## SOLUTIONS

# PROGRESS TEST-6 

## GZR-1901 TO 1907

## GZRK-1901 \& 1902 \& GZBS-1901

(JEE MAIN PATTERN)

## Test Date: 16-09-2017

Corporate Office: Paruslok, Boring Road Crossing, Patna-01 Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd.,
Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

## PHYSICS

1. (D)

Mass of hanging portion is $\frac{\mathrm{M}}{3}$ (one-third) and centre of mass c , is at a distance $\mathrm{h}=\frac{\mathrm{L}}{6}$ below the table top.
Therefore, the required work done is,

2. $(A)$

Work $=$ Force $\times$ Camponent of displacement along force
3. (D)

Tension acts perpendicular to displacement. Hence no work.
4. (B)
$x=\frac{t^{4}}{4}$,
$\therefore \mathrm{v}=\frac{\mathrm{dx}}{\mathrm{dt}}=\mathrm{t}^{3}$
$\therefore \mathrm{a}=\frac{\mathrm{dv}}{\mathrm{dt}}=3 \mathrm{t}^{2}$
$\therefore \mathrm{f}=\mathrm{ma}=1 \times 3 \mathrm{t}^{2}$
$\therefore \mathrm{w}=\int \mathrm{F} . \mathrm{vdt}=\int_{0}^{1} 3 \mathrm{t}^{2} \times \mathrm{t}^{3} \mathrm{dt}=\frac{1}{2} \mathrm{~J}$
5. (C)
$\mathrm{w}=\mathrm{Fs} \cos \theta \Rightarrow 25=5 \times 10 \times \cos \theta \Rightarrow \theta=60^{\circ}$
6. (D)
$K=\frac{\mathrm{p}^{2}}{2 \mathrm{~m}}$
$\therefore \frac{\mathrm{K}^{\prime}}{\mathrm{K}}=\frac{(1.5)^{2}}{1} \Rightarrow \mathrm{~K}^{\prime}=125 \%$
7. (B)
$\overrightarrow{\mathrm{s}}=\hat{\mathrm{i}}+\hat{\mathrm{j}}$
$\therefore \mathrm{w}=\overrightarrow{\mathrm{F}} . \overrightarrow{\mathrm{s}}=8 \mathrm{~J}$
From work - energy theorem
$8=\frac{1}{2} \times 1 \times\left(\mathrm{v}^{2}-2^{2}\right) \Rightarrow \mathrm{v}=4.5 \mathrm{~m} / \mathrm{s}$
8. (A)

For tangential forcé W.D. against gravity as well as friction is independent from path followed.
9. (B)

Clearly $\mathrm{mgh} \times 2=\mathrm{Mg}(\mathrm{H}-\mathrm{h}) \Rightarrow \mathrm{h}=\frac{\mathrm{H}}{3}$
$\& v=\sqrt{2 g\left(H-\frac{H}{3}\right)}=2 \sqrt{\frac{2 H}{3}}$
10. (B)
$v=\frac{d t}{d t}=2 t+2, \quad \therefore v_{i}=6 \mathrm{~m} / \mathrm{s} \quad v_{f}=10 \mathrm{~m} / \mathrm{s}$
$\therefore \mathrm{w}=\frac{1}{2} \mathrm{~m}\left(\mathrm{v}_{\mathrm{f}}^{2}-\mathrm{v}_{\mathrm{i}}^{2}\right)=64 \mathrm{~J}$
11. (D)
$a=\frac{v-0}{t_{1}}$
$\therefore F=\frac{m v}{t_{1}}$
Also $s=0+\frac{1}{2}\left(\frac{v}{t_{1}}\right) t^{2}$
$\therefore \mathrm{w}=\frac{\mathrm{mv}}{\mathrm{t}_{1}} \times \frac{1}{2} \cdot \frac{\mathrm{v}}{\mathrm{t}_{1}} \cdot \mathrm{t}^{2}=\frac{1}{2} \frac{\mathrm{mv}^{2}}{\mathrm{t}_{1}^{2}} \times \mathrm{t}^{2}$
12. $(A)$

