SOLUTIONS

WEEKLY TEST-10

GRS-1801 & GRKS-1801

[Top 170 selected students]

(JEE ADVANCED PATTERN)

Test Date: 29-07-2017

Mentors® Eduserv™

Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd., Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

PHYSICS

1. **(A, B, C)**

2. **(B, D)**

Resistance absorbs energy at the rate of 2W.

Potential difference across AB $\Rightarrow V_{AB}.I = 50$ W

$$V_{AB} = 50 \text{ V}$$

Drop across resistor is 2V, therefore EMF of E is 48 V.

As AB is absorbing energy at the rate of 50 W, 48 W is being absorbed by E. Thus E is on charging.

i.e. current is entering from +ve terminal of E.

3. **(A, B, C, D)**

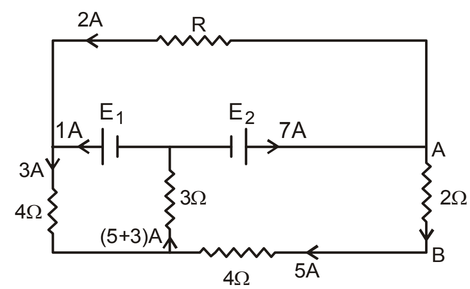
$I_{CD} = 5\,A$

Using KVL

$E_1 = 36\,V$

$E_2 = 54\,V$

$R = 9\,\Omega$



4. **(A, B, C, D)**

A voltmeter should draw minimum current so that current in the circuit is to be altered to minimum possible extent. Hence by using large resistance in series with galvanometer, the accuracy of the voltmeter increases. By using large resistance in series current flowing through the galvanometer decreases, therefore to produce full scale deflection in the galvanometer more potential difference is required across its terminals. Hence range of voltmeter increases.

$\therefore$ (A) and (B) are correct.

Ammeter is connected in series, so it alters the resistance of the circuit. In order to reduce this alteration shunt is connected in parallel to the galvanometer and there by accuracy of the meter is increased. On decreasing shunt resistance, more fraction of current passes through shunt. Therefore to produce full scale deflection in galvanometer more current is required. Hence range increases.

$\therefore$ (C) and (D) are correct.

5. **(A, B, C)**

$I = \frac{dq}{dt} = 3e^{-t}$; $t = 0$ $I = 3$

let potential at P be v.

$\therefore \quad 20 - V = \frac{q}{C} + 3(2)$

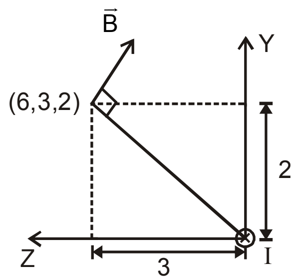
$20 - V = 6$ or $V = 14\,V$

Current thorough capacitive branch = $\frac{18-14}{1}=4A$

$\therefore$ Current through R in 7A & hence $R=\frac{14}{7}=2\Omega$

6. **(C, D)**

$\vec{B}=K(3\hat{j}-2\hat{k})$



So $\vec{v}$ should be parallel or anti-parallel to $\vec{B}$.

7. **(A, B, D)**

When k_2 is closed.

$$C_{eq}=\frac{5C}{3}$$

$$\therefore \quad U_2=\frac{1}{2}\left(\frac{5C}{3}\right)V^2=\frac{5CV^2}{6}.$$

8. **(C,D)**

9. $C=\frac{\in_0 (L-y)L}{L}+\frac{k\in_0 yL}{L}$ **=** $\in_0 (L-y+ky)$ **=** $\frac{\in_0}{9}(t^2-54t+810)$

$\therefore$ **(C)**

10. $q=C\varepsilon$, $i=-\frac{dq}{dt}$, $i=9\in_0 (54-2t)$

$\therefore$ **(B)**

11. **(A)**

12. **(A)**

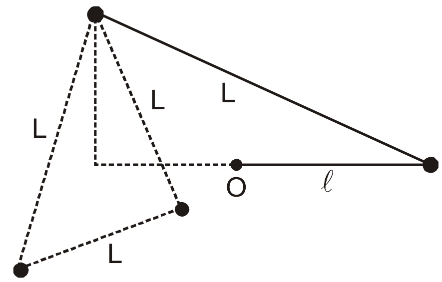
13. **(B)**

In the final state the four charges will lie at the vertices of a tetrahedron with O at geometric centre.

