SOLUTIONS

PROGRESS TEST-6

GZRA-1901, GZR-1901(A)

GZRS-1901

(JEE MAIN PATTERN)

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PHYSICS

1. (D)

$$\frac{\Delta V}{V}\times 100 = 3\left(\frac{\Delta \ell}{\ell}\times 100\right)$$

2. (B)

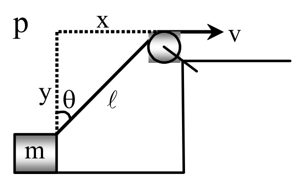
$a_A = 6a_B$

$a_B = \frac{a_A}{6} = \frac{12 m/s^2}{6} = 2 m/s^2$

3. (D)

4. (B)

$x^2 + y^2 = \ell^2$



$2x\frac{du}{dt} + 0 = 2\ell\frac{d\ell}{dt}$ [y = constant]

or $\left|\frac{dx}{dt}\right| = \frac{\left|\frac{d\ell}{dt}\right|}{(x/\ell)} = \frac{v}{\sin\theta}$

5. (C)

t is the time to reach ground.

$h = \frac{1}{2}at^2$; $\left(1-\frac{9}{25}\right)h = \frac{1}{2}a(t-1)^2$

$\left(1-\frac{9}{25}\right) = \frac{(t-1)^2}{t^2}$; $\frac{16}{25} = \frac{(t-1)^2}{t^2}$

or $\frac{4}{5} = \frac{t-1}{t}$ $\therefore$ t = 5 sec

$h = \frac{1}{2}\times 9.8\times 5^2 = 122.5$ m

6. (A)

7. (B)

8. (B)

$\because f = ma_2$

9. **(B)**

10. **(A)**

When pulley is clamped (or masses are stationary)

$W_1 = (m_1 + m_2)g$ ....(i)

When clamp is removed,

$W_2 = 2T = \frac{4m_1m_2}{m_1 + m_2}g$ ....(ii)

$\therefore \Delta W = W_1 - W_2 = \frac{(m_1 - m_2)^2}{m_1 + m_2}g$

11. **(D)** **12. (A)**

13. **(C)**

Given equation is $\left(p + \frac{a}{V^2}\right)(V - b) = RT$

We know that $\left[p + \frac{a}{V^2}\right] = \left[ML^{-1}T^{-2}\right]$, P = pressure.

$\left[\frac{a}{V^2}\right] = [ML^{-1}T^{-2}]$

$a = [L^3]^2\,[ML^{-1}T^{-2}] = [L^6][ML^{-1}T^{-2}] = [ML^5T^{-2}]$ $\left[\because V \equiv \left[L^3\right]\right]$

14. **(B)**

Value of 1 main scale division = a unit

Now (n+1) vernier scale division = n main scale divisions = na units.

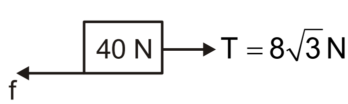
Therefore, value of 1 vernier scale division = $\frac{na}{(n+1)}$ units.

Vernier constant = value of 1 main scale division – value of 1 vernier scale division

$= a - \frac{na}{n+1} = a\left(1 - \frac{n}{n+1}\right) = \frac{a}{(n+1)}$ units.

15. **(D)**

From f.b.d.



$\mu = \frac{8\sqrt{3}}{40} \simeq 0.35$

16. **(A)**

Resolve the applied force and get normal reaction and limiting friction

17. (C)

Common acceleration $a = \frac{KA}{2m}$

$\therefore\ f_r = ma = \frac{KA}{2}$

18. (C)

Clearly $\mu = \tan\theta = \frac{3}{4}$

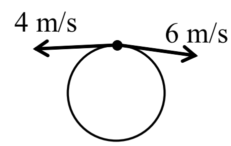
19. (C)

Use homogenity of dimensión and use

$\mu \rightarrow$ Dimension less quantity

$\lambda \rightarrow$ meter

20. (B)



$\therefore\ S_1 + S_2 = 2\pi r$

$\therefore$ 4t + 6t = 2πr = t = $\frac{2\pi r}{10} = \frac{2\times3.14\times4}{10}$ = 2.5 s