Displacement $=$ Area $=50 \mathrm{~m}$ (using similar angle trianyle)
Force $=$ mass $\times$ acceleration $=$ mass $\times$ slope $=6 \mathrm{~N}$
$\therefore \mathrm{w}=6 \times 50=300 \mathrm{~J}$
13. (B)
$\overrightarrow{\mathrm{F}}=30\left(\frac{\hat{\mathbf{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}}{\sqrt{3}}\right)=10 \sqrt{3}(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathrm{k}})$
$\overrightarrow{\mathrm{S}}=\overrightarrow{\mathrm{r}}_{2}-\overrightarrow{\mathrm{r}}_{1}=\hat{\mathrm{i}}+2 \hat{\mathrm{k}}$
$\therefore \mathrm{w}=\overrightarrow{\mathrm{F}} . \overrightarrow{\mathrm{s}}=10 \sqrt{3}(1+2)=30 \sqrt{3} \mathrm{~J}$
14. (D)
$w=\frac{1}{2} k(x+y)^{2}-\frac{1}{2} k x^{2}$
15. (B)

$\mathrm{f}_{\mathrm{r}}=\mathrm{mg} \sin \theta$
Work done by fr $=-m g \sin \theta \cos \theta \mathrm{vt}$
$=-\frac{\mathrm{mg} \sin 2 \theta}{2}(\mathrm{v} \times \mathrm{t})$
16. (B)

Clearly $\frac{\sqrt{3}}{2} v=10 \times \frac{1}{2} \Rightarrow v=\frac{10}{\sqrt{3}}$
$\therefore \mathrm{w}=\frac{5 \sqrt{3}-\frac{5}{\sqrt{2}}}{2}=\frac{5}{\sqrt{3}} \mathrm{rad} / \mathrm{sec}$
17. (C)
$\mathrm{t}=\frac{\sqrt{\left(50 \sin 30^{\circ}\right)^{2}+2 \times 10 \times 70}+50 \sin 30^{\circ}}{10}=7 \mathrm{sec}$
18. (C)

Component of velocity in vertical should be same.
19. (D)

From f.b.d.

$\mu=\frac{8 \sqrt{3}}{40} \simeq 0.35$
20. (A)

Resolve the applied force and get normal reaction and limiting friction
21. (C)
$a=g \sin g 45^{\circ}+\mu g \cos 45^{\circ}$
22. (A)

$$
\mu \times \frac{m v^{2}}{R}=m g \Rightarrow v=\sqrt{\frac{g R}{\mu}}
$$

23. (C)

Common acceleration $a=\frac{K A}{2 m}$
$\therefore \mathrm{f}_{\mathrm{r}}=\mathrm{ma}=\frac{\mathrm{KA}}{2}$
24. (C)

Clearly $\mu=\tan \theta=\frac{3}{4}$
25. (C)

Use homogenity of dimensión and use
$\mu \rightarrow$ Dimension less quantity
$\lambda \rightarrow$ meter
26. (B)
$t=\frac{d}{v}$ is possible only when swimmer moves across the river. But there will be drifting in this case.
27. (B)
$F=m \sqrt{a_{t}^{2}+a_{r}^{2}}$
28. (B)

$\therefore \mathrm{S}_{1}+\mathrm{S}_{2}=2 \pi \mathrm{r}$
$\therefore 4 \mathrm{t}+6 \mathrm{t}=2 \pi \mathrm{r}=\mathrm{t}=\frac{2 \pi \mathrm{r}}{10}=\frac{2 \times 3.14 \times 4}{10}=2.5 \mathrm{~s}$
29. (C)
$v \frac{d v}{d x}=2 x+1$
$v d v=(2 x+1) d x$
$\int_{0}^{v} v d v=\int_{0}^{x}(2 x+1) d x \quad \Rightarrow \frac{v^{2}}{2}=x^{2}+x$
30. (C)
$N=m g \cos \theta$
$\mathrm{kx}=\mathrm{N} \sin \theta$

$$
=m g \cos \theta \times \sin \theta
$$

$\mathrm{x}=\frac{\mathrm{mg} \sin ^{2} \theta}{2 \mathrm{k}}$
31. (B)

Root mean square speed $=\mathrm{V}$
$\sqrt{\frac{3 R T}{M}}=V$
Then $\frac{3 R T}{M}=V^{2}$

$$
\begin{aligned}
& \frac{\frac{3 K T_{1}}{\not M}}{\frac{3 K T_{2}}{M M}}=\frac{v^{2}}{x^{2}} \\
& \frac{140}{560}=\left(\frac{v}{x}\right)^{2} \\
& \frac{v}{x}=\sqrt{\frac{1}{4}} \\
& x=2 V
\end{aligned}
$$

