)

$$\lim_{x\to 0}\left(\frac{\sin 3x}{x^3}+\frac{a}{x^2}+b\right)=\lim_{x\to 0}\left(\frac{\sin 3x+ax+bx^3}{x^3}\right)=\lim_{x\to 0}\left(\frac{3\frac{\sin 3x}{3x}+a+bx^2}{x^2}\right)$$

For existence of limit, 3 + a = 0 $\Rightarrow$ a = – 3

$$\therefore\ l=\lim_{x\to 0}\left(\frac{\sin 3x-3x+bx^3}{x^3}\right)=0$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{\sin 3x-3x}{x^3}+b\right)=0$$

$$\Rightarrow -\frac{27}{6}+b=0 \Rightarrow b=\frac{9}{2}$$

62. (C)

Sine of integral multiple of $\pi = 0$

63. (B)

We have,

$$3f(x)+5f\left(\frac{1}{x}\right)=\frac{1}{x}-3,\forall\,(x\neq 0)\in R$$

....(i)

$$\Rightarrow\quad 3f\left(\frac{1}{x}\right)+5f(x)=x-3 \quad \text{[Replacing x by 1/x]} \qquad ...(ii)$$

Multiplying Eq. (i) by 3 and Eq. (ii) by 5 and subtracting, we get

$$9f(x)-25f(x)=\left(\frac{3}{x}-9\right)-(5x-15)$$

$$\Rightarrow\quad -16f(x)=\frac{3}{x}-5x+6$$

$$\Rightarrow\quad f(x)=\frac{1}{16}\left(-\frac{3}{x}+5x-6\right),\forall\,(x\neq 0)\in R$$

64. (C)

$$-1\le\frac{|x|-2}{3}\le 1$$

$-3 \le |x| - 2 \le 3$

$-1 \le |x| \le 5$

$-5 \le x \le 5$

65. **(A)**

Here $G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right) \equiv \left(1, \frac{2}{3}\right)$.

Now, centroid divides the line segment joining orthocentre H(h, k) and circumcentre into the ratio 2 : 1 internally.

$\therefore \quad G\left(1, \frac{2}{3}\right) \equiv \left(\frac{2 \times 0 + 1 \times h}{2 + 1}, \frac{2 \times 0 + 1 \times k}{2 + 1}\right) \Rightarrow H(h, k) \equiv (3, 2)$.

66. **(B)**

$$\sin^{-1}\left(\frac{4x}{x^2 + 4}\right) + 2\tan^{-1}\left(-\frac{x}{2}\right) = \sin^{-1}\left(\frac{2.\frac{x}{2}}{\left(\frac{x}{2}\right)^2 + 1}\right) - 2\tan^{-1}\frac{x}{2} = 2\tan^{-1}\frac{x}{2} - 2\tan^{-1}\frac{x}{2} = 0$$

Here $\left|\frac{x}{2}\right| \le 1$

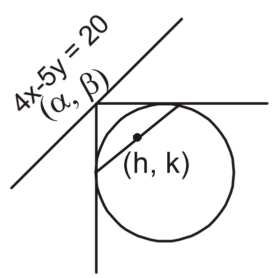
$|x| \le 2 \Rightarrow -2 \le x \le 2$

67. **(A)**

$4\alpha - 5\beta = 20$

$x\alpha + y\beta = 9$

$xh + yk = h^2 + k^2$



$\frac{\alpha}{h} = \frac{\beta}{k} = \frac{9}{h^2 + k^2}$

$\alpha = \frac{9h}{h^2 + k^2}$, $\beta = \frac{9k}{h^2 + k^2}$

$\frac{4(9h)}{h^2 + k^2} - \frac{5(9k)}{h^2 + k^2} = 20 \Rightarrow 36h - 45k = 20(h^2 + k^2) \Rightarrow 20(x^2 + y^2) = 36x - 45y$