21. (C)

$v\frac{dv}{dx}$ = 2x + 1

vdv = (2x + 1) dx

$\int_0^v v\,dv = \int_0^x (2x+1)\,dx \quad \Rightarrow \frac{v^2}{2} = x^2 + x$

22. (A)

23. (C)

The displacement of the body during the time t as it reaches the point of projection again

$\Rightarrow\ S = 0 \qquad \Rightarrow\ v_0 t - \frac{1}{2}gt^2 = 0 \qquad \Rightarrow t = \frac{2v_0}{g}$

During the same time t, the body moves in absence of gravity through a distance

$D' = v_0 t$, because in absence of gravity g = 0

$\Rightarrow\ D' = v_0\left(\frac{2v_0}{g}\right) = \frac{2v_0^2}{g}$ ...(i)

In presence of gravity the total distance covered is

$= D = 2H = 2\frac{v_0^2}{2g} = \frac{v_0^2}{g}$ ...(ii)

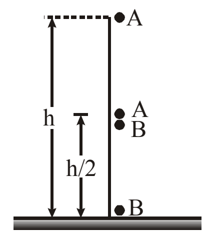
(i) $\div$ (ii) $\Rightarrow D' = 2D$

Hence **(C)**

24. (C)

Time of travel of each stone = t

Distance travelled by each stone $= \frac{h}{2}$



For stone $A, \frac{h}{2} = \frac{1}{2}gt^2$ i.e., $t = \sqrt{\frac{h}{g}}$

For stone B, $\frac{h}{2} = ut - \frac{1}{2}gt^2 = u\sqrt{\frac{h}{g}} - \frac{1}{2}g\left(\frac{h}{g}\right)$

$\Rightarrow \frac{h}{2} = u\sqrt{\frac{h}{g}} - \frac{h}{2} \Rightarrow u\sqrt{\frac{h}{g}} = h$

$\therefore u = h\sqrt{\frac{g}{h}} = \sqrt{gh}$

The correct option is **(C)**

25. (A)

$S = ut + \frac{1}{2}at^2$; $15 = 2t + \frac{1}{2} \times (-0.1)t^2 \Rightarrow 20 \times 15 = 40t - t^2$ or $t^2 - 40t + 300 = 0$

$(t - 30)(t - 10) = 0$; $t = 30$ s

or $t = 10$ s

The particle is at a distance 15 m from starting point at t = 10 s and also t = 30 s

$\therefore$ **(A)**

26. (C)

t is the time to reach ground.

$h = \frac{1}{2}at^2$; $\left(1 - \frac{9}{25}\right) h = \frac{1}{2}a(t-1)^2$

$\left(1 - \frac{9}{25}\right) = \frac{(t-1)^2}{t^2}$; $\frac{16}{25} = \frac{(t-1)^2}{t^2}$

or $\frac{4}{5} = \frac{t-1}{t}$ $\therefore$ t = 5 sec

$h = \frac{1}{2} \times 9.8 \times 5^2 = 122.5$ m

27. (C)

The two stones meet at distance S from top of cliff t seconds after first stone is dropped.

For 1st stone $S = \frac{1}{2}gt^2$; For 2nd stone $S = u(t-2) + \frac{1}{2}g(t-2)^2$

i.e. $\frac{1}{2}gt^2 = ut - 2u + \frac{1}{2}gt^2 - 2gt + 2g$

$0 = (u - 2g)t - 2(u - g)$; $t = \frac{2(u-g)}{u-2g} = \frac{2(30-10)}{30-20} = 4$ s

Distance S at which they meet $= \frac{1}{2} \times g t^2 = \frac{1}{2} \times 10 \times 16$

= 80 m from top of cliff

$\therefore$ **(C)**

28. (A)

For a force of 100 N on 10 kg block, relative motion will take place.