32. (C)
33. (B)

$$
\mathrm{T}_{1}<\mathrm{T}_{2}<\mathrm{T}_{3}
$$

34. (B)
$V_{t}=V_{0}+\frac{t V_{0}}{273}$
$V_{40^{\circ}}=V_{0}+\frac{40 V_{0}}{273}$
$V_{41^{\circ}}=V_{0}+\frac{41 V_{0}}{273}$
$V_{41^{\circ}}-V_{40^{\circ}}=\frac{V_{0}}{273}$
35. (C)

$$
\left(P+\frac{a}{V_{m}^{2}}\right)\left(V_{m}-b\right)=R T
$$

for $b=0$,

$$
P V_{m}+\frac{a}{V_{m}}=R T
$$

$$
P V_{m}=R T-\frac{a}{V_{m}}
$$

From graph,
Slope $=-a=\frac{21.6-20.1}{2-3}$
$\therefore \mathrm{a}=1.5$
36. (D)
$V_{n}=2.18 \times 10^{8} \times \frac{Z}{n} \mathrm{~cm} / \mathrm{sec}$
$V_{n} \propto Z ; \quad V_{n}=K Z$

$V_{n}=K \cdot \frac{1}{n}$

37. (D)
$\mathrm{E} \alpha \frac{1}{\lambda}$
$\lambda$ highest than $E$ is minimum
$\mathrm{E}_{5}-\mathrm{E}_{4}<\mathrm{E}_{5}-\mathrm{E}_{3}<\mathrm{E}_{5}-\mathrm{E}_{2}<\mathrm{E}_{5}-\mathrm{E}_{1}$
38. (C)

Higher $(\mathrm{n}+\mathrm{I})$ means higher energy.
39. (A)
$0.5=\frac{0.4 \times V \times 2+50 \times 0.3}{50+V}$
$\mathrm{V}=33.33 \mathrm{ml}$
40. (D)

Let the water added is V ml

$$
\begin{aligned}
& 16 \times 0.5=(16+v) \times 0.2 \\
& 40=16+V \\
& V=24 \mathrm{ml}
\end{aligned}
$$

41. (B)


5, 6-Diethyl-3-methyldec-4-ene
42. (C)

43. (B)

44. (A)

45. (C)

46. (C)

47. (A)

48. (C)

49. (B)

50. (C)

51. (B)

$$
\begin{align*}
& \mathrm{A}+\mathrm{A} \rightarrow \mathrm{~A}^{-}+\mathrm{A}^{+} \\
& \text {I.P. }-\mathrm{E} . \mathrm{A} .=4 \mathrm{eV}  \tag{i}\\
& \mathrm{~A}^{-} \rightarrow \mathrm{A}^{+} \\
& \mathrm{IP}+\mathrm{EA}=10 \mathrm{eV} \tag{ii}
\end{align*}
$$

From (i) and (ii)
$\mathrm{IP}=7 \mathrm{eV}$
$\mathrm{EA}=3 \mathrm{eV}$
52. (C)
53. (D)

I.E. $=\mathrm{Hg}>\mathrm{Zn}>\mathrm{Cd}$
54. (D)

Ge
$\left.\begin{array}{l}\mathrm{Sn} \\ \mathrm{Pb}\end{array}\right\}$ (Exception) Lanthanide Contraction
I. $E_{1}=\mathrm{Ge}>\mathrm{Pb}>\mathrm{Sn}$
55. (C)
56. (A)
57. (B)
58. (B)
(A)


(B)

(C)

(D)


59. (A)

$$
\mathrm{BeF}_{3}^{-}\left\langle\mathrm{Sp}^{2}\right\rangle
$$

60. (B)


## MATHEMATICS

61. (A)
$x=\frac{1}{1^{4}}+\frac{1}{3^{4}}+\frac{1}{5^{4}}+\ldots .$. to $\infty$
$=\left(\frac{1}{1^{4}}+\frac{1}{2^{4}}+\frac{1}{3^{4}}+\ldots\right.$. to $\left.\infty\right)-\left(\frac{1}{2^{4}}+\frac{1}{4^{4}}+\ldots \ldots\right.$. to $\left.\infty\right)$
$=\frac{\pi^{4}}{90}-\frac{1}{16}\left(\frac{1}{1^{4}}+\frac{1}{2^{4}}+\frac{1}{3^{4}}+\ldots\right.$. to $\left.\infty\right)=\frac{\pi^{4}}{90}-\frac{1}{16} \cdot \frac{\pi^{4}}{90}=\frac{\pi^{4}}{96}$
62. (A)
63. (B)

