SOLUTIONS

WEEKLY TEST-3

RBA

(JEE MAIN PATTERN)

Test Date: 29-07-2017

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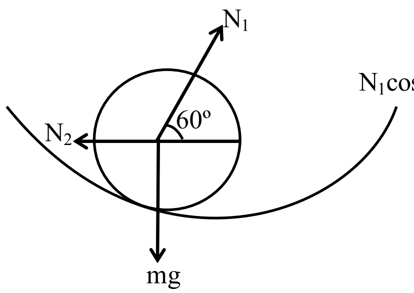
PHYSICS

1. **(A)**

2. **(D)**

3. **[B]**



$$N_1\cos60^\circ = N_2$$

$$\frac{N_1}{N_2} = 2$$

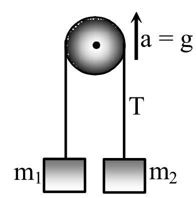
4. **(C)**

5. **(D)**

6. **(A)**

7. **(A)**

$$T = \frac{2m_1m_2(g+a)}{m_1+m_2},$$

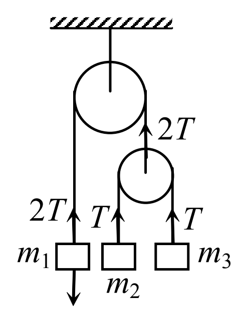


$$T = \frac{2\frac{w_1}{g}.\frac{w_2}{g}(g+g)}{\frac{w_1}{g}+\frac{w_2}{g}} \Rightarrow T = \frac{4w_1w_2}{(w_1+w_2)}$$

8. If m_1 remains at rest

$2T = m_1g$ …(i)

$$T = \frac{2m_2m_3g}{m_1+m_2} \quad \text{…(ii)}$$



From (i) and (ii)

$$\frac{4m_2m_2g}{m_1+m_2} = m_1g$$

$$\frac{1}{m_1} = \frac{m_1+m_2}{4m_2m_3}, \quad \frac{4}{m_1} = \frac{1}{m_2} + \frac{1}{m_3}$$

$\therefore$ **(B)**

9. **(C)**

10. **(C)**

$mg - B = mf$

$B - (m - m')g = (m - m')f$

$\Rightarrow \; m'g = (2m - m')f \;\; \Rightarrow m' = \dfrac{2mf}{g+f}$

$\Rightarrow \; w' = \dfrac{2wf}{g+f}$

11. $2T - Mg = Ma$

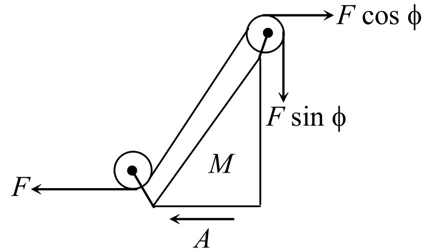
$T = \dfrac{M(g+a)}{2} = 522.5\text{ N}$

$\therefore$ **(A)**

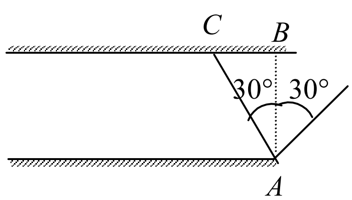
12. $F - F\cos\theta = MA$

$A = \dfrac{F - F\cos\phi}{M}$

$\therefore$ **(B)**



13. From the law of refection



$\tan 30° = \dfrac{BC}{AB} = \dfrac{BC}{0.2}$; $BC = 0.2 \times \dfrac{1}{\sqrt{3}} = 0.115$

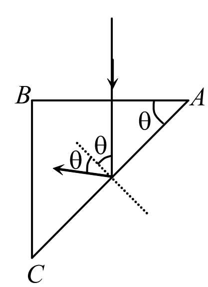
Total no. of reflection = 30

$\therefore$ **(B)**

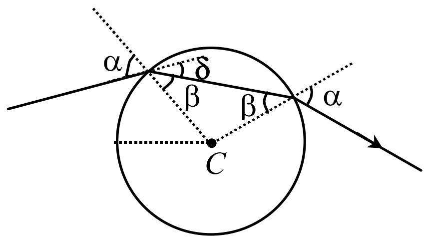
14. $\sin\theta > \sin\theta_C = \dfrac{1}{\mu}$