68. (B)

$$\lim_{x\to\infty}\left(\sqrt[3]{(x+a)(x+b)(x+c)}-x\right)$$

$$=\lim_{x\to\infty}\frac{(x^3+(a+b+c)x^2+(ab+bc+ca)x+abc)-x^3}{((x+a)(x+b)(x+c))^{2/3}+x^2+x((x+a)(x+b)(x+c))^{1/3}}\quad\left(\because\ x-y=\frac{x^3-y^3}{x^2+xy+y^2}\right)$$

$$=\lim_{x\to\infty}\frac{(a+b+c)x^2+(ab+bc+ca)x+abc}{((x+a)(x+b)(x+c))^{2/3}+x^2+x((x+a)(x+b)(x+c))^{1/3}}$$

$=\frac{a+b+c}{3}$ (Dividing numerator and denominator by x^2)

69. (D)

$$\cos^{-1}\frac{6x}{1+9x^2}=-\frac{\pi}{2}+2\tan^{-1}3x$$

$$\Rightarrow\quad \frac{\pi}{2}-\sin^{-1}\frac{6x}{1+9x^2}=-\frac{\pi}{2}+2\tan^{-1}3x$$

$$\Rightarrow\quad \sin^{-1}\frac{6x}{1+9x^2}=\pi-2\tan^{-1}3x \qquad \Rightarrow\quad \sin^{-1}\frac{2.3x}{1+(3x)^2}=\pi-2\tan^{-1}3x$$

Above is true when $3x\ge 1 \quad\Rightarrow\ x\ge\frac{1}{3}$

$$x\in\left[\frac{1}{3},\infty\right)$$

70. (A)

$$\underset{x\to\infty}{\text{Limit}}\frac{\cot^{-1}\left(\frac{\log_a x}{x^a}\right)}{\sec^{-1}\left(\frac{a^x}{\log_a x}\right)}\ ;\ \underset{x\to\infty}{\text{as}}\left(\frac{\log_a x}{x^a}\right)\to 0\ \text{ and }\ \left(\frac{a^x}{\log_a x}\right)\to\infty\ \text{(using L'opital rule)}$$

$$\therefore\ l=\frac{\pi/2}{\pi/2}=1$$

71. (C)

Centre of the circles lies on $x + y = 3a$.

let centres are $(\alpha, 3a-\alpha)$ and $(\beta, 3a-\beta)$

$\Rightarrow \alpha,\beta$ be the roots of equation

$(x-a)^2 + (2a-x)^2 = x^2$

$\Rightarrow x^2 - 6ax + 5a^2 = 0$

$\alpha + \beta = 6a,\ \alpha\beta = 5a^2$

$|\alpha - \beta| = 4a$

$\Rightarrow C_1C_2 =$ distance between centres $= 4\sqrt{2}a$

72. (C)

$$\lim_{x\to 0}\frac{8}{x^8}\left[1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right]$$

$$= \lim_{x\to 0}\frac{8}{x^8}\left[\left(1-\cos\frac{x^2}{2}\right)-\cos\frac{x^2}{4}\left(1-\cos\frac{x^2}{2}\right)\right]$$

$$= \lim_{x\to 0}\frac{8}{x^8}\left(1-\cos\frac{x^2}{2}\right)\left(1-\cos\frac{x^2}{4}\right)$$

$$= \lim_{x\to 0}\frac{8}{x^8}\cdot 2\sin^2\frac{x^2}{4}\cdot 2\sin^2\frac{x^2}{8}$$

$$= \lim_{x\to 0}\frac{32}{x^8}\left(\frac{\sin\frac{x^2}{4}}{\frac{x^2}{4}}\right)^2\left(\frac{x^2}{4}\right)^2\cdot\left(\frac{\sin\frac{x^2}{8}}{\frac{x^2}{8}}\right)^2\cdot\left(\frac{x^2}{8}\right)^2 = \frac{1}{32}$$

