SOLUTIONS

PHASE TEST-1

CDK-1801 & CDS-1801

JEE MAIN PATTERN

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Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd., Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

PHYSICS

1. (C)

Let potential at A = x, applying kirchhoff current law at juction A

$$\frac{x-20-10}{1}+\frac{x-15-20}{2}+\frac{x+45}{2}+\frac{x+30}{1}=0$$

$$\Rightarrow \quad \frac{2x-60+x-35+x+45+2x+60}{2}=0$$

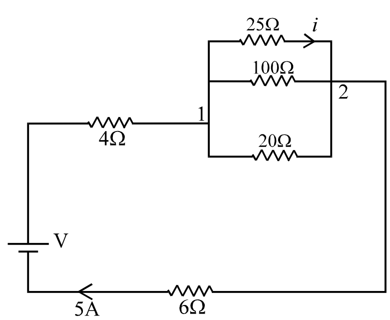
$\Rightarrow$ 6x + 10 = 0 $\qquad\qquad \Rightarrow$ x = –5/3

Potential at A = $\frac{-5}{3}$ V

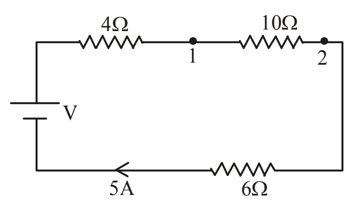
2. (A)

Equivalent circuit is shown in the figure (a) and (b)

(a)



(b)



from the figure (b)

V = 20 × 5 = 100V

Potential difference between (1) and (2) = 50V

i = 50 / 25 = 2A

3. (D)

Resistance of smaller section $=\frac{1}{4}\times 10=2.5\Omega$

Resistance of bigger section $=\frac{3}{4}\times 10=7.5\Omega$

The two resistances are in parallel. Resultant resistance

$$=\frac{7.5\times 2.5}{7.5+2.5}=\frac{7.5\times 2.5}{10}=1.875\Omega$$

$$i=\frac{3}{1+1.875}=\frac{3}{2.875}=\frac{24}{23}A$$

Current in smaller section $=\dfrac{\frac{24}{23}\times 1.875}{2.5}=\dfrac{18}{23}A$

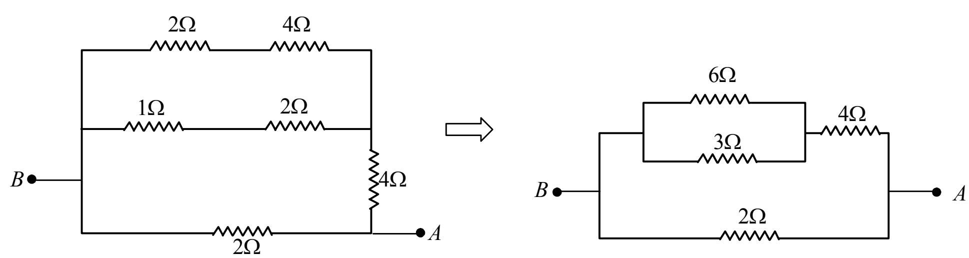
Current in bigger section $=\dfrac{\frac{24}{23}\times 1.875}{7.5}=\dfrac{6}{23}A$.

4. (D)

As the charge distribution remains same on opening the switch, no charge will flow in the circuit. So heat dissipated is zero.

5. (C)

Equivalent circuit diagram of the circuit is



So $R_{eq}=\dfrac{3}{2}\Omega$

6. (B)

In such a problem, term $\alpha\,\Delta T$ will have a larger value so could not be used directly in

$R = R_0\,(1 + \alpha\,\Delta T)$. We need to go for basics as

As we know that $\alpha = \dfrac{dR}{RdT}$

$\Rightarrow \quad \int\frac{dR}{R} = \int \alpha dT \quad \Rightarrow \quad \ell n\frac{R_2}{R_1} = \alpha(T_2 - T_1)$

$\Rightarrow \quad R_2 = R_1 e^{\alpha(T_2-T_1)} \quad \Rightarrow \quad R_2 = 10e^1$