Here $2 \sin \frac{\alpha+\beta}{2} \cdot \cos \frac{\alpha-\beta}{2}=a, 2 \sin \frac{\alpha+\beta}{2} \cdot \sin \frac{\beta-\alpha}{2}=b$
Now, divide and get the value
64. (B)
$\sin n \theta=b_{0}+b_{1} \sin \theta+b_{2} \sin ^{2} \theta+\ldots \ldots$.
This is possible when n is an odd integer.
Put $\theta=0$ to get $b_{0}$. After differentiating w.r.t. $\theta$, put $\theta=0$ to get $b_{1}$.
65. (A)
66. (A)

As the points are in order, the are $\left.=\left|\frac{1}{2}\left\{\left|\begin{array}{ll}4 & 1 \\ 3 & 6\end{array}\right|+\left|\begin{array}{cc}3 & 6 \\ -5 & 1\end{array}\right|+\left|\begin{array}{cc}-5 & 1 \\ -3 & -3\end{array}\right|+\left|\begin{array}{cc}-3 & -3 \\ -3 & 0\end{array}\right|\left|\begin{array}{cc}-3 & 0 \\ 4 & 1\end{array}\right|\right\}\right|\right\}=30$ unit $^{2}$
67. (A)
68. (D)
$\tan \left(180^{\circ}-\theta\right)=$ slope of $A B=-3$
$\therefore \quad \tan \theta=3$
$\therefore \quad \frac{\mathrm{OC}}{\mathrm{AC}}=\tan \theta, \frac{\mathrm{OC}}{\mathrm{BC}}=\cot \theta$
$\Rightarrow \quad \frac{\mathrm{BC}}{\mathrm{AC}}=\frac{\tan \theta}{\cot \theta}=\tan ^{2} \theta=9$
69. (D)

$$
|x|\left(\frac{1+|x|}{x^{2}+x+1}\right) \leq 0 \Rightarrow x=0
$$

70. (C)
71. (C)

For internal point $p(2,8) 4+64-4+32-p<0 \Rightarrow p>96$ and x intercept $=2 \sqrt{1+\mathrm{p}}$ therefore $1+p>0$
$\Rightarrow p>-1$ and $y$ intercept $=2 \sqrt{4+p} \Rightarrow p>-4$
Combining the above conditions on $p$ we get $p>96$ i.e. $p \in(96, \infty)$
72. (A)

Let $d$ be the distance between the centres of two circles of radii $r_{1}$ and $r_{2}$.
These circle intersect at two distinct points if $\left|r_{1}-r_{2}\right|<d<r_{1}+r_{2}$
Here, the radii of the two circles are $r$ and 3 and distance between the centres is 5 .
centres is 5 .
Thus, $|r-3|<5<r+3 \Rightarrow-2<r<8$ and $r>2 \Rightarrow 2<r<8$.
73. (C)

The cosine formula applied to triangle $Q_{1} O Q_{2}$ gives $\cos \angle Q_{2} O Q_{1}=\frac{O Q_{1}^{2}+O Q_{2}^{2}-Q_{1} Q_{2}^{2}}{2 \cdot O Q_{1} \cdot O Q_{2}}$

$$
=\frac{\left(x_{1}-0\right)^{2}+\left(y_{1}-0\right)^{2}+\left(x_{2}-0\right)^{2}+\left(y_{2}-0\right)^{2}-\left[\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y_{2}\right)^{2}\right]}{2 \cdot O Q_{1} \cdot O Q_{2}}=\frac{2\left(x_{1} x_{2}\right)+2\left(y_{1} y_{2}\right)}{2 \cdot O Q_{1} \cdot O Q_{2}}
$$

$\therefore O Q_{1} \cdot O Q_{2} \cos \angle Q_{1} O Q_{2}=x_{1} x_{2}+y_{1} y_{2}$
74. (A)
75. (D)