$\therefore$ The frictional force between 10 kg block and 40 kg block,

$f = \mu mg = 0.4 \times 10 \times 9.8 N$

The acceleration of the slab of 40 kg is

$a = \frac{0.4 \times 10 \times 9.8}{40} = 0.98 \text{ m/s}^2$

29. (C)

Since mgsin30° > μmgcos30°

the block has a tendency to slip downwards. Let F be the minimum force applied on it, so that it does not slip. Then

N = F + mgcos30°

$\therefore$ mgsin30° = μN = μ(F + mgcos30°)

or $F = \frac{mg\sin 30°}{\mu} - mg\cos 30° = \frac{(2)(10)(1/2)}{0.5} - (2)(10)\left(\frac{\sqrt{3}}{2}\right)$

or F = 20 – 17.32 = 2.68 N

30. (B)

CHEMISTRY

31. (B)

Root mean square speed = V

$\sqrt{\frac{3RT}{M}} = V$

Then $\frac{3RT}{M} = V^2$

$\frac{\frac{\cancel{3R}T_1}{\cancel{M}}}{\frac{\cancel{3R}T_2}{\cancel{M}}} = \frac{V^2}{x^2}$

$$\frac{140}{560} = \left(\frac{v}{x}\right)^2$$

$$\frac{v}{x} = \sqrt{\frac{1}{4}}$$

x = 2V

32. (C)

33. (B)

$T_1 < T_2 < T_3$

34. (B)

$$V_{t^0} = V_0 + \frac{t\,V_0}{273}$$

$$V_{40^o} = V_0 + \frac{40\ \ V_o}{273}$$

$$V_{41^o} = V_0 + \frac{41\ \ V_o}{273}$$

$$V_{41^o} - V_{40^o} = \frac{V_0}{273}$$

35. (C)

$$\left(P + \frac{a}{V_m^2}\right)(V_m - b) = RT$$

for b = 0,

$$PV_m + \frac{a}{V_m} = RT$$

$$PV_m = RT - \frac{a}{V_m}$$

From graph,

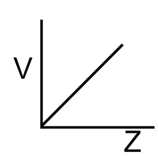
$$\text{Slope} = -a = \frac{21.6 - 20.1}{2 - 3}$$

$\therefore$ a = 1.5

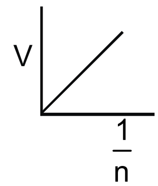
36. (D)

$$V_n = 2.18 \times 10^8 \times \frac{Z}{n}\ \text{cm/sec}$$

$V_n \propto Z; \quad V_n = KZ$



$V_n = K.\frac{1}{n}$



37. (D)

$E \alpha \frac{1}{\lambda}$

λ highest than E is minimum

$E_5 - E_4 < E_5 - E_3 < E_5 - E_2 < E_5 - E_1$

38. (C)

Higher (n + l) means higher energy.

39. (A)

$0.5 = \frac{0.4 \times V \times 2 + 50 \times 0.3}{50 + V}$

V = 33.33 ml

40. (D)

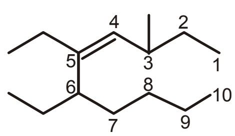
Let the water added is V ml

16 × 0.5 = (16 + v) × 0.2

40 = 16 + V

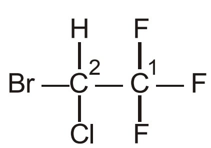
V = 24 ml.

41. (B)

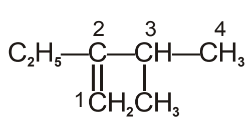


5, 6-Diethyl-3-methyldec-4-ene

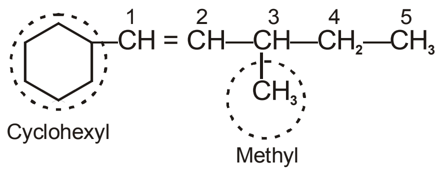
42. (C)



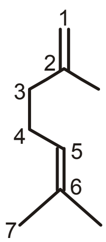
43. (B)



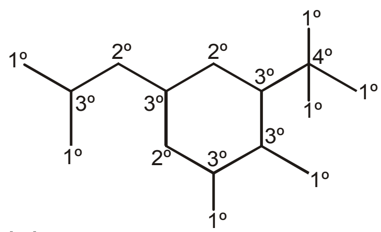
44. (A)