At this state distance between any two charge, will be

$$L=\frac{2\sqrt{2}}{\sqrt{3}}\ell$$

Now final potential energy of system will be



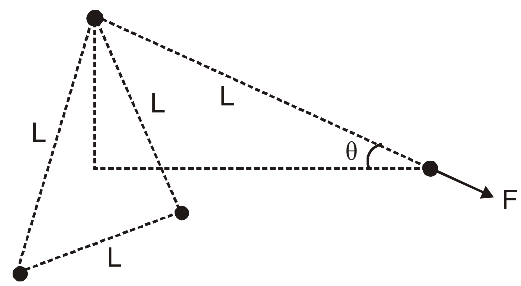
$$U_f = \frac{kq^2}{L} \times 3 \times 4 \times \frac{1}{2} = \frac{6kq^2}{L} = \frac{6kq^2}{\frac{2\sqrt{2}}{\sqrt{3}}\ell} = \frac{3\sqrt{3}\,kq^2}{\sqrt{2}\ell}$$

$$U_i = \frac{kq^2}{\ell} \times 2 \times 3 \times \frac{1}{2} = \frac{3kq^2}{\ell}$$

$\therefore$ work done

$$W_{ext} = U_f - U_i = \frac{3kq^2}{\ell}\left[\frac{\sqrt{3}}{\sqrt{2}} - 1\right]$$

14. (C)



$$T = 3F\cos\theta$$

$$= 3\frac{kq^2}{L^2}\frac{\sqrt{2}}{\sqrt{3}}$$

$$= \frac{3kq^2}{\left(\frac{2\sqrt{2}}{\sqrt{3}}\ell\right)^2}\frac{\sqrt{2}}{\sqrt{3}} \quad = \frac{3\sqrt{3}}{4\sqrt{2}}\frac{kq^2}{\ell^2}$$

15. (B) **16. (D)**

17. (D)

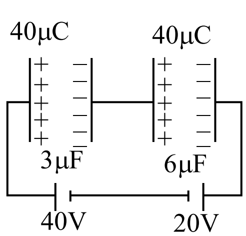
If S_1 is closed, then $\frac{kQ_A}{a} + \frac{kQ}{2a} = 0$ $\quad Q_A = -\frac{Q}{2}$

If S_2 is closed, then $\frac{kQ_B}{2a} = 0$ $\quad Q_B = 0$

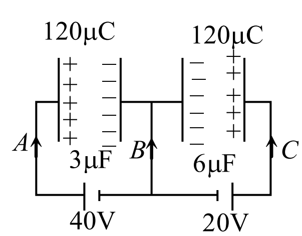
If S_3 is closed, then $\frac{kQ}{3a} + \frac{kQ_C}{3a} = 0$ $\quad Q_C = -Q$

If S_4 is closed, charge on shell *B* is <u>Q</u>

18. (A) A-Q, B-P, C-S, D-R



Before closing



After closing

q_A = 80 mC, q_B = –240 mC q_C = 160 mC

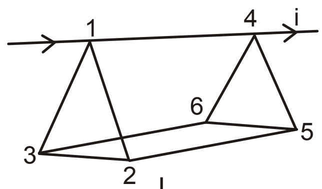
W_1 = 40 ´ 80 = 3200 mJ

W_2 = 20 ´ 160 = 3200 mJ

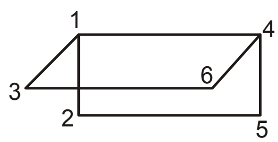
$\therefore$ A-Q, B-P, C-S, D-R

19. **(A)** A-P,R,S; B-P,R; C-P,R,S; D-P,S

20. **(B)** (A- R, S) ; (B- Q,T) ; (C- P) ; (D- P)



can be transformed as



2 & 3 are on same potential
5 & 6 are on same potential.

