# SOLUTIONS

# WEEKLY TEST-9

## GZRA-1901 & 1902

## (JEE ADVANCED PATTERN)

## Test Date: 29-07-2017

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# PHYSICS

**1. (A, D)**

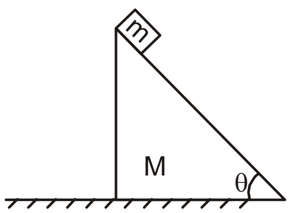
**2. (A, B)**

**3. (A, B, D)**

**4. (A, B)**

Let the acceleration of wedge be $a_0$ in backward direction.

Consider the motion of smaller block from the frame of wedge.



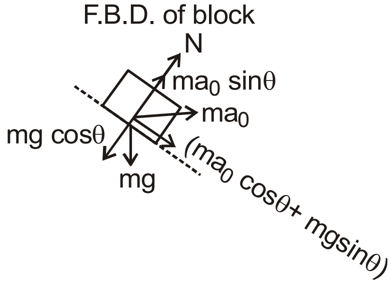

Component of force parallel to incline; $ma_0\cos\theta + mg\sin\theta = ma$

$\Rightarrow\ a = a_0\cos\theta + g\sin\theta.$ .....(i)

Component of force perpendicular to incline

$N + ma_0\sin\theta = mg\cos\theta$ .....(ii)

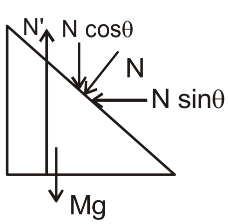
F.B.D. of block




Horizontal components give,

$N\sin\theta = Ma_0 \Rightarrow N = Ma_0/\sin\theta$ .....(iii)

F.B.D. of wedge



Putting in (ii);

$$\frac{Ma_0}{\sin\theta} + Ma_0 \sin\theta = mg\cos\theta$$

$$\Rightarrow a_0 = \frac{mg\sin\theta\cos\theta}{M + m\sin^2\theta} \text{ and } a = \frac{(M+m)g\sin\theta}{M + m\sin^2\theta} \text{ in wedge frame.}$$

**5. (A,C)**

T = kx

$m_2 g = kx$

Just after cutting the string S, tension in it will be zero but spring will not loss its force instantaneously.

so $a_1 = \frac{kx}{m_1} = \frac{m_2 g}{m_1} = \left(\frac{m_2 g}{m_1}\right)$ and so block of mass $m_2$ will have acceleration $a_2 = 0$

**6. (A, B, C)**

$\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{A} + \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \hat{i}(0) + \hat{j}(1-2) + \hat{k}(1) = \left(-\hat{j} + \hat{k}\right)$$

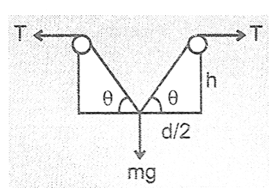
$\therefore$ Correct choices are (A), (B) and (C).

**7. (A, D)**

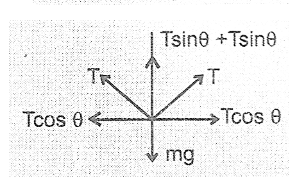
If initially acceleration of A is greater than that of B, then there will be extension and if that of B is greater than A, then there will be compression in the spring. Otherwise the length of spring will remain same

**8. (B, C)**

$$\tan\theta = \frac{2h}{d}$$




$$\cos\theta = \frac{d}{2\sqrt{\frac{d^2}{4} + h^2}}$$




$$\sin\theta = \frac{h}{\sqrt{h^2 + \frac{d^2}{4}}}$$

as man moves slowly $2T\sin\theta = mg$

$$T = \frac{mg}{2\sin\theta}$$

as man moves upward $\theta$ becomes small

$\therefore$ $\sin\theta$ decreases

$\Rightarrow$ T increases

$$T = \frac{mg}{2\times h}\sqrt{h^2 + \frac{d^2}{4}} = \frac{mg\sqrt{d^2+4h^2}}{4h}$$