The equation of the common chord of the circles $x^{2}+y^{2}-4 x-4 y=0$ and $x^{2}+y^{2}=16$ is $x+y$ $=4$ which meets the circle $x^{2}+y^{2}=16$ at points $A(4,0)$ and $B(0,4)$. Obviously $O A \perp O B$. Hence the common chord AB makes a right angle at the centre of the circle $x^{2}+y^{2}=16$.
Hence (D) is the correct answer.
76. (D)

It can be seen that the given points $P(p, q), C\left(\frac{p}{2}, \frac{q}{2}\right)$ and the origin are collinear which implies that line OP where O is the origin is a diameter of the given circle. Therefore, equation of the given circle is

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$$
\bar{x}(x-p)+y(y-q)=0
$$

i.e. $x^{2}+y^{2}-p x-q y=0$

Let $M(a, 0)$ be the mid-point of a chord AP (see fig.). Then, we have

$$
\mathrm{CM} \perp \mathrm{AP}
$$

i.e. slope of $C M \times$ slope of $A P=-1 \Rightarrow \frac{\frac{q}{2}}{\frac{p}{2}-a} \times \frac{q}{p-a}=-1$
i.e. $q^{2}+(p-2 a)(p-a)=0$
i.e. $2 a^{2}-3 p a+p^{2}+q^{2}=0$

Equation (2) which is a quadratic equation in a shows that there will be two real and distinct values of a if the discriminant is $>0$
i.e. if $(3 p)^{2}-4 \times 2\left(p^{2}+q^{2}\right)>0$
i.e. if $p^{2}>8 q^{2}$
which is the desired result.
Aliter. Equation of the given circle is $x^{2}+y^{2}-p x-q y=0 \ldots$


Equation of any line through $P(p, q)$ can be written as
$y-q=m(x-p)$ (where $m$ is a variable)
i.e. $x=\frac{y+(m p-q)}{m} \ldots$
putting the value of $x$ from equation (2) in equation (1) will give the ordinate of the intersection points of the line and the given circle as

$$
\left\{\frac{y+(m p-q)}{m}\right\}^{2}+y^{2}-p\left\{\frac{y+(m p-q)}{m}\right\}-q y=0
$$

i.e. $\{y+(m p-q)\}^{2}+m^{2} y^{2}-m p\{y+(m p-q)\}-m^{2} q y=0$
i.e. $\left(1+m^{2}\right) y^{2}+\left\{2(m p-q)-m p-m^{2} q\right\} y+(m p-q)^{2}-m p(m p-q)=0$
i.e. $\left(1+m^{2}\right) y^{2}+\left(p m-2 q-q m^{2}\right) y-q(m p-q)=0$

The above equation gives the Y coordinates of the intersection points of the chord and the given circle. According to the given condition, the mid-point of this intercept lies on the X-axis, therefore we have sum of the roots of equation (3) $=0$
i.e. $p m-2 q-q m^{2}=0$
i.e. $q m^{2}-p m+2 q=0$.

The above equation shows that there will be two real and distinct values of $m$ if $p^{2}>8 q^{2}$ which is the desired result.
77. (A)

Homogenize the equation $3 x^{2}+4 x y-4 x(2 x+y)+(2 x+y)^{2}=0$, now, coefficient of $x^{2}+$ coefficient of $y^{2}=0$

Thus angle between lines is $\frac{\pi}{2}$
78. (A)
$S=1+\frac{2}{3}+\frac{6}{3^{2}}+\frac{10}{3^{3}}+\frac{14}{3^{4}} \cdots \ldots \infty$
$\frac{1}{3} S=\frac{1}{3}+\frac{2}{3^{2}}+\frac{6}{3^{3}} \ldots \ldots \ldots \infty$
from equation (i) and (ii)
$S\left(1-\frac{1}{3}\right)=1+\frac{1}{3}+\frac{4}{3^{2}}+\frac{4}{3^{3}} \cdots \ldots \infty$
$S\left(\frac{2}{3}\right)=\frac{4}{3}+\frac{4}{9}\left(1+\frac{1}{3}+\frac{1}{3^{2}} \cdots \ldots \infty\right)$
$S\left(\frac{2}{3}\right)=2 \Rightarrow S=3$
79. (B)
$a=1+10+10^{2}+\ldots+10^{54}$
$=\frac{10^{55}-1}{10-1}=\frac{10^{55}-1}{10^{5}-1} \times \frac{10^{5}-1}{10-1}=b c$
80. (A)
$x=1+a+a^{2}$ $\qquad$
$x=\frac{1}{1-a} \Rightarrow 1-a=\frac{1}{x} \Rightarrow a=\frac{x-1}{x}$
$y=1+b+b^{2}$ $\qquad$ .$\infty$
$y=\frac{1}{1-b} \Rightarrow b=\frac{y-1}{y}$
Given