73. (B)

$$\lim_{n\to\infty}\frac{\sqrt[4]{n^5+2}-\sqrt[3]{n^2+1}}{\sqrt[5]{n^4+2}-\sqrt[2]{n^3+1}} = \lim_{n\to\infty}\frac{n^{5/4}\sqrt[4]{1+\frac{2}{n^5}}-n^{2/3}\sqrt[3]{1+\frac{1}{n^2}}}{n^{4/5}\sqrt[5]{1+\frac{2}{n^4}}-n^{3/2}\sqrt[2]{1+\frac{1}{n^3}}} = \lim_{n\to\infty}\frac{\frac{n^{5/4}}{n^{3/2}}\sqrt[4]{1+\frac{2}{n^5}}-\frac{n^{2/3}}{n^{3/2}}\sqrt[3]{1+\frac{1}{n^2}}}{\frac{n^{4/5}}{n^{3/2}}\sqrt[5]{1+\frac{2}{n^4}}-\frac{n^{3/2}}{n^{3/2}}\sqrt[2]{1+\frac{1}{n^3}}}$$

[On dividing the numerator and denominator by the highest power of n is, $n^{3/2}$]

$$= \lim_{n\to\infty}\frac{\frac{1}{n^{1/4}}\sqrt[4]{1+\frac{2}{n^5}}-\frac{1}{n^{5/6}}\sqrt[3]{1+\frac{1}{n^2}}}{\frac{1}{n^{7/10}}\sqrt[5]{1+\frac{2}{n^4}}-\frac{n^{3/2}}{n^{3/2}}\sqrt[2]{1+\frac{1}{n^3}}} = \frac{0-0}{0-1} = 0$$

74. (A)

Area of ΔABC is maximum when C is farthest from AB, i.e. when it is isosceles.

75. (B)

76. (D)

Centre (0, 0), radius = $\frac{1}{2}.\ \frac{3}{4}.\ 4a = \frac{3a}{2}$

Equation of the circle is $4(X^2 + y^2) = 9a^2$ ....(i)

Equation of the parabols is $y^2 = 4ax$ ....(ii)

Solving (i) and (ii) $x^2 + 4ax - \frac{9a^2}{4} = 0$

$$x = \frac{-4a \pm \sqrt{16a^2 + 9a^2}}{2} = \frac{-4a \pm 5a}{2}$$

$\therefore$ $x = a/2$

$\Rightarrow$ $y = \pm \sqrt{2}\,a$

$\therefore$ The double ordinate = $2\sqrt{2}\,a$

$\therefore$ area of the trapezium $PL_1\,L_2\,Q = \frac{1}{2}\left(2\sqrt{2}a + 4a\right)\frac{a}{2} = \left(\frac{2+\sqrt{2}}{2}\right)a^2$

77. (A)

Slope of tangent at P is $\frac{1}{t_1}$ $\Rightarrow \cot\theta_1 = t_1$

and at Q = $\frac{1}{t_2} \Rightarrow \cot\theta_2 = t_2$

Slope of PQ = $\frac{2}{t_1 + t_2} \Rightarrow$ Slope of OR is $-\frac{t_1 + t_2}{2} = \tan\phi$

(Note : angle in a semicircle is 90°)

$\Rightarrow$ $\tan\phi = \frac{1}{2}(\cot\theta_1 + \cot\theta_2) \Rightarrow \cot\theta_1 + \cot\theta_2 = -2\tan\phi$

78. (A)

$$\lim_{x \to -\infty}\left(\sqrt{x^2 + 4x} + x\right)$$

$= \lim\limits_{x\to -\infty} \dfrac{4x}{-x\left(\sqrt{1+\dfrac{4}{x}}+1\right)}$

$= -2$

79. **(C)**

$y^2 = 6\left(x - \dfrac{3}{2}\right)$

Equation of directrix $x - \dfrac{3}{2} = -\dfrac{3}{2}$ i.e. $x = 0$

Let coordinate P be $\left(\dfrac{3}{2}+\dfrac{3}{2}t^2, 3t\right)$

$\therefore$ Coordinate of M are (0, 3t)