**9.** **(A)**

**10.** **(A)**

**From constraint relation ;**

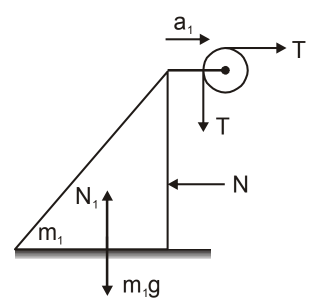
$Tx_1 + Tx_2 + Tx_3 = 0$

by double diff$^{n.}$

$Ta_1 + Ta_2 + Ta_3 = 0$

$\therefore$ $a_1 + a_2 + a_3 = 0$ ...(A)

**F.B.D. of wedge of mass $m_1$ ;**




$T - N = m_1a_1$

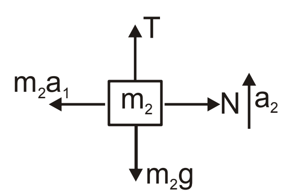
$N = m_2a_1$

$\therefore$ $T = m_1a_1 + m_2a_1$

$= a_1(1+2) = 3a_1$

T = $3a_1$ ...(i)

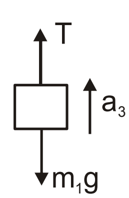
**F.B.D. mass $m_2$ ;**




$T - m_2g = m_2a_2$

$T - 2g = 2a_2$ ...(ii)

**F.B.D. of mass $m_3$ ;**




$T - m_3g = m_3a_3$

$T - 3g = 3a_3$ ...(iii)

**From (i) , (ii) & (iii) and putting in equation (A)**

$$\frac{T}{3}+\frac{T}{2}-g+\frac{T}{3}-g=0$$

$$\frac{2T}{3}+\frac{T}{2}=2g \Rightarrow \frac{4T+3T}{6}=2g$$

$$\Rightarrow \quad T=\frac{6\times2\times10}{7}=\frac{120}{7}N$$

$$a_1=\frac{T}{3}=\frac{40}{7}\ ms^{-2}$$

$$a_2=\frac{T}{2}-g=\frac{60}{7}-10=\frac{-10}{7}ms^{-2}$$

so, $acc^n$ of mass $m_2$;

$$\sqrt{a_2^2+a_1^2}=\sqrt{\left(\frac{40}{7}\right)^2+\left(\frac{10}{7}\right)^2}$$

$$=\frac{10}{7}\sqrt{16+1}=\frac{\sqrt{17}}{7}g\ ms^{-2}$$

$acc^n$ of mass $m_3$;

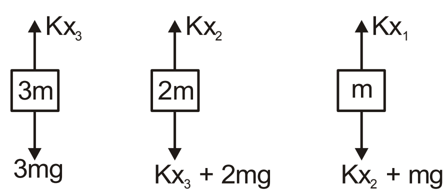
$$a_3=\frac{T}{3}-g=\frac{40}{7}-10$$

$$=-\frac{30}{7}ms^{-2}$$

**11. (A)**

**12. (A)**

**Concept :** Spring force does not change instantaneously. As a first step find tension in all the springs.




(b)

Form FBD of blocks we get

Block C $3mg = Kx_3$ ...(1)

Block B $2mg + Kx_3 = Kx_2$

$2mg + 3mg = Kx_2 \Rightarrow 5mg = Kx_2$ ...(2)

Block A $Kx_1 = Kx_2 + mg$ ...(3)

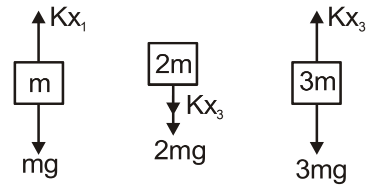
when spring 2 is cut spring force in other two strings remain unchanged, at that instant.

$Kx_1 - mg = ma_3$

$\Rightarrow \quad a_3 = 5g \uparrow$

$Kx_3 + 2mg = 2ma_2$

$\Rightarrow a_2 = \frac{5g}{2} \downarrow$




(c)

acceleration of 3 m will be zero.