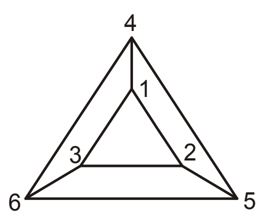
$$\therefore \quad \frac{1}{Reff} = \frac{1}{R} + \frac{1}{3R} + \frac{1}{3R}$$

$$\Rightarrow \quad Reff = \frac{3R}{5} = \frac{3}{5} \times 15 = 9\Omega$$

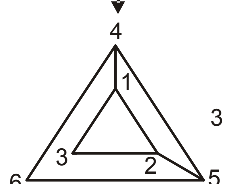
$\therefore$ Current through cell

$$i = \frac{\varepsilon}{Reff} = \frac{24}{9} = \frac{8}{3} A$$

Between 1 & 2

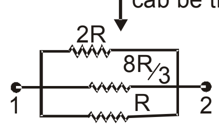


cab be transformed as



3 & 6 are same potential

cab be transformed as



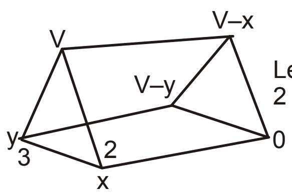
$\therefore \quad \frac{1}{Reff} = \frac{1}{R} + \frac{1}{2R} + \frac{3}{8R}$

$\Rightarrow \quad \frac{1}{Reff} = \frac{8R}{15} = 8\Omega$

$\therefore$ Current through cell

$$i = \frac{24V}{8\Omega} = 3A$$

Between 1 & 5



Let potential of 2 & 3 be 'x' & 'y' respt.

KCL at Node 2 :

$$\frac{x-V}{R} + \frac{x-y}{R} + \frac{x}{12} = 0$$

$\Rightarrow \quad 3x - y = V$ ...(i)

KCL at Node 3 :

$$\frac{y-V}{R} + \frac{y-x}{R} + \frac{2y-V}{R} = 0$$

$\Rightarrow 4y - x = 2V$ ...(ii)

Solving (i) & (ii)

$$x = \frac{18}{33}V$$

$$y = \frac{17}{11}V$$

$\therefore$ Current through cell = $\frac{22V}{11\Omega} = 2A$

CHEMISTRY

21. (A), (B), (C)

Higher the solvation, lower will be the ionic mobility

solvation increases in the order $Li^+ > Na^+ > Cs^+$ and $F^- > Cl^- > Br^- > I^-$

The ionic moblity of H^+ is very high due to proton jump.

Hence, correct order is :

$H^+ > Ci^+ > Na^+ > Li^+$

$I^- > Br^- > Cl^- > F^-$

On increasing temperature, electrolytic resistance decreases, hence, electrolytic conductance increases.

Kohlrausch's law is applicable for both strong as well as weak electrolyte.

22. (A,D)

23. (C), (D)

E_{cell} should be +ve.