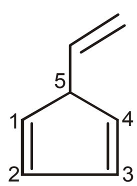
45. (C)



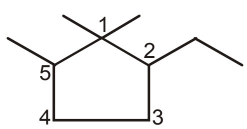
46. (C)



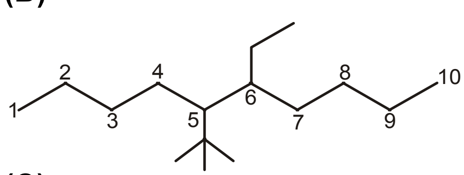
47. (A)



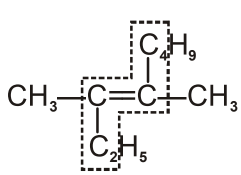
48. (C)



49. (B)



50. (C)



51. **(B)**

$A + A \rightarrow A^{-} + A^{+}$ 4eV

I.P. – E.A. = 4eV ....(i)

$A^{-} \rightarrow A^{+}$

$IP + EA = 10eV$ ....(ii)

From (i) and (ii)

IP = 7eV

EA = 3eV

52. **(C)**

53. **(D)**

r ↓ Zn, Cd ↑ I.E.

Hg ⟶ I.E. = Max. due to Lanthanide Contraction

I.E. = Hg > Zn > Cd

54. **(D)**

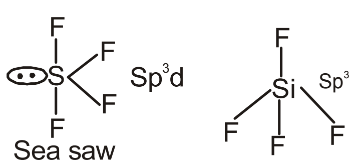
Ge

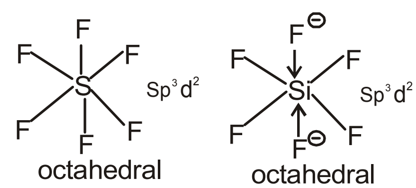
Sn, Pb } (Exception) Lanthanide Contraction

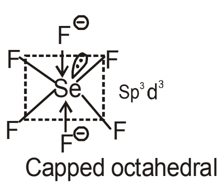
$I.E_1$ = Ge > Pb > Sn

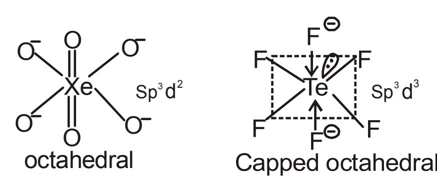
55. **(C)** **56.** **(A)** **57.** **(B)**

58. **(B)**

(A) 

(B) 

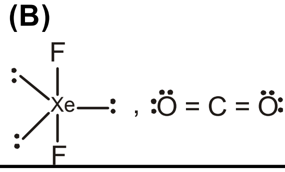
(C) 

(D) 

59. **(A)**

BeF_3^{-} $\langle Sp^2 \rangle$

60. **(B)**



MATHEMATICS

61. (A)

$$x=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+......\text{to } \infty$$

$$=\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+.....\text{to } \infty\right)-\left(\frac{1}{2^4}+\frac{1}{4^4}+......\text{to } \infty\right)$$

$$=\frac{\pi^4}{90}-\frac{1}{16}\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+.....\text{to } \infty\right)=\frac{\pi^4}{90}-\frac{1}{16}.\frac{\pi^4}{90}=\frac{\pi^4}{96}$$

62. (A)

63. (B)

Here $2\sin\frac{\alpha+\beta}{2}.\cos\frac{\alpha-\beta}{2}=a, 2\sin\frac{\alpha+\beta}{2}.\sin\frac{\beta-\alpha}{2}=b$

Now, divide and get the value

64. (B)

$\sin n\theta=b_0+b_1\sin\theta+b_2\sin^2\theta+.......$

This is possible when n is an odd integer.

Put $\theta=0$ to get b_0. After differentiating w.r.t. θ, put $\theta=0$ to get b_1.