$\Rightarrow \quad R_2 = 10\,e\,\Omega$ **Ans.**

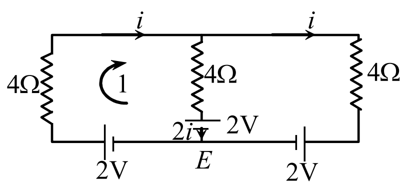
7. (B)

$I = \frac{V}{R_e} = \frac{2}{15}$ A (I is current in each branch)

$V_C - V_B = \frac{4}{3}$ V

8. (D)

By KVL in loop 1 $2 - 4i - 8i - 2 = 0$



$\Rightarrow$ $i = 0$

9. (A)

$I = \frac{n\varepsilon}{nr} = \frac{\varepsilon}{r}$, $V = \varepsilon - Ir = 0$

10. (C)

For series connection $x = nR$.

For parallel connection $y = \frac{R}{n}$.

Therefore $xy = nR \times \frac{R}{n} = R^2$.

11. (C)

12. (C)

$$E = \frac{V}{d} = \frac{5\times10^3}{10\times10^{-3}} = 5\times10^5 \text{ V/m}$$

13. (A)

When one plate is fixed, the other is attracted towards the first with a force

$$F = \frac{q^2}{2A\varepsilon_0} = \text{constant}$$

Hence, an external force of same magnitude will have to be applied in opposite direction to increase the separation between the plates.

$$\therefore \quad W = F(2d - d) = \frac{q^2 d}{2A\varepsilon_0}$$

14. (B)

$$\frac{q_1}{C_1}=\frac{q_2}{C_2}\ ;\ q_1+q_2=2Q_0$$

$$C_1=\frac{\varepsilon_0 A}{d_0+vt};\ C_2=\frac{\varepsilon_0 A}{d_0-vt}$$

$$\frac{q_1}{q_2}=\frac{d_0-vt}{d_0+vt}$$

$$q_2\left(\frac{d_0-vt}{d_0+vt}\right)+q_2=2Q_0$$

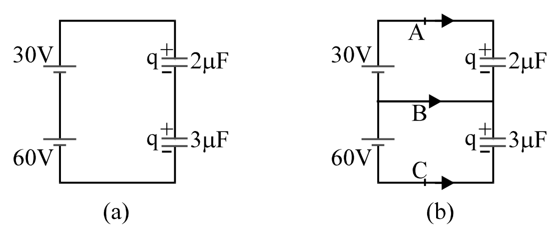
$$q_2\left[\frac{2d_0}{d_0+vt}\right]=2Q_0$$

$$q_2=\frac{2Q_0}{2d_0}(d_0+vt)$$

$$I=\frac{dq_2}{dt}=\frac{Q_0 v}{d_0}$$ = 20 amp

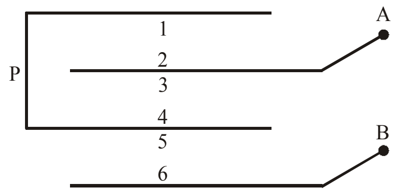
15. (A)

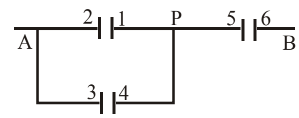
Let us draw two figures and find the charge on both the capacitors before closing the switch and after closing the switch.



16. (D)

Let us call the isolated plate as P. A capacitor is formed by a pair of parallel plates facing each other. Hence we have three capacitor formed by the pairs (1, 2), (3, 4) and (5, 6). The surface 2 and 3 are at same potential as that of A. The arrangement can be redrawn as a network of three capacitors.





$$C_{AB} = \frac{2C.C}{2C+C} = \frac{2C}{3}$$

$$= \frac{2}{3}\frac{\varepsilon_0 A}{d}$$

17. (D)	**18. (B)**	**19. (A)**	**20. (C)**
21. (B)	**22. (D)**	**23. (B)**	**24. (C)**
25. (A)	**26. (B)**	**27. (C)**	**28. (D)**
29. (B)	**30. (A)**		