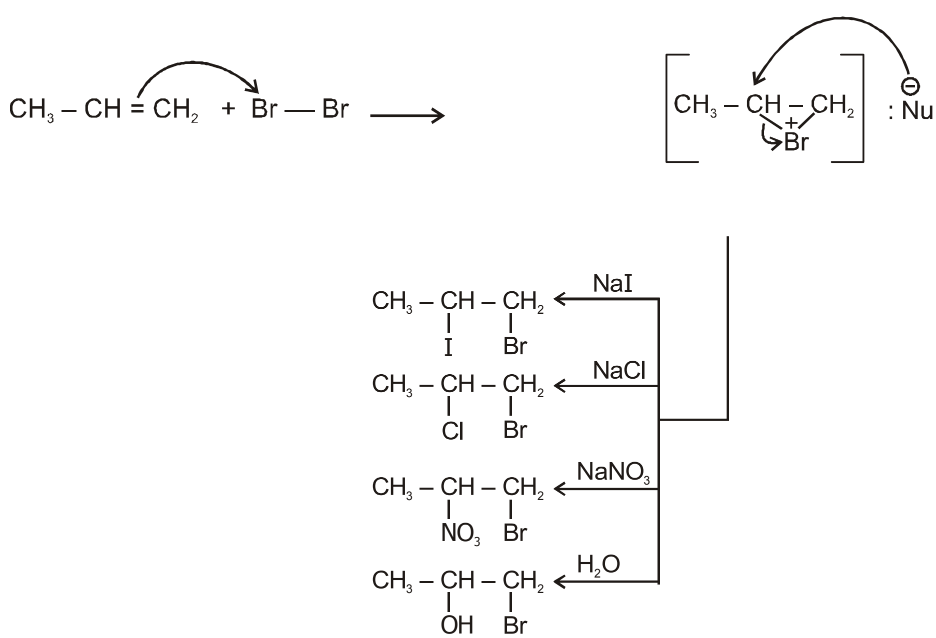
E_{cell} will be negative with F^-/F_2 .

24. (A,B,C)

25. (A, B, C)

More the hyper-conjugation more stable is the compound and less is the $\Delta H_{C.}$

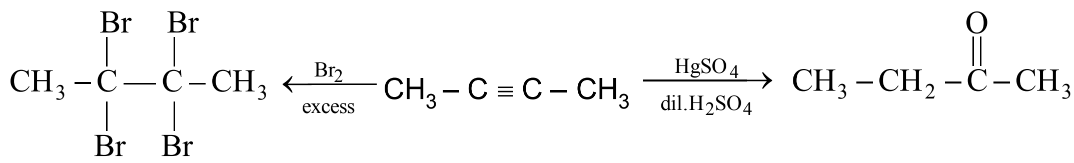
26. (C), (D)

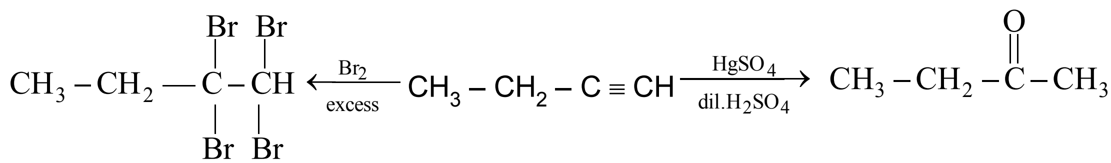


All these products are formed by attack of nucleophile at more stable carbocation in 2nd step.

27. (D)

$C_4H_6 \longrightarrow C_4H_8O$





28. (A), (C), (D)

$\lambda_m = \dfrac{1000 \times K}{M}$, $T \uparrow M \downarrow \lambda_m \uparrow$

Polarity of solvent $\uparrow$ no. of ions $\uparrow \lambda_m \uparrow$

29. (A)

Levigation is process by which lighter earthy particle are free from heavier ore particle by washing with H_2O.

30. (B)

Baeyer process : NaOH

Hall process : Na_2CO_3

31. (A)

When N is connected to SHE electron flow from N to SHE i.e. E^o_{oxi} of N is positive

$N^{+2} + 2e^- \rightarrow N \qquad E_{N^{+2}/N} = -0.25V$

Also reduction potential of M is greater than reduction potential of N (as e^- flow from N to M)

$E^o_{M^{+2}/M} = +0.34V, E^o_{N^{+2}/N} = -0.25V$

Now, $N_{(s)} | N^{+2}(0.1M) || M^{+2}(1M) | M_{(s)}$

$$Ecell = E^o cell - \frac{0.0591}{2}\log_{10}\frac{[N^{+2}]}{[M^{+2}]}$$

$$= (0.34 + 0.25) - \frac{0.0591}{2}\log_{10}\frac{0.1}{1}$$

$Ecell = 0.62V$

32. (D)