65. (A)

66. (A)

As the points are in order, the are

$$=\left|\frac{1}{2}\left\{\begin{vmatrix}4 & 1\\3 & 6\end{vmatrix}+\begin{vmatrix}3 & 6\\-5 & 1\end{vmatrix}+\begin{vmatrix}-5 & 1\\-3 & -3\end{vmatrix}+\begin{vmatrix}-3 & -3\\-3 & 0\end{vmatrix}\begin{vmatrix}-3 & 0\\4 & 1\end{vmatrix}\right\}\right|=30\text{ unit}^2$$

67. (A)

68. (D)

$\tan(180°-\theta)=\text{slope of } AB=-3$

$\therefore \quad \tan\theta=3$

$\therefore \quad \frac{OC}{AC}=\tan\theta, \frac{OC}{BC}=\cot\theta$

$\Rightarrow \quad \frac{BC}{AC}=\frac{\tan\theta}{\cot\theta}=\tan^2\theta=9$

69. (D)

$|x|\left(\frac{1+|x|}{x^2+x+1}\right)\le 0 \quad \Rightarrow x=0$

70. (C)

Δ ABC is right angled at A

$\therefore$ Eqn. of circle is (x + 1) (x – 5) + (y – 1) (y – 5) = 0

T = 0 gives the required tangent.

71. (C)

For internal point p(2, 8) 4 + 64 – 4 + 32 – p < 0 $\Rightarrow$ p > 96 and x intercept = $2\sqrt{1+p}$ therefore

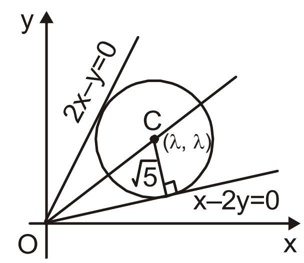
1 + p > 0 $\Rightarrow$ p > – 1 and y intercept = $2\sqrt{4+p}$ $\Rightarrow$ p > – 4

Combining the above conditions on p we get p > 96 i.e. $p \in (96, \infty)$

72. (A)

Pair of lines are $x - 2y = 0$ and $2x - y = 0$,equation of the bisectors of the pair of lines will be

$\frac{x^2 - y^2}{2-2} = \frac{xy}{-5/2}$ i.e. $y = \pm x$



Centre $C \equiv (\lambda, \lambda)$

$\therefore \left| \frac{\lambda - 2\lambda}{\sqrt{1^2 + 2^2}} \right| = \sqrt{5}$

$\therefore \lambda = \pm 5$

Circle lies in the first quadrant $\because \lambda = 5$

$\therefore$ eqn. of circle $(x-5)^2 + (y-5)^2 = (\sqrt{5})^2 \Rightarrow x^2 + y^2 - 10x - 10y + 45 = 0$

73. (C)

The cosine formula applied to triangle Q_1OQ_2 gives $\cos \angle Q_2OQ_1 = \frac{OQ_1^2 + OQ_2^2 - Q_1Q_2^2}{2 \cdot OQ_1 \cdot OQ_2}$

$= \frac{(x_1-0)^2 + (y_1-0)^2 + (x_2-0)^2 + (y_2-0)^2 - [(x_1-x_2)^2 + (y_1-y_2)^2]}{2 \cdot OQ_1 \cdot OQ_2} = \frac{2\,(x_1x_2) + 2(y_1y_2)}{2 \cdot OQ_1 \cdot OQ_2}$

$\therefore \quad OQ_1 \cdot OQ_2 \cos \angle Q_1OQ_2 = x_1x_2 + y_1y_2$

74. (C)

$P \equiv \frac{x}{\cos\frac{\pi}{4}} = \frac{y}{\sin\frac{\pi}{4}} = 6\sqrt{2} \quad \Rightarrow x = 6,\ y = 6$

Since P(6,6) lie on circle

72 + 12 (g + f) + c = 0 ......(i)

Since y = x touches the circle, then

$2x^2 + 2x(g+f) + c = 0$ has equal roots D = 0

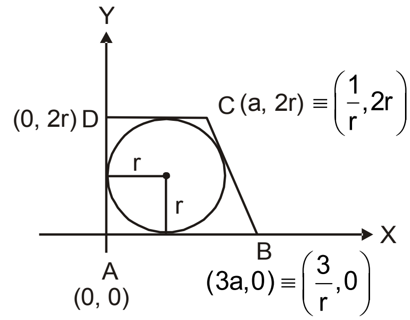
4 $(g + f)^2$ = 8c $\Rightarrow$ $(g + f)^2$ = 2c ......(ii)