$\sin\theta > \dfrac{1}{\dfrac{3/2}{4/3}}$, $\sin\theta > \dfrac{8}{9}$

$\therefore$ **(A)**



15. **(B)**

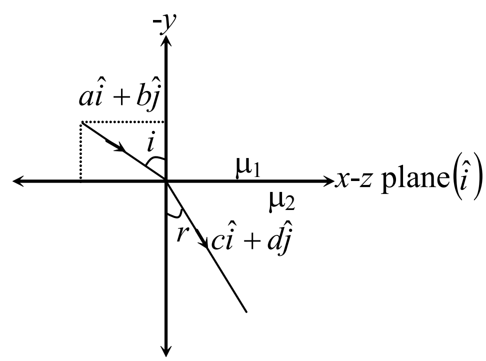


Total angle of deviation $= 2(\alpha - \beta)$

16. For refraction at spherical surface $\frac{\mu}{v} - \frac{1}{\infty} = \frac{\mu - 1}{R} \Rightarrow v = \frac{\mu}{\mu - 1}R = 3R$

$\therefore$ **(B)**

17. Applying Snell's law $\mu_1 \sin i = \mu_2 \sin r$



$$\left(\frac{3}{2}\right)\left(\frac{a}{\sqrt{a^2 + b^2}}\right) = 2\left(\frac{c}{\sqrt{c^2 + d^2}}\right)$$

Here $\sqrt{a^2 + b^2} = \sqrt{c^2 + d^2} = 1$

$\therefore \quad \frac{a}{c} = \frac{4}{3}$

$\therefore$ **(A)**

18. **(D)**

For refraction

$\frac{1.5}{v_1} - \frac{1}{-2R} = \frac{1.5 - 1}{R}$

$\Rightarrow \quad v_1 = \infty$ i.e. parallel beam

For Reflection

$\frac{1}{v_2} + \frac{1}{-\infty} = \frac{1}{-R/2}$

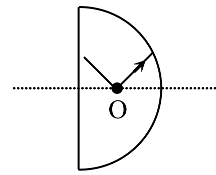
$\Rightarrow v_2 = -R/2$

For Final Refraction

$\frac{1}{v_3} - \frac{1.5}{-3R/2} = \frac{1 - 1.5}{-R}$

$\Rightarrow v_3 = -2R$ i.e. at pole of silvered part

19. (D)



$$d' = \frac{d}{n_{ref}}$$

$$d' = \frac{4}{1.6} = 2.5 \text{ cm}$$

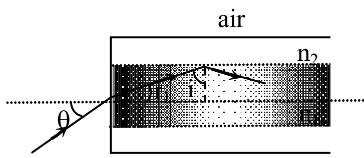
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$$

$$\frac{1}{v} - \frac{1.6}{-4} = \frac{1-1.6}{-8}$$

$$v = \frac{-40}{13} = -3\text{cm (Approx)}$$

hence distance between images = 8 – (3 + 2.5) = 2.5 cm

20. (A)



r + i = 90

For θ max. ⇒ r should be max.

i will be minimum for TIR minimum value of i

i = c

r = 90 – c …… (i)

$$\sin r = \sin(90 - c) = \cos c = \sqrt{1-\sin^2 c} = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$$

$$\sin c = \frac{n_2}{n_1}$$

$$\sin r = \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \quad \text{……(ii)}$$

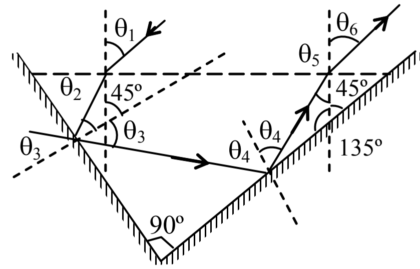
by snell's law

$$1 \times \sin\theta = n_1 \sin r = \sqrt{n_1^2 - n_2^2}$$

$$\theta = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

Option (A) is correct.

21. [B]



$\theta_2 + \theta_3 + 135^\circ = 180^\circ$

$\theta_3 = 45^\circ - \theta_2$

$\theta_3 + \theta_4 = 90^\circ$

$45^\circ - \theta_2 + \theta_4 = 90^\circ$

$\theta_4 = 45^\circ + \theta_2$

$90^\circ - \theta_4 + \theta_5 + 135^\circ = 180^\circ$

$45^\circ - \theta_2 + \theta_5 + 135^\circ = 180^\circ$

$\theta_5 = \theta_2$

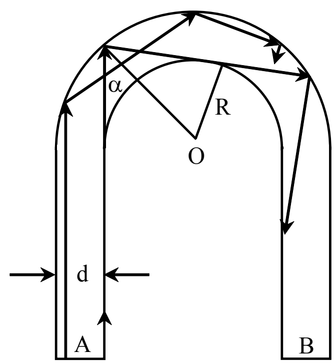
$\therefore \theta_6 = \theta_1$ (By Snell law)

$\therefore \delta = 180^\circ$

22. (B)

23. (A)

Consider the representative rays shown in Fig. A ray entering the glass through surface A and passing along the inner side of the rod will be reflected by the outer side with the smallest angle α, at which the reflected ray is tangent to the inner side. We have to consider the conditions under which the ray will undergo total internal reflection before reaching B.