$$
1+a b+a^{2} b^{2} \ldots \ldots \infty=\frac{1}{1-a b}
$$

$$
\begin{aligned}
& =\frac{1}{1-\frac{(x-1)(y-1)}{x y}} \\
& =\frac{x y}{x+y-1}
\end{aligned}
$$

81. (C)

If $a_{1}, a_{2}, a_{3}, \ldots . . a_{n}$ are in H.P.
$\therefore \frac{1}{a_{1}}, \frac{1}{a_{2}}, \frac{1}{a_{3}} \ldots \ldots . \frac{1}{a_{n}}$ are in A.P.
$\therefore \frac{1}{a_{2}}-\frac{1}{a_{1}}=d$ where $d$ is common difference of A.P.

$$
\begin{align*}
& \Rightarrow a_{1}-a_{2}=a_{1} a_{2} d  \tag{i}\\
& \Rightarrow \frac{1}{a_{3}}-\frac{1}{a_{2}}=d \Rightarrow a_{2}-a_{3}=d a_{3} a_{2} \tag{ii}
\end{align*}
$$

$$
\begin{equation*}
\Rightarrow \frac{1}{a_{n}}-\frac{1}{a_{n-1}}=d \Rightarrow a_{n-1}-a_{n}=d a_{n-1} a_{n} \tag{iii}
\end{equation*}
$$

adding all these equation
$\left(a_{1}-a_{2}\right)+\left(a_{2}-a_{3}\right)+\left(a_{3}-a_{4}\right)+\ldots . .+\left(a_{n-1}-a_{n}\right)=d\left(a_{1} a_{2}+a_{2} a_{3} \ldots .+a_{n} a_{n-1}\right)$
$a_{1}-a_{n}=d\left(a_{1} a_{2}+a_{2} a_{3} \ldots . .+a_{n} a_{n-1}\right)$
$(n-1) a_{1} a_{n} d=d\left(a_{1} a_{2}+a_{2} a_{3} \ldots . .+a_{n} a_{n-1}\right)$
$(n-1) a_{1} a_{n}=\left(a_{1} a_{2}+a_{2} a_{3}+a_{3} a_{4} \ldots .+a_{n} a_{n-1}\right)$
82. (B)

As $\log 2, \log \left(2^{x}-1\right)$ and $\log \left(2^{x}+3\right)$ are in A.P.,

$$
\begin{aligned}
& 2 \log \left(2^{x}-1\right)=\log 2+\log \left(2^{x}+3\right) \Rightarrow\left(2^{x}-1\right)^{2}=2\left(2^{x}+3\right) \\
\Rightarrow & 2^{2 x}-4 \times 2^{x}-5=0 \Rightarrow\left(2^{x}-5\right)\left(2^{x}+1\right)=0
\end{aligned}
$$

As $2^{x}$ cannot be negative, we get $2^{x}=5$ or $x=\log _{2} 5$.
Hence (B) is the correct answer.
83. (A)

Let $T_{r}$ be the $r$ th term of the given series. Then,

$$
\begin{aligned}
& T_{r}=\frac{2 r+1}{1^{2}+2^{2}+\ldots+r^{2}} \\
& =\frac{6(2 r+1)}{(r)(r+1)(2 r+1)} \\
& =6\left(\frac{1}{r}-\frac{1}{r+1}\right)
\end{aligned}
$$

So, sum is given by

$$
\begin{aligned}
& \sum_{r=1}^{50} T_{r}=6 \sum_{r=1}^{50}\left(\frac{1}{r}-\frac{1}{r+1}\right) \\
& =6\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{50}-\frac{1}{51}\right)\right] \\
& =6\left[1-\frac{1}{51}\right] \\
& =\frac{100}{17}
\end{aligned}
$$