**13. (A)**

**14. (D)**

**15. (A)**

**16. (C)**

**17. (A)**

$F = (m_1 + m_2)a$ ...(i)

$T\sin\theta = m_2 a$ ...(ii)

$T\cos\theta = m_2 g \quad \Rightarrow T = m_2 g\sec\theta$

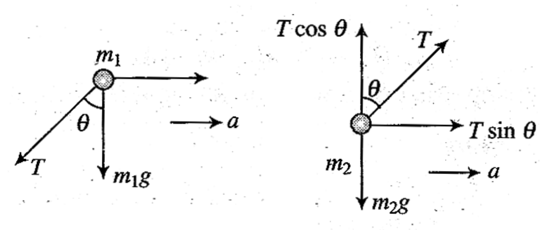
From (ii) and (iii), $a = g\tan\theta$

Put in (i), $F = (m_1 + m_2)g\tan\theta$

Net force acting on $m_2 = m_2 a = \frac{m_2 F}{m_1 + m_2}$

Force acting on $m_1$ by wire.

$m_1 g + T\cos\theta = m_1 g + m_2 g$

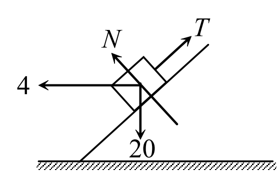



**18. (B)**

Acceleration of 2kg block relative to wedge = 2 m/s²

Acceleration of 2 kg block relative to ground

$= \sqrt{2^2 + 2^2 + (2 \times 2 \times 2 \times \cos 120^\circ)}$ = 2 m/s²




$20\sin 60^\circ + 4\cos 60^\circ - T = 4$

$T = 10\sqrt{3} - 2$

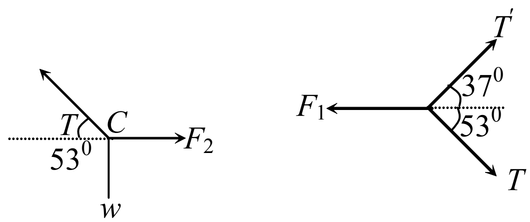
$N = 20\cos 60° - 4\sin 60° = 10 - 2\sqrt{3}$ N

Net force = 2 × 2 = 4 N

$\therefore$ **(A) – Q , (B) – P , (C) – R , (D) – S**

**19. (D)**

$$T = \frac{w}{\sin\left(180 - 53^0\right)} = \frac{5w}{4}$$




$$F_2 = T\sin\left(180 - 37^0\right) = \frac{3w}{4}$$

and $F_1 = \dfrac{T}{\sin\left(180 - 37^0\right)} = \dfrac{5T}{3} = \dfrac{25}{12}w$

$$\frac{T'}{\sin\left(180^0 - 53^0\right)} = \frac{F_1}{\sin 90^0} \quad \Rightarrow \quad T' = \frac{25}{12}w \times \frac{4}{5} = \frac{5w}{3}$$

$\therefore$ **(A)- Q, (B)- S, (C)- P, (D)-R**

**20. (A)**

In condition (a), by constraint relation $\dfrac{a_1}{a_2} = 4$

In condition (b), by constraint relations $\dfrac{a_1}{a_2} = \dfrac{1}{3}$

In condition (a), tension in string of $m_2$ is $T = \dfrac{16m_1m_2g}{(16m_1 + m_2)} = 32$ N

In condition (b), tension in string connecting $m_2$ is $T = \dfrac{4m_1m_2g}{(m_1 + 9m_2)} = \dfrac{160}{37}$ N

$\therefore$ **(A) – Q; (B) – P; (C) – R; (D) – T**

# CHEMISTRY

**21. (A,B)**

Higher $\rho$ means lower h,V.P. of Hg is very low.

**22. (A,B,D)**

P = $\rho$ gh (P independent of A) and $\rho \alpha \frac{1}{h}$

more volatile liquid means higher V.P. so more error. mg force acts always downward.