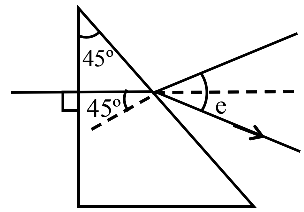
If $\alpha > \theta_c$, the critical angle, at which total internal reflection occurs, all the incident beam will emerge through the surface B. Hence we require $\sin\alpha > \frac{1}{n}$.

The geometry gives $\sin\alpha = \frac{R}{(R+d)}$.

Therefore $\frac{R}{R+d} \geq \frac{1}{n}$,

or $\left(\frac{R}{d}\right)_{min} = \frac{1}{n-1} = \frac{1}{1.5-1} = 2.$

24. **[C]**



sin e = (1 + 0.4 t) × sin 45º

$\cos e \times \frac{de}{dt} = \frac{1}{\sqrt{2}} \times 0.4$

at t = 1 sec μ = 1.4 $\therefore$ $\sin e = \frac{1.4}{\sqrt{2}}$

$\cos e = 0.141 \therefore \frac{de}{dt} = \frac{0.4}{0.141} \times \frac{1}{\sqrt{2}} = 2$ rad/sec.

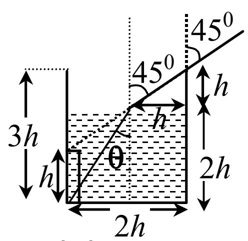
25. Ray inside medium *AB* is parallel to ray inside medium *CD*

$\therefore$ **(D)**

26. $\mu \sin\theta = \sin 45^0$

$\frac{\mu h}{h\sqrt{5}} = \frac{1}{\sqrt{2}}$

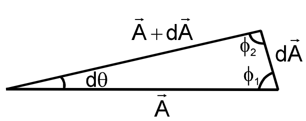
$\mu = \sqrt{\frac{5}{2}}$



$\therefore$ **(B)**

27. **(A)**

Let $\vec{A}$ at any instant of time t change to $\vec{A} + d\vec{A}$ at instant t + dt as shown in diagram.



$\because$ magnitude of vector does not change

$|\vec{A}| = |\vec{A} + d\vec{A}|$

Hence $\phi_1 = \phi_2 = \pi / 2$ [as $d\theta \to 0$]

or $d\vec{A} \perp \vec{A}$ or $\frac{d\vec{A}}{dt} \perp \vec{A}$

Alternate solution :

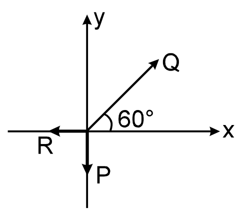
Let $\vec{A} = r\hat{e}_r$

$\therefore \frac{d\vec{A}}{dt} = r\omega\hat{e}_t$ if magnitude of $\vec{A}$ does not change.

Where $\hat{e}_r$ and $\hat{e}_t$ vectors in radial and normal directions.

$\therefore \frac{d\vec{A}}{dt} \perp \vec{A}$

28. (D)



The particle is at rest under action of forces P, Q and R.

$\therefore$ Q sin 60° = P and Q cos 60° R

$\Rightarrow \frac{2}{\sqrt{3}}$ P = Q and 2R = Q

$\Rightarrow$ P : Q : R = $\sqrt{3}$: 2 : 1

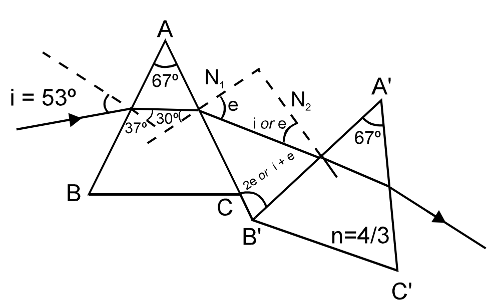
29. (C)

30. (A)

Let be the angle of emergence from the first prism be 'e'

calculating step by step we get e = $\sin^{-1}\frac{2}{3}$

Then for net deviation to be double, the incident ray on side A′ B′ of second prism should make angles i or e with normal.



Hence the angle between the given then will be 2e or i + e.