From, we get

$(12(g+f))^2 = [-(c+72)]^2 \Rightarrow$ 144 (2c) = $(c + 72)^2$ $\Rightarrow$ $(c - 72)^2 = 0$ $\Rightarrow$ c = 72

75. (B)

Area of trapezium ABCD $= \frac{1}{2}(a+3a)(2r) = 4 \Rightarrow ar = 1$



Equation of line BC is $y = -r^2\left(x - \frac{3}{r}\right)$

or, $y + r^2x - 3r = 0$

$\because$ BC is the tangent to the circle

$\Rightarrow \frac{|r + r^3 - 3r|}{\sqrt{1+r^4}} = r \Rightarrow r^4 + 4 - 4r^2 = 1 + r^4 \Rightarrow r = \frac{\sqrt{3}}{2}$

76. (D)

It can be seen that the given points P(p, q), $C\left(\frac{p}{2}, \frac{q}{2}\right)$ and the origin are collinear which implies that line OP where O is the origin is a diameter of the given circle. Therefore, equation of the given circle is

$x(x - p) + y(y - q) = 0$

i.e. $x^2 + y^2 - px - qy = 0$.......(1)

Let M(a, 0) be the mid-point of a chord AP (see fig.). Then, we have

$CM \perp AP$

i.e. slope of CM × slope of AP = – 1 $\Rightarrow \frac{\frac{q}{2}}{\frac{p}{2} - a} \times \frac{q}{p-a} = -1$

i.e. $q^2 + (p - 2a)(p - a) = 0$

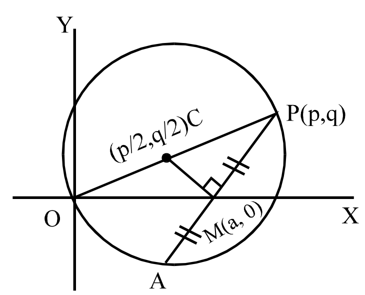
i.e. $2a^2 - 3pa + p^2 + q^2 = 0$ .... (2)

Equation (2) which is a quadratic equation in a shows that there will be two real and distinct values of a if the discriminant is > 0

i.e. if $(3p)^2 - 4 \times 2(p^2 + q^2) > 0$

i.e. if $p^2 > 8q^2$

which is the desired result.



Aliter. Equation of the given circle is

$x^2 + y^2 - px - qy = 0$....... (1)

Equation of any line through P(p, q) can be written as

$y - q = m(x - p)$ (where m is a variable)

i.e. $x = \frac{y + (mp - q)}{m}$ .... (2)

putting the value of x from equation (2) in equation (1) will give the ordinate of the intersection points of the line and the given circle as

$$\left\{\frac{y + (mp-q)}{m}\right\}^2 + y^2 - p\left\{\frac{y + (mp-q)}{m}\right\} - qy = 0$$

i.e. $\{y + (mp - q)\}^2 + m^2y^2 - mp\{y + (mp - q)\} - m^2qy = 0$

i.e. $(1 + m^2)y^2 + \{2(mp - q) - mp - m^2q\}y + (mp - q)^2 - mp(mp - q) = 0$

i.e. $(1 + m^2)y^2 + (pm - 2q - qm^2)y - q(mp - q) = 0$ ....(3)

The above equation gives the Y coordinates of the intersection points of the chord and the given circle. According to the given condition, the mid-point of this intercept lies on the X-axis, therefore we have sum of the roots of equation (3) = 0

i.e. $pm - 2q - qm^2 = 0$

i.e. $qm^2 - pm + 2q = 0$.......(4)

The above equation shows that there will be two real and distinct values of m if $p^2 > 8q^2$ which is the desired result.