**23. (A,B,C)**

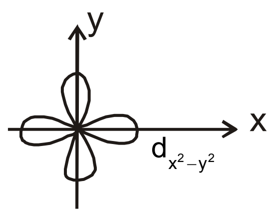
$\log_{10}^{P} = -\log_{10}^{V} + \log_{10}^{K}$

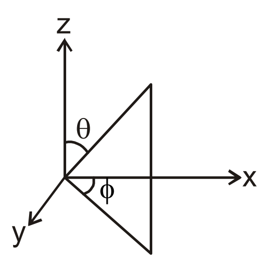
$P = \frac{K}{V}$ or $\frac{P}{V} = \frac{K}{V^2}$

PV = constant

$\frac{1}{P} \times \frac{1}{V} = \frac{1}{K}$

**24. (A,B,D)**




**25. (A,B,C)**

**26. (A,B,C)**

Electron has wave character (de-Broglie), $\lambda = \frac{h}{mv}$

**27. (A,B,C)**

**28. (A,B,C,D)**

**29. (A)**

**30. (B)**

**31. (A)**

**32. (C)**

**33. (B)**

Difference in vertical height = 5 cm.

**34. (D)**

Vertical difference of height is 5 cm.

Now,

$$\rho_1 h_1 = \rho_2 h_2$$

$$5 \times \rho_{Hg} = \frac{\rho_{Hg}}{2} \times h_2$$

or, $h_2$ = 10 cm

$$l = \frac{10}{\sin 30^o} = 20\text{cm}$$

**35. (A)**

$P_{bottom}$ = (76 + 5 + 10) cm of Hg

= 91 cm of Hg

$P_X$ = (76 + 5) cm of Hg = 81 cm of Hg

**36. (B)**

$$V_X = \frac{P_1 V_1}{P_X}$$

$$V_Y = \frac{P_1 V_1}{P_Y}$$

$$\frac{V_X}{V_Y} = \frac{P_Y}{P_X} = \frac{76}{81}$$

**37. (A)**

**(A) — (R); (B) — (P); (C) — (S); (D) — (Q)**

Given ratio = $\frac{\ell}{n - \ell - 1}$

**38. (B)**

**(A) — (Q); (B) — (S); (C) — (P); (D) — (R)**

**39. (D)**

**(A) — (R); (B) — (S); (C) — (P); (D) — (Q)**

**40. (C)**

**(A) — (R); (B) — (Q); (C) — (P); (D) — (S)**

# MATHEMATICS

**41. (B, D)**

A(– 5, 0); B(3, 0); C(a, a – 2)

Area = $\frac{1}{2}\begin{vmatrix} -5 & 0 & 1 \\ 3 & 0 & 1 \\ a & (a-2) & 1 \end{vmatrix} = \pm 20$ $\Rightarrow \begin{vmatrix} -5 & 0 & 1 \\ 8 & 0 & 0 \\ a-3 & a-2 & 0 \end{vmatrix} = \pm 40$

$\Rightarrow$ 8a – 16 = 40 or 8a – 16 = – 40 $\Rightarrow$ (a = 7) or (a = – 3)

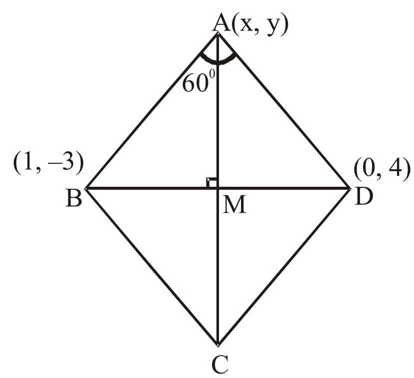
$\therefore$ C(7, 5) or C(– 3, – 5)

**42. (A, B, D)**

$$P(x) = \left(1+\cos\frac{\pi}{6x}\right)\left(1+\sin\frac{\pi}{6x}\right)\left(1-\sin\frac{\pi}{6x}\right)\left(1-\cos\frac{\pi}{6x}\right)$$

$$P(x) = \frac{1}{4}\sin^2\left(\frac{\pi}{3x}\right)$$

**43. (A,B)**




BD = $\sqrt{1^2+7^2} = 5\sqrt{2}$

BM = $\frac{5}{\sqrt{2}}$

$\frac{AM}{BM}$ = tan 60º

$M\left(\frac{1}{2},\frac{1}{2}\right)$

for point A

$$\frac{x-\frac{1}{2}}{\cos\theta} = \frac{y-\frac{1}{2}}{\sin\theta} = \pm\frac{5\sqrt{6}}{2}$$