CHEMISTRY

31. **(D)**

$$Na_2\overset{+2}{S_2}O_3 + Cl_2 + H_2O \longrightarrow Na_2\overset{+6}{S}O_4 + H_2\overset{+6}{S}O_4 + HCl$$

$\therefore$ x factor for $Na_2S_2O_3$ = 2|(2 – 6)| = 8

$\therefore$ equivalent weight of $Na_2S_2O_3 = \frac{\text{Mol.wt}}{8}$

32. **(B)**

The equiv. wt. of $P_4 = \frac{31\times4}{5\times4} = \frac{31}{5}$

$\therefore$ 62 gm $P_4 = \frac{62\times5}{31}$ equiv. of P_4 = 10 equiv. of P_4

The equiv. wt. of $HNO_3 = \frac{\text{Mol.wt}}{1} = \frac{63}{1}$

$\therefore$ the wt. of HNO_3 required

= 10 × 63 = 630 gm

33. **(B)**

The reaction are

$MnO^-{}_4 + 8H^+ + 5e^- \rightarrow Mn^{+2} + 4H_2O$

$A^{+n} + 3H_2O \rightarrow AO_3^- + 6H^+ + (5–n)\, e^-$

Amount of electrons involved in the given amount of MnO_4^- = 5 x 1.6 x 10^{-3} mol.

Equating these two we get 5 × 1.6 × 10^{-3} = (5–n) 2.7 × 10^{-3}

$\therefore$ n = 2 (approx.)

34. **(A)**

	Na_2CO_3	$NaHCO_3$
Let	a meq.	b meq.

when HPh is used as indicator

$Na_2CO_3 + H_2SO_4 \rightarrow NaHCO_3 + NaHSO_4$

then $\frac{1}{2}$ meq. of Na_2Cl_3 = meq. of H_2SO_4

$\frac{a}{2} = 2.5 \times 0.1 \times 2 \Rightarrow a = 1$

MeOH is added after the first end point the solution

Contains $NaHCO_3$ original & $NaHCO_3$ produced.

$NaHCO_3 + H_2SO_4 \rightarrow H_2CO_3 + NaHSO_4$

meq. of H_2SO_4 = meq. of $NaHCO_3$ original + meq. of $NaHCO_3$ produced

$2.5 \times 0.2 \times 2 = b + 1/2$ meq. of Na_2CO_3

$= b + a/2$

$b + a/2 = 1$

$b = 1 - 0.5 = 0.5$

wt of Na_2CO_3/lit $= a \times 10^{-3} \times \frac{106}{2} \times \frac{1}{10} \times 1000$

$= 1 \times \frac{53}{10} = 5.3$ gm

wt of $NaHCO_3$/lit $= b \times 10^{-3} \times 84 \times \frac{1}{10} \times 1000 = 4.2$ gm

35. (B)

Mass of HCO_3^- in 1 kg or 10^6 mg water = 244 mg

Millimoles of $HCO_3^- = \frac{244}{61} = 4$

$2HCO_3^- + CaO \longrightarrow CaCO_3 + H_2O + CO_2 + 2e^-$

millimoles of CaO = 2

mass of CaO = 56 × 2 = 112 mg

36. (A)

Molecular mass of chloride of metal = weight of 22,400 ml vapour of metal at STP

$= \frac{0.72 \times 22,400}{100} = 161.28\text{g}$

100g of metal chloride contains = 65.5 g chloride

$\therefore$ 161.28g metal chloride contains $= \frac{65.5 \times 161.28}{100} = 105.6$g

Therefore, the number of mole of chlorine atoms per mole of metal chloride

= 105.6/35.5 = 3

Hence the molecular formula of metal chloride is MCl_3

37. (B)

Milli eq. of HCl initially = $10 \times 0.5 = 5$

Milli eq. of NaOH consumed = Milli eq. of HCl in excess

$= 10 \times 0.2 = 2$

$\therefore$ Milli eq. of HCl consumed = Milli eq. of $Ba(OH)_2$ = 5-2=3

$\therefore$ eq. of $Ba(OH)_2$ = $3/1000 = 3 \times 10^{-3}$

Mass of $Ba(OH)_2$ = $3 \times 10^{-3} \times (171/2) = 0.2565$ g.