$M^{+} + e^{-} \rightarrow M \quad -(1) \qquad E_1^0 = ? \qquad \Rightarrow \Delta G_1^o = -1 \times F \times E_1^o$

$M^{+2} + e^{-} \rightarrow M^{+} - (2) \qquad E_2^0 = 0.15V \qquad \Rightarrow \Delta G_2^o = -1 \times F \times 0.15$

$M^{+2} + 2e^{-} \rightarrow M - (3) \qquad E_3^0 = 0.34V \qquad \Rightarrow G_3^o = -2 \times F \times 0.34$

add eq. (3) - eqn. (2)

$M^{+} + e^{-} \rightarrow M \qquad \Delta G_1^0 = \Delta G_3^0 - \Delta G_2^0$

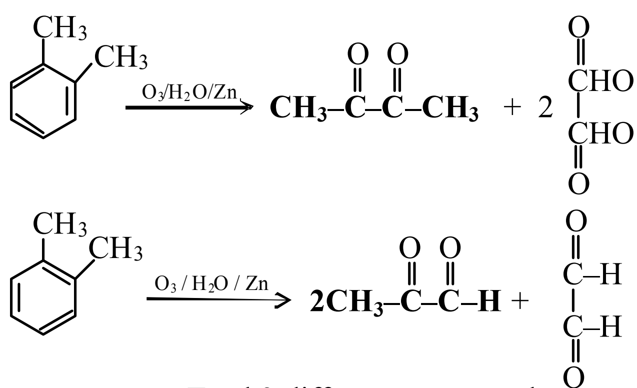
$-1 \times F \times E_1^0 = -2F(0.34) - (-1F \times 0.15)$

$E_1^0 = +0.68 - 0.15$

$E_1^0 = +0.53V$

33. (A)

34. (C)



Total 3 different compounds

35. (C)

36. (C)

37. (B)

(A) — (Q) ; (B) — (R) ; (C) — (S) ; (D) — (P)

(A) For concentration cell,

$$E^o_{cell} = 0,\ E_{cell} = -\frac{0.0591}{n}\log\frac{[\text{Cation}]_{anode}}{[\text{Cation}]_{cathode}}$$

(B) E^o_{red} depend upon nature of substance

(C) Daniel cell : Galvanic cell

(D) H_2 & O_2 is used in fuel cell

38. (C)

(A) — (S); (B) — (Q); (C) — (R); (D) — (P)

Temp. co-efficient = $\frac{\Delta Ecell}{\Delta T} = \frac{(.21-.23)}{308-288K}$ = – 1 × $10^{-3}VK^{-1}$ = –1 mVK^{-1}

$$\Delta G = \Delta H + T\left[\frac{d\Delta G^0}{dT}\right]$$

$$\Delta S = \frac{nFdE^0_{cell}}{dt} = -1 \times 96500 \times 1 \times 10^{-3} = -96.5\ JK^{-1}$$

$$\Delta G^o = \Delta H^o - T\Delta S^o$$

$$-n\overset{0}{\underset{cell}{E}}F = \Delta H^o - T\Delta S^o$$

$$-1 \times 96500 \times 0.23 = \Delta H^o - 288(-96.5)$$

$$\Delta H = -50000J = -50KJ$$

E^o_{cell} = .22V $\qquad$ $AgCl + e \rightarrow Ag^+ + Cl^-$ $\quad E^o = .22V$

$E^o_{cell} = \frac{.0591}{1}\log KSp$ $\qquad$ $Ag \rightarrow Ag^+ + e$ $\quad E^o = -.80V$

$AgCl \rightarrow Ag^+ + Cl^{-1}$ $\quad E^o = -.58V$

$$\log Ksp = \frac{-.58}{.0591} = -9.81$$

39. (A)