77. (A)

Homogenize the equation $3x^2 + 4xy - 4x(2x + y) + (2x + y)^2 = 0$, now, coefficient of x^2 + coefficient of $y^2 = 0$

Thus angle between lines is $\frac{\pi}{2}$

78. (A)

$$S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4}.......\infty \qquad .... (i)$$

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3}..........\infty \qquad ....(ii)$$

from equation (i) and (ii)

$$S\left(1 - \frac{1}{3}\right) = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3}......\infty$$

$$S\left(\frac{2}{3}\right) = \frac{4}{3} + \frac{4}{9}\left(1 + \frac{1}{3} + \frac{1}{3^2}......\infty\right)$$

$$S\left(\frac{2}{3}\right) = 2, \Rightarrow S = 3$$

79. (B)

$$a = 1 + 10 + 10^2 + ... + 10^{54}$$

$$= \frac{10^{55} - 1}{10 - 1} = \frac{10^{55} - 1}{10^5 - 1} \times \frac{10^5 - 1}{10 - 1} = bc$$

80. (A)

$x = 1 + a + a^2$ ....∞

$$x = \frac{1}{1 - a} \Rightarrow 1 - a = \frac{1}{x} \Rightarrow a = \frac{x - 1}{x} \qquad ....(i)$$

$$y = 1 + b + b^2..........\infty$$

$$y = \frac{1}{1 - b} \Rightarrow b = \frac{y - 1}{y} \qquad ....(ii)$$

Given

$$1+ab+a^2b^2......\infty=\frac{1}{1-ab}$$

$$=\frac{1}{1-\frac{(x-1)(y-1)}{xy}}=\frac{xy}{x+y-1}$$

81. (C)

If $a_1, a_2, a_3, ... a_n$ are in H.P.

$\therefore \frac{1}{a_1},\frac{1}{a_2},\frac{1}{a_3}......\frac{1}{a_n}$ are in A.P.

$\therefore \frac{1}{a_2}-\frac{1}{a_1}=d$ where d is common difference of A.P.

$\Rightarrow a_1-a_2=a_1a_2d$ (i)

$\Rightarrow \frac{1}{a_3}-\frac{1}{a_2}=d \Rightarrow a_2-a_3=da_3a_2$ (ii)

$\Rightarrow \frac{1}{a_n}-\frac{1}{a_{n-1}}=d \Rightarrow a_{n-1}-a_n=da_{n-1}a_n$ (iii)

adding all these equation

$$(a_1-a_2)+(a_2-a_3)+(a_3-a_4)+.....+(a_{n-1}-a_n)=d(a_1a_2+a_2a_3.....+a_na_{n-1})$$

$$a_1-a_n=d(a_1a_2+a_2a_3.....+a_na_{n-1})$$

$$(n-1)a_1a_nd=d(a_1a_2+a_2a_3.....+a_na_{n-1})$$

$$(n-1)a_1a_n=(a_1a_2+a_2a_3+a_3a_4.....+a_na_{n-1})$$

82. (A)

83. (A)

Let T_r be the rth term of the given series. Then,

$$T_r=\frac{2r+1}{1^2+2^2+...+r^2}=\frac{6(2r+1)}{(r)(r+1)(2r+1)}=6\left(\frac{1}{r}-\frac{1}{r+1}\right)$$

So, sum is given by

$$\sum_{r=1}^{50}T_r=6\sum_{r=1}^{50}\left(\frac{1}{r}-\frac{1}{r+1}\right)$$

$$=6\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+...+\left(\frac{1}{50}-\frac{1}{51}\right)\right]$$

$$=6\left[1-\frac{1}{51}\right]=\frac{100}{17}$$

84. (D)

$$m_1 + m_2 = \frac{-2h}{b}, m_1 m_2 = \frac{a}{b} \Rightarrow m + 4m = \frac{-10}{1} \Rightarrow m = -2$$

$$\text{and } m \times 4m = \frac{a}{1} \Rightarrow 4(-2)^2 = a \Rightarrow a = 16$$

85. (B)

The given circle $x^2 + y^2 - 4x - 6y - 12 = 0$ has its centre at (2, 3) and radius equal to 5.