% $Ba(OH)_2$ = $(0.2565/2) \times 100 = 12.8\%$

38. (C)

Milli equivalents of HCl = N × V (ml) = $\frac{1 \times 40}{10} = 4$

Milli equivalents of KOH = N × V (ml) = $\frac{1 \times 60}{20} = 3$

One milli equivalent of an acid neutralizes one milli equivalent of a base

Milli equivalent of HCl left = 4 – 3 = 1

Total volume of the solution = 40 + 60 = 100 ml

Milli equivalents of HCl = N × V (ml)

1 = N × 100

Normality (N) of HCl left in solution = 0.01

Salt formed = Milli equivalent of acid or base neutralized

Milli equivalents of the salt formed = N × V (ml)

3 = N × 100

Normality (N) of salt formed = 0.03

39. (A)

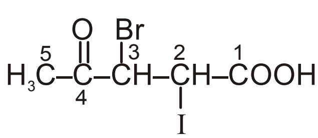
Meq equlivalent for KMnO4 is 300 x (1/12) = 25

Meq for H_2O_2 is 25 Normality = 25/20 = 1.25N

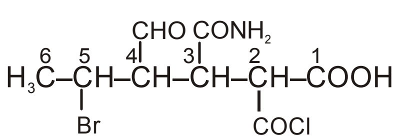
volume stenght = 5.6 x 1.25 = 7

40. (A)

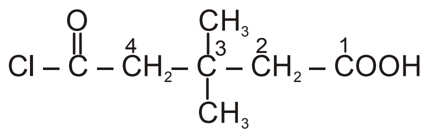
41. (C)



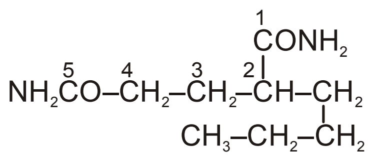
42. (B)



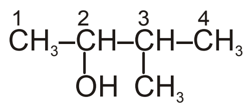
43. (C)



44. (C)



45. (C)



3-Methylbutan-2-ol

46. (A)

47. (C)

More stable resonating structure contributed higher in R.S.

48. (B)

Lone pair of electrons of $H_2C=\ddot{N}-CH_3$ is in sp^2 hybrid orbital.

49. (A)

Due to delocalization of π electron in benzene.

50. (A)

In resonance position of atoms does not change.

51. (A)

52. (B)

53. (A)

54. (D)

55. (D)

56. (A)

57. (D)

Oxide and hydroxide of Zn, Al, Be, Pb are amphoteric.

58. (D)

59. (A)

60. (B)

Option (B). is not correct due to same reason as in above question

MATHEMATICS

61. (D)

$$f(x)+f(1-x)=\frac{1}{27}$$

$$=\frac{1}{27}\times 54=2$$

62. (B)

Odd Extension from [0, 1] to [–1, 1] means the function which satisfies the condition $f(-x) = -f(x)$. Now $|-x| = |x|$

$f(-x) = x^2 - x - \sin x + \log(1 + |x|)$

$= -(-x^2 + x + \sin x - \log(1 + |x|))$

$\therefore$ (b) is correct.

63. (B)

$$f(x)=\alpha+5x-x^2=\alpha+\frac{25}{4}-\left(x-\frac{5}{2}\right)^2 \Rightarrow \alpha+\frac{25}{4}=5$$

64. (C)

$$\left[\sqrt{p^2+1}\right]=p \Rightarrow p+r\le\sqrt{p^2+q}<p+r+1 \Rightarrow p^2+r^2+2pr\le p^2+q<p^2+2pr+2p+r^2+2r+1$$

$$\Rightarrow r^2+2pr\le q<(r^2+2pr)+(2p+2r+1)$$

Hence q can take '2p + 2r + 1' different values.

65. (D)

Let equation of AB be $y = x + a$

$\therefore$ A(1 – a, 1) and B(2, 2 + a)

$\therefore$ equation of AD is

$y - 1 = -1(x - 1 + a)$

$\therefore$ D (– 2, 4 – a)

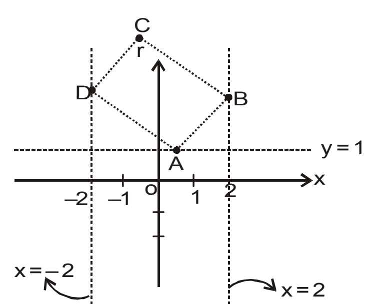
Let C(h,k)

$\Rightarrow h + 1 - a = 2 - 2 \Rightarrow h = a - 1$

and $k + 1 = 2 + a + 4 - a$

$\Rightarrow k = 5$

$\therefore$ Locus of C(h,k) is $y = 5$



66. (D)

$$\tan(180°-\theta)=\text{slope of } AB=-3$$

$\therefore \quad \tan\theta = 3$

$$\therefore \quad \frac{OC}{AC} = \tan\theta, \frac{OC}{BC} = \cot\theta$$

$$\Rightarrow \quad \frac{BC}{AC} = \frac{\tan\theta}{\cot\theta} = \tan^2\theta = 9$$