(A) — (Q); (B) — (R); (C) — (P); (D) — (S)

40. (B)

(A) – Q; (B) – S; (C) – R; (D) – P

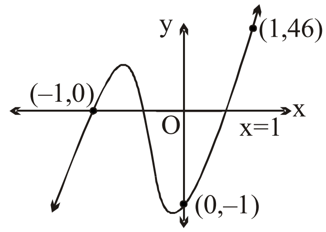
MATHEMATICS

41. **(A,D)**

(A) Clearly $f(-1) = 0$, $f(0) = -1$, $f(1) = 46$

(By using intermediate value theorem)

$\Rightarrow$ All zeroes of the cubic $f(x) = 12x^3 + 24x^2 + 11x - 1$ are confined in $[-1, 1]$



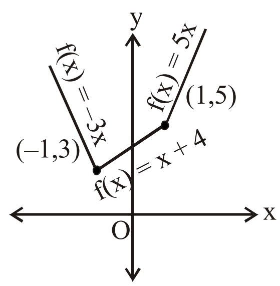
(B) Clearly $f(x) = x - \frac{1}{x}$ is continuous as well as differentiable in the interval $\left[\frac{1}{2}, 3\right]$

Hence LMVT is applicable .

(C) Let $f(x) = x + |2(x + 1)| + 2|x - 1|$

$$\text{Then } f(x) = \begin{cases} x - 2(x+1) - 2(x-1), & x < -1 \\ x + 2(x+1) - 2(x-1), & -1 \le x \le 1 \\ x + 2(x+1) + 2(x-1), & x > 1 \end{cases}$$

$$f(x) = \begin{cases} -3x, & x < -1 \\ x + 4, & -1 \le x \le 1 \\ 5x, & x > 1 \end{cases}$$



Graph of $y = f(x)$ is as shown.

Clearly $y = k$ can intersect

$y = f(x)$ at exactly one point only if $k = 3$ **Ans.**]

(D) Let $g(x) = e^x f(x)$; then $g'(x) = e^x (f(x) + f'(x)) \le e^x$.

Integrating with respect to x from 0 to 1, we get

$g(1) - g(0) = \int_0^1 g'(x)dx \le \int_0^1 e^x dx = e - 1$. But $g(1) - g(0) = e \cdot f(1)$, so we get $f(1) \le \frac{(e-1)}{e}$

42. **(B)**

$x . \int \frac{dx}{x} = x(ln|x| + C) = x\, ln|x| + Cx$

43. **(A)**

$ln(2(1 + x)) = t$; $\frac{1}{1+x} dx = dt$

44. **(B)**

$f(f(x)) = [1 - \{(1 - x^n)^{1/n}\}^n]^{1/n}$

$\Rightarrow\ f(f(x)) = x,$

$\Rightarrow\ f(f(f(x))) = f(x)$

$\Rightarrow\ f(f(f(f(x)))) = f(f(x)) = x$, and so on

$\therefore\ I = \int x\,dx = \frac{x^2}{2} + C$

45. (B, D)

$$\int \frac{\cos x + x\sin x}{x(x+\cos x)}dx = \int \frac{(x+\cos x) - x + x\sin x}{x(x+\cos x)}dx$$

$= \int \frac{1}{x}dx - \int \frac{1-\sin x}{x+\cos x}dx$ = log |x| – log |x+cosx| + c. Hence $f'\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$.

46. (A,C,D)

$f(x+y) = 2^x f(y) + 4^y f(x)$

Interchaning x and y, we get $f(x+y) = 2^y f(x) + 4^x f(y)$

$$\frac{f(x)}{4^x - 2^x} = \frac{f(y)}{4^y - 2^y} = k$$

$f(x) = k\left(4^x - 2^x\right)$

since $f'(0) = \ell n2$ we get k = 1

$f(x) = 4^x - 2^x$

47. (B, C)

$f'(x) = e^{\tan x}(1 - \sin 2x) \geq 0\ \forall\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ f is increasing in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$f'(x) = 0 \quad \Rightarrow\ x = \frac{\pi}{4}$