84. (D)
$m_{1}+m_{2}=\frac{-2 h}{b}, m_{1} m_{2}=\frac{a}{b} \Rightarrow m+4 m=\frac{-10}{1} \Rightarrow m=-2$
and $m \times 4 m=\frac{a}{1} \Rightarrow 4(-2)^{2}=a \Rightarrow a=16$
85. (B)

The given circle $x^{2}+y^{2}-4 x-6 y-12=0$ has its centre at $(2,3)$ and radius equal to 5 .
Let ( $h, k$ ) be the coordinates of the centre of the required circle. Then, the point ( $h, k$ ) divides the line joining $(-1,-1)$ to $(2,3)$ in the ratio $3: 2$, where 3 is the radius of the required circle. Thus, we have

$$
\mathrm{h}=\frac{3 \times 2+2(-1)}{3+2}=\frac{4}{5} \text { and } \mathrm{k}=\frac{3 \times 3+2(-1)}{3+2}=\frac{7}{5}
$$

Hence, the equation of the required circle is

$$
\left(x-\frac{4}{5}\right)^{2}+\left(y-\frac{7}{5}\right)^{2}=3^{2} \Rightarrow 5 x^{2}+5 y^{2}-8 x-14 y-32=0
$$

86. (C)

We have $a-\frac{a^{2}}{4}>0$ and $a-\frac{a^{2}}{2}<0$

$\Rightarrow\left(a-\frac{a^{2}}{4}\right)\left(a-\frac{a^{2}}{2}\right)<0 \Rightarrow a \in(2,4)$
87. (A)

The equation of the given circle is

$$
\begin{aligned}
& x^{2}+y^{2}+8 x-16 y+64=0 \\
\Rightarrow & \left(x^{2}+8 x+16\right)+\left(y^{2}-16 y+64\right)=16 \\
\Rightarrow & (x+4)^{2}+(y-8)^{2}=4^{2} \\
\Rightarrow & \{x-(-4)\}^{2}+(y-8)^{2}=4^{2} .
\end{aligned}
$$

Clearly, its centre is at $(-4,8)$ and radius $=4$.


The image of this circle in the line mirror has its center $C_{1}(4,8)$ and radius 4 . So, its equation is $(x-4)^{2}+(y-8)^{2}=4^{2} \mathrm{or}, x^{2}+y^{2}-8 x-16 y+64=0$.
88. (B)

Clearly from the figure, the origin is contained in the acute angle. Writing the equations of the lines as $2 x-y+4=0$ and $-x+2 y+1=0$, the required bisector is $\frac{2 x-y+4}{\sqrt{5}}=\frac{-x+2 y+1}{\sqrt{5}}$

89. (C)

As we know that diagonals of a square are perpendicular to each other.
Let the equation of other diagonal is

$$
x+7 y=k
$$

Also, passes through $(-4,5)$.

$$
\begin{array}{ll}
\therefore & -4+35=k \\
\Rightarrow & k=31
\end{array}
$$

$\therefore$ Required equation is $x+7 y-31=0$
90. (A)

Let the two perpendicular lines be the coordinate axes. Let AB be rod of length $l$ and the coordinates of $A$ and $B$ be $(a, 0)$ and $(0, b)$ respectively.

Let $P(h, k)$ be the mid point of the $\operatorname{rod} A B$ in one of the infinte position it attains, then

$$
\mathrm{h}=\frac{\mathrm{a}+0}{2} \text { and } \mathrm{k}=\frac{0+\mathrm{b}}{2}
$$

$\Rightarrow \mathrm{h}=\frac{\mathrm{a}}{2}$ and $\mathrm{k}=\frac{\mathrm{b}}{2} \ldots$ (i)


From $\triangle \mathrm{OAB}$, we have

$$
\begin{aligned}
& A B^{2}=O A^{2}+\mathrm{OB}^{2} \\
\Rightarrow & \mathrm{a}^{2}+\mathrm{b}^{2}=\mathrm{l}^{2} \\
\Rightarrow & (2 \mathrm{~h})^{2}+(2 \mathrm{k})^{2}=\mathrm{l}^{2} \\
\Rightarrow & 4 \mathrm{~h}^{2}+4 \mathrm{k}^{2}=\mathrm{l}^{2} \\
\Rightarrow & \mathrm{~h}^{2}+\mathrm{k}^{2}=\frac{l^{2}}{4}
\end{aligned}
$$