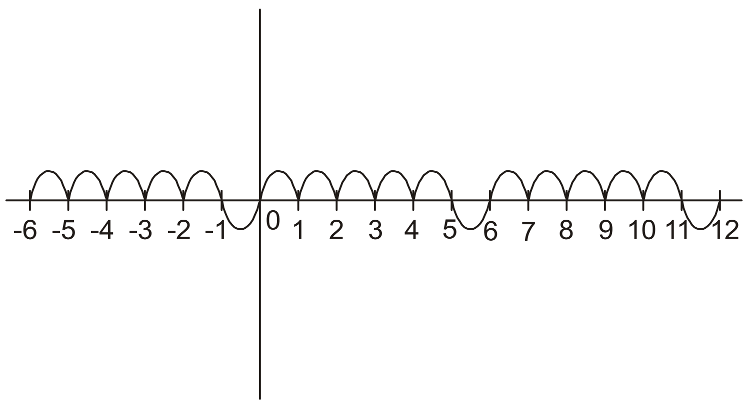
67. (C)

Let A(a, 0), B(0, b) and O(0, 0) are vertices of a right angled triangle, then vertices of the triangle made by reflection of A,B,O into opposite sides will be

$A'(-a,0), B'(0,-b)$ and $O'\left(\frac{2ab^2}{a^2+b^2}, \frac{2a^2b}{a^2+b^2}\right)$. Then the value k is 3.

68. (A)

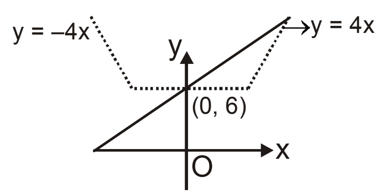
Graph of f(x) is given by



Therefore period of f(x) is 6 and |f(x)| is 1

$\Rightarrow \quad T_1^2 + T_2^2 = 37$

69. (B)



When

(i) P = 0 then it has infinte solution

(ii) if – 4 < P < 0 or 0 < P < 4 then it intersect at 2 points

(iii) P $\geq$ 4 or P $\leq$ –4 then it has only one solution

70. (D)

(1997, 0) lies on y = mx + c

$\Rightarrow$ 0 = 1997m + c $\Rightarrow$ c = –1997 m

$\Rightarrow$ mc = –1997 m^2 $\leq$ 0

which is not possible.

71. (C)

$f(x-c) = g(x) = -g(-x)$ $\quad(\because g(x)$ is odd)

$= -f(-x-c) = -f(x+c)$ $\quad(\because f$ is even)

$\therefore\ f(x+c) = -f(x-c)$

$\Rightarrow\ f(x+2c) = -f(x+c-c) = -f(x)$

$\Rightarrow\ f(x+4c) = -f(x+2c) = -(-f(x)) = f(x)$

$\Rightarrow$ f is periodic with period 4c.

72. (C)

$f(x) = \left|4\frac{(\sqrt{\cos}x-\sqrt{\sin}x)(\sqrt{\cos}x+\sqrt{\sin}x)}{(\cos x+\sin x)}\right|$ is defined only if $\cos x \geq 0$, $\sin x \geq 0$

Therefore, x lies in first quadrant only.

$$f(x) = \left|4\frac{(\cos x-\sin x)}{(\cos x+\sin x)}\right| = \left|4\tan\left(\frac{\pi}{4}-x\right)\right| = \left|4\tan\left(x-\frac{\pi}{4}\right)\right|$$

$\text{Now}, 0 \leq x \leq \frac{\pi}{2}$

or $\ -\frac{\pi}{4} \leq x-\frac{\pi}{4} \leq \frac{\pi}{4}$

or $\ -1 \leq \tan\left(x-\frac{\pi}{4}\right) \leq 1$

or $\ -4 \leq 4\tan\left(x-\frac{\pi}{4}\right) \leq 4$

$0 \leq \left|4\tan\left(x-\frac{\pi}{4}\right)\right| \leq 4$

73. (C)

74. (A)

f(x) is defined if $\log_{|\sin x|}\left(x^2-8x+23\right)-\frac{3}{\log_2|\sin x|} > 0$

$\Rightarrow\ \log_{|\sin x|}\left(\frac{x^2-8x+23}{8}\right) > 0\ \left\{\text{as } \frac{3}{\log_2|\sin x|} = \frac{\log_2 8}{\log_2|\sin x|} = \log_{|\sin x|} 8\right\}$