48. (B)

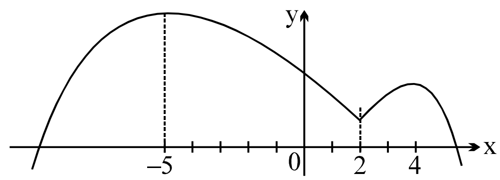
$I = \int \tan^3 x(\sec^2 x - 1)\,dx$

$= \int \tan^3 x \sec^2 x\,dx - \int \tan^3 x\,dx$

$= \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \ell n|\sec x| + C$

49. (D)

50. (C)



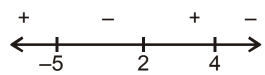
$\because$ $f''(x)$ is negative in $R - \{2\}$

$\therefore$ f' is decreasing in $(-\infty, 2)$ and in $(2, \infty)$

$f'(-5) = 0 \Rightarrow f'(x)$ is positive in $(-\infty, 5)$ and negative in (5, 2)

also $f'(4) = 0 \Rightarrow f'(x)$ is positive in (2, 4) and negative in $(4, \infty)$

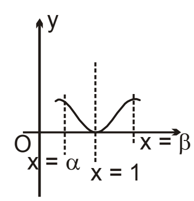
sign of f'



$\therefore$ f has a local maxima at x = –5, local minima at x = 2 and local maxima at x = 4.

51. (D)

Since f(x) is symmetric about x = 1 and it is twice differentiable. so $f'(x)$ must have one root at x = 1.



$f'(x) = a(x-1)(x-\alpha)(x-\beta) = a(x-1)\left(x^2 - (\alpha+\beta)x + \alpha\beta\right)$

Here $\frac{\alpha+\beta}{2} = 1$ so $\alpha + \beta = 2$

$f'(x) = a\left(x^3 - 3x^2 + (\alpha\beta + 2)x - \alpha\beta\right)$

$f''(2) = 0 \Rightarrow \alpha\beta = -2$

$\therefore$ Sum of roots of $f'(x) = 0$ is 1 + α + β i.e., 3.

52. (C)

$f'(x) = a(x-1)\left(x^2 - 2x - 2\right) = a(x-1)\left((x-1)^2 - 3\right)$

$$f(x) = a\left(\frac{(x-1)^4}{4} - \frac{3}{2}(x-1)^2\right) + C$$

$\because f(1) = 0$ so $c = 0$; $f(2) = 1$ so $a = -\frac{4}{5}$

$$f(x) = -\frac{4}{5}\left[\frac{(x-1)^4}{4} - \frac{3}{2}(x-1)^2\right]$$

$$f(3) = \frac{8}{5}$$

57. (B)

Let Asinx + Bcosx = a (3sinx – 4cosx) + b(3cosx + 4sinx)

$\Rightarrow$ 3a + 4b = A and – 4a + 3b = B

I = ax + b ln |3sinx – 4cosx| + C

If a = 1 and b = 1 then A = 7, B = –1

If a = –1 and b = –1, then A = –7, B = 1

If a = 3 and b = –1, then A = 5, B = –15

If a = 2 and b = –2, then A = –2, B = –14

58. (A)

(P) Let $\lim\limits_{x\to\infty} f(x) = l$, then $l = 2l - \dfrac{4}{l}$ $\Rightarrow$ $l = 2$

(Q) $$\lim_{h\to 0}\frac{f(h^2+h+2)-f(2)}{f(1-2h)-f(1)} = \lim_{h\to 0}\frac{f'(h^2+h+2)(2h+1)}{f'(1-2h)(-2)} = \frac{f'(2)}{-2f'(1)} = \frac{-1}{4}$$