CHEMISTRY

31. (C)

$$\lim_{x_A \to 1} \frac{P_T}{x_A} = P_A^0$$

$$P = 120 x_A + 140$$

$$x_A \to 1 \quad P_A^0 = 120 \times 1 + 140$$

= 260 mm

32. (D)

$$1 \text{ atm} = P_A^0 \times \frac{2}{4} + P_B^0 \times \frac{2}{4} \quad .....(i)$$

$$P_A^0 \times \frac{1}{4} + P_B^0 \times \frac{3}{4} > 1$$

$$P_A^0 \times \frac{1}{8} + P_B^0 \times \frac{3}{8} + 0.8 \times \frac{4}{8} = 1$$

$$P_A^0 \times \frac{1}{4} + P_B^0 \times \frac{3}{4} + .8 = 2$$

$$\frac{P_A^0}{4} + P_B^0 \times \frac{3}{4} = 1.2 \qquad ..... (ii)$$

Solving (i) and (ii)

$P_A^o = 0.6$; $P_B^o = 1.4$

33. (B)

34. (D)

35. (C)

$$x_A(\text{ideal}) = \frac{P_A}{P_A^0} \quad P_A^0 < P_B^0$$

$$P_T = P_B^0 + (P_A^0 - P_B^0)x_A$$

$$x_A' = \frac{P_A}{P_T} = \frac{P_A^0 x_A}{P_B^0 + (P_A^0 - P_B^0)x_A}$$

$$\frac{x_A'}{x_A} = \frac{P_A^0}{P_B^0 + (P_A^0 - P_B^0)x_A}$$

$x_A < 1$

$$\frac{x_A'}{x_A} < \frac{P_A^0}{P_B^0 + P_A^0 - P_B^0}$$

$$\frac{x_A'}{x_A} < 1$$

36. (C)

$$\frac{P_A}{P_B} = \frac{P_A^0 x_A}{P_B^0 x_B} = \frac{P_y y_A}{P_7 y_B}$$

$$\frac{y_A}{y_B} = \frac{P_A^0 \times A}{P_B^0 \times B}$$

$$P_A^0 > P_B^0$$

$$\frac{P_A^0}{P_B^0} > 1$$

$$\therefore \quad \frac{y_A}{Y_B} > \frac{x_A}{x_B}$$

37. (A)

$$\frac{P^0 - P}{P} = \frac{n_B}{n_A} i = 4$$

$$\frac{17.25 - 17.20}{17.20} = \frac{5 \times 4}{55.55}$$

$$S = \frac{.05}{17.2} \times \frac{55.55}{4} = 4.04 \times 10^{-2}$$

38. (C)

For 1st reaction

Ea_f = 800 Cal/mol $\qquad k_f = A_1 e^{\frac{-800}{RT}}$ ....(i)

Ea_r = 200 Cal/mol

A_1

for 2nd reaction

$Ea_f' = 200$ Cal / mol $\qquad k_f' = A_2 e^{\frac{-200}{RT}}$ ....(ii)

Ea_r = 200 Cal/mol

A_2

$$\frac{k_f}{k_f'} = \frac{A_1}{A_2} e^{\left(\frac{200-800}{RT}\right)} \qquad \text{(depends on T)}$$

for 1st reaction,

$$k_{eq} = \frac{k_f}{k_r} = \frac{A\, e^{-\frac{800}{RT}}}{A_1' e^{-\frac{200}{RT}}} = A\, e^{\frac{-600}{RT}}$$

for 2nd reaction,

$$k_{eq}' = \frac{k_f'}{k_r'} = A' e^{\frac{600}{RT}}$$

So, $k_{eq} \times k_{eq}' = AA' e^{o}$ = const. (independent of T)

at 300 k

for 1st reaction,

$$k_{eq} = Ae^{\frac{-600}{2\times 300}} = Ae^{-1}$$

for 2nd reaction,

$$k_{eq} = A' e^{\frac{-600}{2\times 300}} = A' e$$

[A is not same for forward reverse reaction]

39. (D)

When $[OH^-]$ = 0.1 M

rate = o.1 k_1 [RX] + k_2 [RX]