$m_{BD}$ = $\frac{7}{-1}$

$m_{AC} = \frac{1}{7}$ = tan θ

$\sin\theta = \frac{1}{\sqrt{50}} = \frac{1}{5\sqrt{2}}$

$\cos\theta = \frac{7}{5\sqrt{2}}$

**44. (B, C)**

$y - y_1 = mx - mx_1$

$\Rightarrow mx - y - mx_1 + y_1 = 0$

A set of parallel lines

If $x = x_1$,

$mx_1 - y - mx_1 + y_1 = 0$

$\Rightarrow \; y = y_1$

Hence, for all values of $y_1$, all will intersect

$x = x_1$

**45. (A, C)**

The given equation can be written as

$$\log_5\left(\frac{5^{1/x}+125}{6}\right) = 1+\frac{1}{2x} \Rightarrow \frac{5^{\frac{1}{x}}+125}{6} = 5^{1+\frac{1}{2x}} \Rightarrow 5^{\frac{1}{x}}+125 = 6.5.5^{\frac{1}{2x}}$$

Let $t = 5^{\frac{1}{2x}}$, then $t^2 + 125 = 30t \Rightarrow t^2 - 30t + 125 = 0 \Rightarrow t = 25$ or $t = 5$

**46. (A, B, C, D)**

$|x^2 - 9x| + |x^2 - 5x + 4| > 4|x+1|$ is true if

$(9x - x^2)(x^2 - 5x + 4) < 0$


$\therefore x \in (-\infty, 0) \cup (1, 4) \cup (9, \infty)$

**47. (A, B, C)**

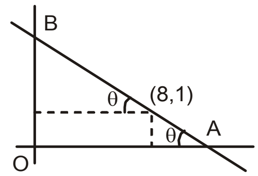
(B) $\log_{\cos\frac{7\pi}{4}}\left(\sin\frac{5\pi}{6}\right) = \log_{\frac{1}{\sqrt{2}}}\left(\frac{1}{2}\right) > 0$

(C) $\log_{\tan\frac{4\pi}{3}}\left(\cot\frac{7\pi}{6}\right) = \log_{\sqrt{3}}\sqrt{3} = 1 > 0$

**48. (A,C,D)**

**49. (D)**

OA = 8 + cotθ ; OB = 1 + 8tanθ




$\Delta = \frac{1}{2}(1 + 8\tan\theta)(8 + \cot\theta)$

$= 8 + \frac{1}{2}(64\tan\theta + \cot\theta)$

For Δ to be minimum tanθ = 1/8

$\therefore\ \Delta_{min} = 16$

**50. (A)**

$z = AB = \text{cosec}\theta + 8\sec\theta \Rightarrow \frac{dz}{d\theta} = 0$; cotθ = 2

$\therefore\ AB_{min} = 5\sqrt{5}$

**51. (A)**

x – 1 = 3 cosθ

y – 2 = 4sinθ

x + y = 3 + 3cosθ + 4sinθ

maximum value = 3 + 5 = 8

**52. (B)**

$$\sin x\cos\frac{\pi}{6} + \cos x.\sin\frac{\pi}{6} + \cos x = \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x + \cos x = \frac{\sqrt{3}}{2}\sin x + \frac{3}{2}\cos x$$

maximum value $\sqrt{\frac{3}{4} + \frac{9}{4}} = \sqrt{3}$

**53. (B)**

Image of A(1, 3) in line x + y = 2 is $\left(1 - \frac{2(2)}{2}, 3 - \frac{2(2)}{2}\right) \equiv (-1,\ 1)$

So line BC passes through (–1, 1) and $\left(-\frac{2}{5}, -\frac{2}{5}\right)$.