$\therefore\ |\sin x| \neq 0, 1$ and $\frac{x^2-8x+23}{8} < 1$

$\left\{\text{as } |\sin x| < 1 \Rightarrow \log_{|\sin x|} a > 0 \Rightarrow a < 1\right\}$

Now, $\dfrac{x^2 - 8x + 23}{8} < 1 \Rightarrow x^2 - 8x + 15 < 0$

$\therefore \quad x \in (3, 5) - \left\{\pi, \dfrac{3\pi}{2}\right\}$

Hence domain of a function

$= (3, \pi) \cup \left(\pi, \dfrac{3\pi}{2}\right) \cup \left(\dfrac{3\pi}{2}, 5\right).$

75. (D)

Minimum value of $|PA - PB|$ is zero. It can be attained, if PA = PB. that means 'P' must lie on the right bisector of AB.

Equation of right bisector of AB is $y - \dfrac{1}{2} = 2(x - 1)$ i.e., $y = 2x - \dfrac{3}{2}$

Solving with given line, we get $P \equiv \left(-\dfrac{9}{20}, -\dfrac{12}{5}\right)$

76. (A)

Extremities of the given diagonal are (4, 0) and (0, 6)

$\Rightarrow$ slope of this diagonal = $-\dfrac{3}{2}$

$\Rightarrow$ slope of other diagonal = $\dfrac{2}{3}$

$\Rightarrow$ equation of the other diagonal is $\dfrac{x - 2}{\frac{3}{\sqrt{13}}} = \dfrac{y - 3}{\frac{2}{\sqrt{13}}} = r$

for the extermities of the diagonal r = $\pm\sqrt{13}$

$\Rightarrow \; x - 2 = \pm 3,\; y - 3 = \pm 2 \;\; \Rightarrow$ x = 5, –1 and y = 5, 1

$\Rightarrow$ the extremities of the diagonal are (5, 5), (–1, 1).

77. (D)

$\because$ L_1, L_2, L_3 are 3 non-concurrent lines

$\therefore$ A triangle will be formed by the lines

$\therefore$ Incentre and 3-ex-centres are the points which are equidistant from L_1, L_2 and L_3

Hence (D) is the correct answer.

78. (A)

$3x + 4y = 9,\ y = mx + 1$

$3x + 4(mx + 1) = 9 \Rightarrow x = \dfrac{5}{(3+4m)}$

Since $x \in I \Rightarrow 3 + 4m = -1, 1, 5, -5 \Rightarrow m = -1, -2$

79. (D)

'm' of PS = $\dfrac{1-2}{\frac{13}{2}-2} = -\dfrac{2}{9}$

Equation to parallel line through (1, –1) is $y + 1 = -\dfrac{2}{9}(x-1)$

$2x + 9y + 7 = 0.$

80. (B)

$$y = \frac{x-1}{x^2-3x+3}$$

$\Rightarrow x^2y - 3xy + 3y = x - 1$

$\Rightarrow x^2y - x(3y + 1) + 3y + 1 = 0$

$\therefore\ D \geq 0$

$\Rightarrow (3y+1)^2 - 4y(3y+1) \geq 0 \quad \Rightarrow -3y^2 + 2y + 1 \geq 0$

$\Rightarrow 3y^2 - 2y - 1 \leq 0 \quad \Rightarrow y^2 - \dfrac{2y}{3} - \dfrac{1}{3} \leq 0$

$\Rightarrow \left(y-\dfrac{1}{3}\right)^2 - \dfrac{1}{9} - \dfrac{1}{3} \leq 0 \quad \Rightarrow \left(y-\dfrac{1}{3}\right)^2 \leq \dfrac{4}{9} \quad \Rightarrow -\dfrac{2}{3} \leq y - \dfrac{1}{3} \leq \dfrac{2}{3}$

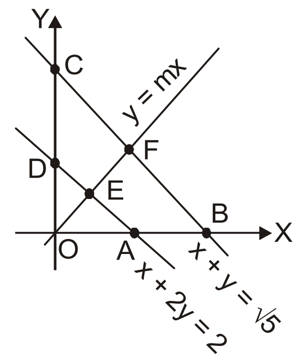
$\therefore \quad -\dfrac{1}{3} \leq y \leq 1$

$\therefore \quad y \in \left[-\dfrac{1}{3}, 1\right]$

81. (B)

Here $A \equiv (2,0), B \equiv (\sqrt{5},0)$

$C \equiv (0,\sqrt{5}), D \equiv (0,1)$



$E \equiv \left(\frac{2}{1+2m}, \frac{2m}{1+2m}\right)$ and $F \equiv \left(\frac{\sqrt{5}}{1+m}, \frac{\sqrt{5}m}{1+m}\right)$.