Let (h, k) be the coordinates of the centre of the required circle. Then, the point (h, k) divides the line joining (– 1, – 1) to (2, 3) in the ratio 3 : 2, where 3 is the radius of the required circle. Thus, we have

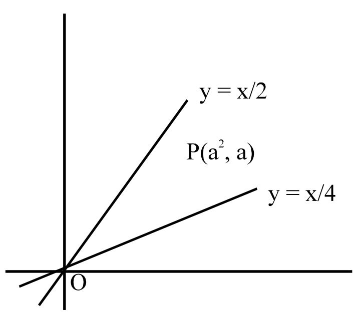
$$h = \frac{3 \times 2 + 2(-1)}{3+2} = \frac{4}{5} \text{ and } k = \frac{3 \times 3 + 2(-1)}{3+2} = \frac{7}{5}$$

Hence, the equation of the required circle is

$$\left(x - \frac{4}{5}\right)^2 + \left(y - \frac{7}{5}\right)^2 = 3^2 \Rightarrow 5x^2 + 5y^2 - 8x - 14y - 32 = 0.$$

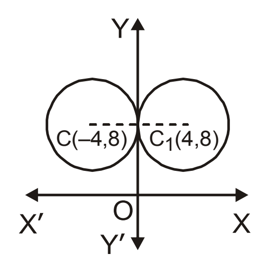
86. (C)

We have $a - \frac{a^2}{4} > 0$ and $a - \frac{a^2}{2} < 0$



$$\Rightarrow \left(a - \frac{a^2}{4}\right)\left(a - \frac{a^2}{2}\right) < 0 \Rightarrow a \in (2, 4)$$

87. (A)



The equation of the given circle is

$x^2 + y^2 + 8x - 16y + 64 = 0$

$\Rightarrow (x^2 + 8x + 16) + (y^2 - 16y + 64) = 16$

$\Rightarrow (x + 4)^2 + (y - 8)^2 = 4^2$

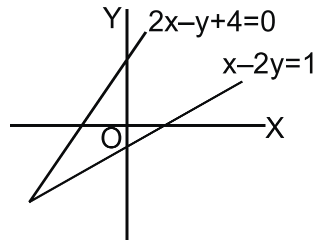
$\Rightarrow \{x - (-4)\}^2 + (y - 8)^2 = 4^2.$

Clearly, its centre is at (–4, 8) and radius = 4.

The image of this circle in the line mirror has its center C_1 (4, 8) and radius 4. So, its equation is $(x - 4)^2 + (y - 8)^2 = 4^2$ or, $x^2 + y^2 - 8x - 16y + 64 = 0$.

88. (B)

Clearly from the figure, the origin is contained in the acute angle. Writing the equations of the lines as 2x – y + 4 = 0 and –x + 2y + 1 = 0, the required bisector is $\frac{2x-y+4}{\sqrt{5}} = \frac{-x+2y+1}{\sqrt{5}}$



89. (C)

As we know that diagonals of a square are perpendicular to each other.

Let the equation of other diagonal is

$$x + 7y = k.$$

Also, passes through (–4, 5).

$\therefore$ $-4 + 35 = k$

$\Rightarrow$ $k = 31$

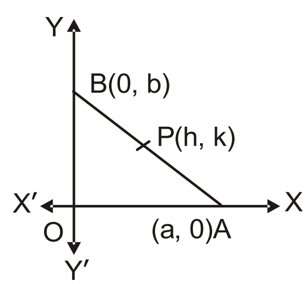
$\therefore$ Required equation is $x + 7y - 31 = 0$

90. (A)

Let the two perpendicular lines be the coordinate axes. Let AB be rod of length l and the coordinates of A and B be (a, 0) and (0, b) respectively.

Let P (h, k) be the mid point of the rod AB in one of the infinte position it attains, then

$$h = \frac{a+0}{2} \text{ and } k = \frac{0+b}{2}$$



$\Rightarrow$ $h = \frac{a}{2}$ and $k = \frac{b}{2}$ ... (i)

From ΔOAB, we have

$AB^2 = OA^2 + OB^2$

$\Rightarrow$ $a^2 + b^2 = l^2$

$\Rightarrow$ $(2h)^2 + (2k)^2 = l^2$

$\Rightarrow$ $4h^2 + 4k^2 = l^2$

$\Rightarrow$ $h^2 + k^2 = \frac{l^2}{4}$.