(R) $$\lim_{x\to 0}\left[\frac{x^3}{x-\sin x}\right] = \lim_{x\to 0}\left[\frac{x^3}{x-\left(x-\frac{x^3}{\lfloor 3}+\frac{x^5}{\lfloor 5}-....\right)}\right] = \lim_{x\to 0}\left[\frac{1}{\frac{1}{\lfloor 3}-\frac{x^2}{\lfloor 5}+...}\right] = 6$$

(S) $$\lim_{x\to 0^+}\frac{\sqrt{\tan x - x} - ax^{3/2}}{x^b} = \lim_{x\to 0^+}\frac{\sqrt{\frac{1}{3}x^3+\frac{2}{15}x^5+...}-ax^{3/2}}{x^b}$$

$$= \lim_{x\to 0^+}\frac{\frac{1}{\sqrt{3}}\left(1+\frac{2}{5}x^2+...\right)^{1/2}-a}{x^{b-\frac{3}{2}}} = \lim_{x\to 0^+}\frac{\frac{1}{\sqrt{3}}\left(1+\frac{1}{5}x^2+...\right)-a}{x^{b-\frac{3}{2}}}$$

$$= \lim_{x\to 0^+}\frac{\frac{1}{5\sqrt{3}}x^2+...}{x^{b-\frac{3}{2}}} \qquad \left(\frac{1}{\sqrt{3}}-a=0 \Rightarrow a=\frac{1}{\sqrt{3}}\right)$$

$\therefore\ b-\dfrac{3}{2}=2 \Rightarrow b=\dfrac{7}{2}$ $\qquad\qquad$ $\therefore\ a^2+b=\dfrac{1}{3}+\dfrac{7}{2}=\dfrac{23}{6}$

59. (B)

(P) If $\dfrac{\pi}{4} < x < \dfrac{3\pi}{8}$, then sin x > cos x

$\therefore$ $\displaystyle\int\frac{\sin x-\cos x}{|\sin x-\cos x|}\,dx = \int 1.\,dx = x+c$

(Q) $\int \frac{x^2\,dx}{(x^3+1)(x^3+2)} = \frac{1}{3}\int 3x^2\left(\frac{1}{x^3+1}-\frac{1}{x^3+2}\right)dx = \frac{1}{3}\,\ell n\left|\frac{x^3+1}{x^3+2}\right| + c$

$\therefore\ f(x) = \ell n\ |x|$

(R) $f(x) = \int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx = \int (\tan x)^{\frac{-1}{2}}\sec^2 x\,dx = 2\sqrt{\tan x} + c$

(S) $\int \frac{dx}{x\ell n|x|} = \ell n\ |\ell n|x|| + c$

$\therefore\ f(x) = \ell n|x|$

60. (D)

(P) $f(x) = x^{4/3} - 4x^{1/3}$

$f'(x) = \frac{4}{3}\left(\frac{x-1}{x^{2/3}}\right)$

Sign of $f'(x)$: $-$ | $-$ | $+$ at points 0, 1

(Q) $f(x) = 5x^{2/5} - x^2$

$f'(x) = 2\left(\frac{1-x^{8/5}}{x^{3/5}}\right) = 2\left(\frac{(1-x^{1/5})(1+x^{1/5})(1+x^{2/5})(1+x^{4/5})}{x^{3/5}}\right)$

Sign of $f'(x)$: $+$ | $-$ | $+$ | $-$ at points -1, 0, 1

(R) $f(0) = \lim_{x\to 0}\frac{1}{x}\log\left(\frac{e^x-1}{x}\right) = \lim_{x\to 0}\frac{1}{\frac{e^x-1}{x}}\cdot\frac{xe^x-(e^x-1)}{x^2} = \frac{1}{2}$

(S) $f(x) = 3x^{2/3} - x^2$

$f'(x) = 2\left(\frac{1-x^{4/3}}{x^{1/3}}\right) = 2\frac{(1-x^{1/3})(1+x^{1/3})(1+x^{2/3})}{x^{1/3}}$

Sign of $f'(x)$: $+$ | $-$ | $+$ | $-$ at points -1, 0, 1