$\therefore$ $MS = \sqrt{9+9t^2}$

$MP = \dfrac{3}{2} + \dfrac{3}{2}t^2$

$\therefore$ $9 + 9t^2 = \left(\dfrac{3}{2}+\dfrac{3}{2}t^2\right)^2 = \dfrac{9}{4}(1 + t^2)^2$

$\therefore$ $4 = 1 + t^2$

$\therefore$ Length of side = 6

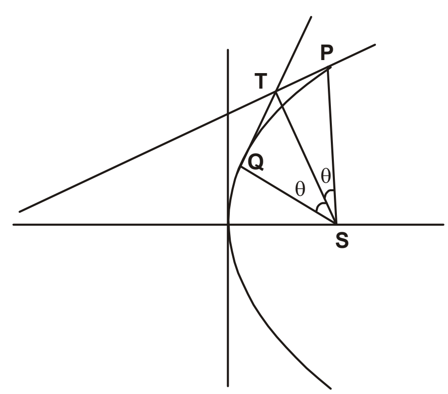
80. **(A)**

$\lim\limits_{x\to\infty}(\sqrt{x+1}-\sqrt{x}) = 0 \Rightarrow \lim\limits_{x\to\infty}\cot^{-1}(\sqrt{x+1}-\sqrt{x}) = \dfrac{\pi}{2}$

and $\lim\limits_{x\to\infty}\left(\dfrac{2x+1}{x-1}\right)^x = \infty \Rightarrow \lim\limits_{x\to\infty}\sec^{-1}\left(\dfrac{2x+1}{x-1}\right)^x = \dfrac{\pi}{2}$

$\therefore$ Required limit = 1

81. **(B)**



Since $\angle TSP = \angle TSQ$

and $ST^2 = SP.SQ$

Now in ΔTSP and ΔTSQ

$\dfrac{ST}{\sin\angle TQS} = \dfrac{TQ}{\sin\angle TSQ}$ ------ (1)

and $\dfrac{ST}{\sin\angle TPS} = \dfrac{TP}{\sin\angle TSP}$ ------ (2)

divide (2) by (1) $\dfrac{\sin\angle TQS}{\sin\angle TPS}=\dfrac{TP}{TQ}=2$ or $\dfrac{1}{2}$

which is given by option (B)

82. (C)

$$f(x)=\sin^{-1}x-\left(\frac{\pi}{2}-\tan^{-1}x\right)+(x+1)^2+5\ =\sin^{-1}x+\tan^{-1}x-\frac{\pi}{2}+(x+1)^2+5$$

Domain of f(x) is [–1, 1] and f(x) is increasing in [–1, 1]

$$f(-1)=\frac{-5\pi}{4}+5,\ \ f(1)=\frac{\pi}{4}+9$$

$\therefore$ Range of f(x) is $\left[\dfrac{-5\pi}{4}+5,\ \dfrac{\pi}{4}+9\right]$

$\Rightarrow a+b=-\pi+14 \qquad \Rightarrow [a+b]=10$

83. (D)

Sami latus rectum is H.M. of two segments of focal chord.

$2a=\dfrac{2.3.5}{8} \quad \Rightarrow a=\dfrac{15}{8}$

84. (A)

a = 2

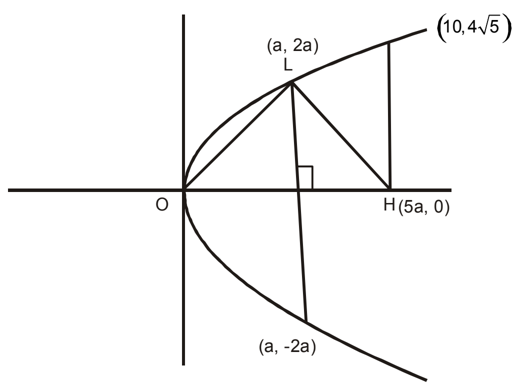
Equation of LH

$y-2a=-\dfrac{1}{2}(x-a)$

$y=0\ ;\ x=5a$

$y^2=40a=80$

$y=\pm4\sqrt{5}$



85. (B)

parabola's $y = x^2 + 1$ and $x = y^2 + 1$ are symmetrical about y = x

$\therefore$ tangent of point A is paralle to y = x

$\dfrac{dy}{dx}=2x \qquad \Rightarrow\ 2x=1 \Rightarrow x=\dfrac{1}{2}$ and $y=\dfrac{5}{4}$