So, % RX which reacts by 2nd order mechanism

$$=\frac{0.1\,k_1\times100}{0.1\,k_1+k_2}=\frac{10\,k_1}{0.1k_1+k_2}$$

$$=\frac{100\;k_1}{k_1+10\;k_2}$$

40. (B)

$$\frac{1}{2}A\longrightarrow 2B$$

$$\text{rate}=\frac{-1}{½}\frac{d[A]}{dt}=\frac{1}{2}\frac{d[B]}{dt}$$

$$\text{or, }\frac{-d[A]}{dt}=\frac{1}{4}\frac{d[B]}{dt}$$

41. (B)	**42. (B)**	**43. (C)**	**44. (C)**
45. (D)	**46. (D)**	**47. (D)**	**48. (C)**
49. (A)	**50. (D)**		

51. (B)

$O^{2-} > Mg^{2+} > Al^{3+}$; (Z/e) ratio.

52. (B)

I.E. of Mg > I.E. of Al (Due to electronic configuration)

53. (C)

$CsBr_3$ exist as $Cs^+\ Br_3^-$, due to lattice energy effect (large cations stabilises by large anion)

54. (C)	**55. (D)**	**56. (C)**	**57. (A)**
58. (C)	**59. (D)**	**60. (D)**	

MATHEMATICS

61. (B)

$$\sin^3 x\cdot\sin 3x=\frac{1}{2}\sin^2 x(\cos 2x-\cos 4x)$$

$$=\frac{1}{4}(1-\cos 2x)(\cos 2x-\cos 4x)$$

$$=\frac{1}{4}(\cos 2x-\cos 4x-\cos^2 2x+\cos 2x.\cos 4x)$$

$$=\frac{1}{4}\left(\cos 2x-\cos 4x-\frac{1+\cos 4x}{2}+\frac{1}{2}(\cos 6x+\cos 2x)\right)$$

$$=\frac{1}{4}\left(\frac{3}{2}\cos 2x-\frac{3}{2}\cos 4x+\frac{1}{2}\cos 6x-\frac{1}{2}\right)=-\frac{1}{8}+\frac{3}{8}\cos 2x-\frac{3}{8}\cos 4x+\frac{1}{8}\cos 6x$$

$\Rightarrow \quad n=6$

62. (B)

use $x^2-5x+7<1$ and $x^2-5x+7>0$

63. (A)

For y to be defined, we must have

(a) $\log_{10}\left(\frac{5x-x^2}{4}\right)\ge 0 \Rightarrow \frac{5x-x^2}{4}\ge 10^0$

$\Rightarrow 5x-x^2\ge 4 \Rightarrow x^2-5x+4\le 0 \Rightarrow (x-1)(x-4)\le 0 \Rightarrow 1\le x\le 4$

(b) $\frac{5x-x^2}{4}>0 \Rightarrow 5x-x^2>0 \Rightarrow x(x-5)<0 \Rightarrow 0<x<5$

From (a) and (b), we get the domain of f = $[1, 4]\cap(0, 5) = [1, 4]$

64. (B)

$\sqrt{9-x^2}$ is defined for

$9-x^2\ge 0 \Rightarrow (3-x)(3+x)\ge 0$

$\Rightarrow (x-3)(x+3)\le 0$

$\Rightarrow -3\le x\le 3$ ... (1)

$\sin^{-1}(3-x)$ is defined for

$-1\le 3-x\le 1 \Rightarrow -4\le -x\le -2 \Rightarrow 2\le x\le 4$ ... (2)

Also, $\sin^{-1}(3-x)=0 \Rightarrow 3-x=0$ or $x=3$ ... (3)

From (1), (2) and (3), we get the domain of f :

$([-3, 3]\cap[2, 4])\setminus\{3\} = [2, 3)$.

65. (D)

$|4-3x|\le\frac{1}{2} \Rightarrow -\frac{1}{2}\le 4-3x\le\frac{1}{2}$

$4-3x\le\frac{1}{2}$ and $4-3x\ge-\frac{1}{2}$

$\frac{7}{2} - 3x \le 0;\ \frac{9}{2} - 3x \ge 0$

$x \ge \frac{7}{6};\ x \le \frac{3}{2}$

$x \in \left[\frac{7}{6}, \frac{3}{2}\right]$

$\therefore$ Option (D) is correct.