Equation of line BC is $y - 1 = \frac{-2/5 - 1}{-2/5 + 1}(x + 1) \Rightarrow 7x + 3y + 4 = 0$

**54. (C)**

Vertex B is point of intersection of 7x + 3y + 4 = 0 and x + y = 2

i.e. B = (–5/2, 9/2)

**55. (B)**

G1 G2 ....... Gn = $\left(\sqrt{1\times 1024}\right)^n = 2^{5n}$

$\therefore$ $2^{5n} = 2^{45}$

$\therefore$ n = 9

**56. (B)**

$A_1 + A_2 + A_3 + \ldots\ldots + A_{m-1} + A_m = 1025 \times 171$

$\therefore$ $m\left(\frac{-2+1027}{2}\right) = 1025 \times 171$

$\therefore$ m = 342

**57. (A)**

**P).** Let $x = 4\cos\theta, y = 3\sin\theta,$ than $x + y = 4\cos\theta + 3\sin\theta \le 5$

$\therefore$ $\log_5(x+y) \le 1$

**Q).** $\because$ $2^{\sqrt{\log_2 3}} = \left(2^{\log_2 3}\right)^{\frac{1}{\sqrt{\log_2 3}}} = 3^{\sqrt{\log_3 2}}$ and $3^{\log_3 2} - 2^{\log_2 3} = 2 - 3 = -1$

$\therefore$ $\alpha + \beta = 3$ and $\alpha\beta = 2$

$\Rightarrow$ $\alpha^2 + \alpha\beta + \beta^2 = (\alpha+\beta)^2 - \alpha\beta = 7$

**R).** $3\log_{18} 96 \log_{12} 3 + \log_{18} 96 + 3\log_{12} 3$

$= \left(\log_{18} 96 + 1\right)\left(3\log_{12} 3 + 1\right) - 1$

$= \log_{18}(96 \times 18).\ \log_{12}(27 \times 12) - 1$

$= \left(3\log_{18} 12\right)\left(2\log_{12} 18\right) - 1 = 6 - 1 = 5$

**S).** $2^{\log_x 3} = y^{\log_5 y} \Rightarrow \log_x 3\ \ln 2 = \log_5 y \ln y$

$\Rightarrow (\ln x)\ (\ln y)^2 = \ln 2 \ln 3 \ln 5$

similarly, $3^{\log_y 5} = x^{\log_2 x} \Rightarrow (\ln x)^2 (\ln y) = \ln 2 \ln 3 \ln 5$

$\therefore$ $(\ln x)(\ln y)^2 = (\ln x)^2(\ln y) \Rightarrow \ln y = \ln x \Rightarrow x = y$

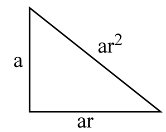
$\therefore$ $\dfrac{\left(x^{\log_y x} + y^{\log_x y}\right)^2}{\left(x^{\log_x y}\right)^2 + \left(y^{\log_y x}\right)^2} = \dfrac{(x+x)^2}{x^2 + x^2} = 2$

**58. (B)**

**(P)** $b + c = a + d = 2 \cdot 10$

$\Rightarrow a + b + c + d = 40$

**(Q)** $(ar^2)^2 = a^2 + a^2r^2$ where $a = 2$




$\therefore r^4 = 1 + r^2 \Rightarrow r^4 - r^2 - 1 = 0$

let $r^2 = t$

$t^2 - t - 1 = 0 \Rightarrow t = \frac{1 \pm \sqrt{5}}{2} \Rightarrow r^2 = \frac{1+\sqrt{5}}{2}$

$\therefore$ hypotenuse is $2 \times \left(\frac{1+\sqrt{5}}{2}\right) = 1 + \sqrt{5}$

comparing with $a + \sqrt{b}$

$a = 1,\ b = 5 \qquad \therefore a^2 + b^2 = 1 + 25 = 26$

**(R)** $a, ar, ar^2 \rightarrow$ G.P. $|r| < 1$

$a + ar + ar^2 = 70$

$\therefore 10ar = 4a + 4ar^2$

$10r = 4 + 4r^2 \Rightarrow 2r^2 - 5r + 2 = 0 \Rightarrow 2r^2 - 4r - r + 2 = 0$

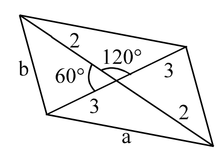
$(2r - 1)(r - 2) = 0 \Rightarrow \therefore r = 1/2$