A $\left(\dfrac{1}{2},\dfrac{5}{4}\right)$ and B $\left(\dfrac{5}{4},\dfrac{1}{2}\right)$

$\therefore$ Radius = $\frac{1}{2}\sqrt{\left(\frac{1}{2}-\frac{5}{4}\right)^2+\left(\frac{5}{4}-\frac{1}{2}\right)^2}$ = $\frac{1}{2}\sqrt{\frac{9}{16}+\frac{9}{16}}$ = $\frac{3}{8}\sqrt{2}$

$\therefore$ Area = $\pi.\ \frac{9}{32}$

86. **(C)** $\therefore$ the length of focal chord = 4a cosec$^2\ \alpha$ $\qquad \therefore\ \alpha \in \left(0,\frac{\pi}{4}\right]$

$\therefore$ minimum length = 4a. 2 = 8a

87. **(C)**

The circle having focal chord PQ as diameter will touch the directrix.

Also, $t_1t_2 = -1$

Now, CT = 4 (C is mid point of PQ)

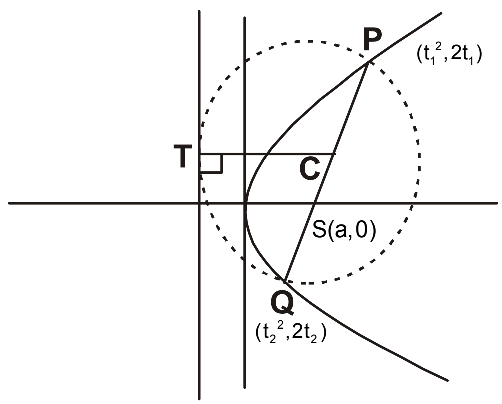
$$\left|\frac{t_1^2+t_2^2}{2}+1\right|=4$$

$\Rightarrow t_1^2+t_2^2=6$

$\Rightarrow (t_1+t_2)^2=4$

$\Rightarrow t_1+t_2=\pm 2$

$\Rightarrow k=\pm\ 2$



88. **(D)**

$$\lim_{x\to\infty} x\left\{\tan^{-1}(x+1)-\tan^{-1}(x+5)\right\}+\tan^{-1}(x+1)-5\tan^{-1}(x+5)$$

$$= \lim_{x\to\infty} x\cdot\tan^{-1}\left(\frac{(x+1)-(x+5)}{1+(x+1)(x+5)}\right)+\frac{\pi}{2}-\frac{5\pi}{2}$$

$$= \lim_{x\to\infty}\frac{x\cdot\left(\tan^{-1}\frac{-4}{1+(x+1)(x+5)}\right)}{\left(\frac{-4}{1+(x+1)(x+5)}\right)\cdot\left(\frac{1+(x+1)(x+5)}{-4}\right)}-2\pi$$

$= 0-2\pi=-2\pi$

89. (B)

$$\cot\left\{\cot^{-1}3+\cot^{-1}7+\cot^{-1}13+\cot^{-1}21\right\}$$

$$=\cot\left\{\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}+\tan^{-1}\frac{1}{21}\right\}$$

$$=\cot\left\{\tan^{-1}2-\tan^{-1}1+\tan^{-1}3-\tan^{-1}2+...+\tan^{-1}5-\tan^{-1}4\right\}$$

$$=\cot\left[\tan^{-1}5-\tan^{-1}1\right]=\frac{3}{2}$$

90. (C)

$$\lim_{x\to m}\frac{\left|ax^2+bx+c\right|}{ax^2+bx+c}=1 \text{ when, } ax^2+bx+c>0$$

Which is possible for (i) a < 0 if $\alpha<m<\beta$ and for (ii) $a>0$ if $m<\alpha$ or $m>\beta$