Now, $2 \times ar(\square ABFE) = ar(\square ABCD)$

$\Rightarrow \quad 2\{ar(\Delta OBF) - ar(\Delta OAE)\} = ar(\Delta OBC) - ar(\Delta OAD)$

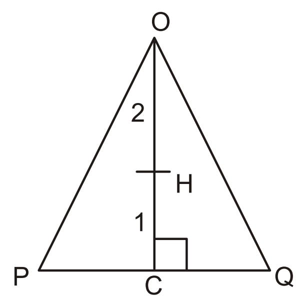
$\Rightarrow \quad 2\left\{\frac{1}{2} \times \sqrt{5} \times \frac{\sqrt{5}m}{1+m} - \frac{1}{2} \times 2 \times \frac{2m}{1+2m}\right\} = \frac{1}{2} \times \sqrt{5} \times \sqrt{5} - \frac{1}{2} \times 2 \times 1$

$\Rightarrow \quad m = \frac{3}{2} \qquad (\because m > 0)$

82. (A)

In an equilateral triangle the orthocentre and the centroid are the same. OPQ is the equilateral triangle in which $OC \perp PQ$.

Clearly, the point H which divides OC internally in the ratio 2:1 is the orthocentre.



Clearly, $OC = \frac{1}{\sqrt{2}}$. So, $OH = \frac{2}{3} \times \frac{1}{\sqrt{2}}$

$\therefore \quad H = \left(\frac{2}{3\sqrt{2}}\cos 45°, \frac{2}{3\sqrt{2}}\sin 45°\right)$

83. (D)

Orthocentre of triangle BCH is the vertex A(–1, 0).

84. (C)

It is obvious that a, b and c are the roots of the equation $mt^3 + (l - p)\,t - kq = 0$, where (p, q) is the point of concurrency.

Obviously sum of roots = a + b + c = 0

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

85. (B)

$f(f(x)) = x$

$$\frac{a\left(\frac{ax+b}{cx+d}\right)+b}{c\left(\frac{ax+b}{cx+d}\right)+d}=x$$

$$c\left(a+d\right)x^2+\left(d^2-a^2\right)x-b\left(a+d\right)=0$$

$$\Rightarrow a+d=0\Rightarrow a=-d$$

$$\text{Now,} f\left(1\right)=1\Rightarrow c=2a+b.$$

$$\& \ f\left(5\right)=5\Rightarrow 25c=10a+b$$

$$\& \text{ hence } a=3c\Rightarrow b=-5c$$

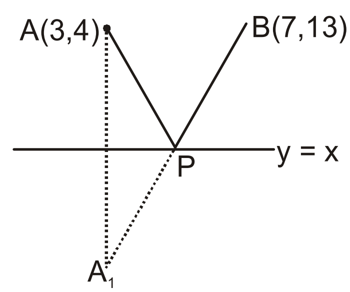
$$\therefore\ f\left(x\right)=\frac{3x-5}{x-3}$$

86. (A)

$xy>0 \Rightarrow$ P either lies in first quadrant or in third quadrant.

$x+y<1 \Rightarrow$ P lies below line x + y = 1

87. (C)



Let A_1 be the reflection of A in y = x

A_1 (4, 3)

Now PA + PB = A_1P + PB

which is minimum if A_1, P, B are collinear.

Equation of A_1B is 3y = 10x – 31

Solving it with y = x, we get $P\left(\frac{31}{7},\frac{31}{7}\right)$

88. (C)

$$f(x)+f(x+1)+f(x+2)+...+f(x+2015)=0$$

$$f(x+1)+f(x+2)+...+f(x+2015)+f(x+2016)=0$$

substracting we get

$$\Rightarrow f(x+2016)=f(x)\ \forall\, x\in R$$

Period of f(x) is 2016

89. **(D)**

Use $\frac{x^2-5x+6}{x^2+x+1}>0 \Rightarrow x \in (-\infty,2)\cup(3,\infty)$

and $[x^2-1]>0 \Rightarrow x \in (-\infty,-\sqrt{2}]\cup[\sqrt{2},\infty)$

90. **(A)**

f(7) + f(– 7) = – 10

or, f(7) = – 17

or, f(7) + 17 cos x = – 17 + 17 cos x

which has the range [–34, 0]