66. (D)

Since f(x) is an odd function,

$\left[\frac{x^2}{a}\right] = 0$ for all $x \in [-10, 10] \Rightarrow 0 \le \frac{x^2}{a} < 1$ for all $x \in [-10, 10]$

$\Rightarrow$ a > 100 Hence, (D) is the correct answer

67. (C)

68. (A)

$$\lim_{x \to 2^-} \frac{\cos(2x-4) - 33}{2} = -16$$

$$\lim_{x \to 2^-} \frac{x^2 |4x-8|}{x-2} = -16$$

$\therefore$ By sandwich theorem ; $\lim_{x \to 2^-} f(x) = -16$

69. (C)

$$f(x) = \begin{cases} 1 & ; \quad x \in I \\ 0 & ; \quad x \notin I \end{cases}.$$

70. (C)

$$f(x) = \begin{cases} -1 & ; \quad -1 < x < 1 \\ 0 & ; \quad x = 1, -1 \\ 1 & ; \quad |x| > 1 \end{cases}$$

71. (B)

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{f(h)+|x|h+x\cdot h^2}{h}$$

Also x = y = 0 $\Rightarrow f(0)=0$

$$\therefore f'(x)=\lim_{h\to 0}\left(\frac{f(h)-f(0)}{h}+|x|+xh\right) \qquad \Rightarrow f'(x)=f'(0)+|x|$$

72. (C)

For differentiablility at x = 1; $g'(1^+)=g'(1^-)$

$$\Rightarrow a=6x-\frac{4}{2\sqrt{x}}\Rightarrow a=6-2=4$$

For continuity at x = 1; a + b = 3 – 4 + 1 = b = –4

73. (A)

$$\tan^{-1}\frac{1}{\sqrt{2}}-\tan^{-1}\frac{\sqrt{\left(\sqrt{3}-\sqrt{2}\right)^2}}{1+\sqrt{3}.\sqrt{2}}=\tan^{-1}\frac{1}{\sqrt{2}}-\tan^{-1}\sqrt{3}+\tan^{-1}\sqrt{2}$$

$$=\cot^{-1}\sqrt{2}+\tan^{-1}\sqrt{2}-\tan^{-1}\sqrt{3} \qquad =\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$$

74. (D)

f is not one-one as f(0) = 0 and f(–1) = 0. f is also not onto as for y = 1 there is no x $\in$ R such that f(x) = 1. If there is such an x $\in$ R, then $e^{|x|}-e^{-x}=e^x+e^{-x}$ .Clearly x $\neq 0$.For x > 0, this equation gives e^{-x} = 0 which is not possible and for x < 0, $\frac{e^{2x}+1}{e^x}=0$, which is also not possible.

Hence (D) is the correct answer.

75. (D)

$$\cos\left(\tan^{-1}x\right)=x$$

$$\Rightarrow\frac{1}{\sqrt{1+x^2}}=x$$

$\Rightarrow x^2 = \frac{\sqrt{5}-1}{2} \quad \Rightarrow \frac{x^2}{2} = \frac{\sqrt{5}-1}{4} = \sin\frac{\pi}{10}.$

76. (C)

$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Range of right hand side is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow -\frac{\pi}{2} \le 2\sin^{-1}x \le \frac{\pi}{2} \Rightarrow -\frac{\pi}{4} \le \sin^{-1}x \le \frac{\pi}{4} \quad \Rightarrow x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right].$

Hence (C) is the correct answer.