$\Rightarrow a + \frac{a}{2} + \frac{a}{4} = 70 \Rightarrow a + \frac{3a}{4} = 70$ C $\qquad \Rightarrow a = 40$

$\therefore$ series is 40, 20, 10

$\therefore$ first term of G.P. is 40 Ans.

**(S)** Using cosine rule




$a^2 = 9 + 4 - 2 \cdot 2 \cdot 3 \cdot \left(-\frac{1}{2}\right) = 13 + 6 = 19$

$a^2 = 19 \Rightarrow a = \sqrt{19}$

$b^2 = 9 + 4 - 2 \cdot 2 \cdot 3 \cdot \left(\frac{1}{2}\right)$

$b^2 = 7 \Rightarrow b = \sqrt{7}$

$\therefore$ P = $2\left(\sqrt{19} + \sqrt{7}\right) \Rightarrow a + b = 26$

**59. (A)**

(P) Lines are concurrent $\begin{vmatrix} 1 & -2 & -6 \\ 3 & 1 & -4 \\ \lambda & 4 & \lambda^2 \end{vmatrix} = 0$

$\Rightarrow \lambda^2 + 2\lambda - 8 = 0 \Rightarrow \qquad \lambda = 2, -4$

(Q) points are collinear $\begin{vmatrix} \lambda+1 & 1 & 1 \\ 2\lambda+1 & 3 & 1 \\ 2\lambda+2 & 2\lambda & 1 \end{vmatrix} = 0$

$\Rightarrow 2\lambda^2 - 3\lambda - 2 = 0 \Rightarrow \quad \lambda = 2, -1/2$

(R) point of intersection of $x - y + 1 = 0$ and $3x + y - 5 = 0$ is (1, 2).

it will satisfy $x + y - 1 - \lambda = 0 \Rightarrow \lambda = 2$

(S) midpoint of (1, – 2) and (3, 4) will satisfy $y - x - 1 + \lambda = 0 \Rightarrow \lambda = 2$

**60. (A)**

(P) $x^{\log_{10} x} = 100x$

$\log_{10}x \cdot \log_{10}x = \log_{10}100 + \log_{10}x$

$\log_{10}x = t$

$t^2 - t - 2 = 0$

$t^2 - 2t + t - 2 = 0$

$t = -1, t = 2$

$x = 10^t$

$x|_{t=-1} = 10^{-1} = 1/10$

$x|_{t=2} = 10^2 = 100$

(Q) $\Rightarrow \log_2(9 - 2^x) = 3 - x$

$9 - 2^x = 2^{3-x}$

$9.2^x - 2^x 2^x = 8$ ; Let $2^x = t$

$t^2 - 9t + 8 = 0$

$t = 8, 1$

$2^x = 2 \quad t = 1$ ; $t = 8 \Rightarrow 2^x = 8$

$x = 0$ ; $x = 3$

(R) $\Rightarrow \log_{1/8}\log_{1/4}\log_{1/2} x = \frac{1}{3}$

$\log_{1/4}.\log_{1/2}x = (1/8)^{1/3}$

$\log_{\frac{1}{2}} x = (1/4)^{1/2}$

$x = (1/2)^{1/2}$

$= 1/\sqrt{2}$

(S) $\log_b a = 3$

$\Rightarrow a = b^3$

$\log_b c = -4 \Rightarrow c = b^{-4}$

now $a^{3x} = c^{x-1}$

$(b^3)^{3x} = (b^{-4})^{x-1}$

$9x = -4x + 4$

$13x = 4 \Rightarrow x = 4/13$

p = 4, q = 13 ; are relatively prime

$\therefore$ p + q = 17