77. (C)

We have, $\sin^{-1}x > \cos^{-1}x$

$\Rightarrow \sin^{-1}x > \frac{\pi}{2} - \sin^{-1}x$

$2\sin^{-1}x > \frac{\pi}{2} \Rightarrow \sin^{-1}x > \frac{\pi}{4}.$

$\Rightarrow \sin(\sin^{-1}x) > \sin\frac{\pi}{4} \quad \Rightarrow x > \frac{1}{\sqrt{2}} \Rightarrow x \in \left(\frac{1}{\sqrt{2}}, 1\right] \quad$ since $-1 \le x \le 1$

78. (C)

Sine of integral multiple of $\pi = 0$

79. (B)

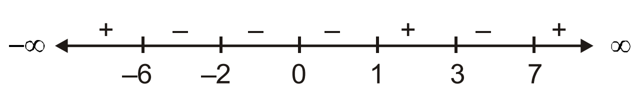
$\frac{|1+\{x\}|}{1+\{x\}} = 1$

80. (C)

$x^{\log_x a \times \log_a y \times \log_y z} = x^{\frac{\log a}{\log x}\cdot\frac{\log y}{\log a}\cdot\frac{\log z}{\log y}} = x^{\log_x z} = z$

81. (A)

$(x-1)^3(x+2)^4(x-3)^5(x+6) \ge 0$



$\therefore x \in (-\infty, -6] \cup [1,3] \cup (7,\infty) \cup \{-2\}$

82. (A)

$$f(-x) = \frac{\cos(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}} = \frac{\cos x}{-\left[\frac{x}{\pi}\right]-1+\frac{1}{2}} \qquad \left(\text{as } x \neq n\pi \Rightarrow \frac{x}{\pi} \notin I, \text{ so as} \left[-\frac{x}{\pi}\right] = -\left[\frac{x}{\pi}\right]-1\right)$$

$$= -\frac{\cos x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} = -f(x) \Rightarrow f(x) \text{ is an odd function.}$$

83. (B)

$$f(x) = \sqrt{3}\sin x - \cos x + 2 = 2\sin\left(x - \frac{\pi}{6}\right) + 2$$

Since f(x) is one–one and onto, f is invertible.

Now $fof^{-1}(x) = x \Rightarrow 2\sin\left(f^{-1}(x) - \frac{\pi}{6}\right) + 2 = x$

$$\Rightarrow \sin\left(f^{-1}(x) - \frac{\pi}{6}\right) = \frac{x}{2} - 1 \Rightarrow f^{-1}(x) = \sin^{-1}\left(\frac{x}{2}-1\right) + \frac{\pi}{6}$$

Because $\left|\frac{x}{2}-1\right| \le 1$ for all $x \in [0, 4]$

84. (A)

$3x^2 - 10x + 3 = 0 \Rightarrow x = 3, 1/3$

or, $|x - 5| = 1 \Rightarrow x = 6, 4.$

85. (D)

$$\tan\left(\frac{2\pi}{5} - \frac{\pi}{15}\right) = \frac{\tan\frac{2\pi}{5} - \tan\frac{\pi}{15}}{1 + \tan\frac{2\pi}{5}\cdot\tan\frac{\pi}{15}}$$

$$\sqrt{3}\left(1 + \tan\frac{2\pi}{5}\cdot\tan\frac{\pi}{15}\right) = \tan\frac{2\pi}{5} - \tan\frac{\pi}{15}$$

86. (B)

We have,

$$\cos\theta \cos2\theta \cos2^2\theta \ldots \cos2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n \sin\theta} = \frac{\sin(\pi - \theta)}{2^n \sin\theta} \qquad [\because 2^n\theta = \pi - \theta]$$

$$= \frac{1}{2^n}$$

87. (C)

$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$$

$$\therefore \cos\frac{5\pi}{8}=\cos\left(\pi-\frac{3\pi}{8}\right)=-\cos\frac{3\pi}{8}$$

$$\cos\frac{7\pi}{8}=\cos\left(\pi-\frac{\pi}{8}\right)=-\cos\frac{\pi}{8}$$

$$\therefore \left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$$

$$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)=\left(\sin^2\frac{\pi}{8}\right)\left(\sin^2\frac{3\pi}{8}\right)$$

$$=\frac{\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\frac{3\pi}{4}\right)}{4}=\frac{\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)}{4}=\frac{1-\frac{1}{2}}{4}=\frac{1}{8}$$

88. (A)

$\sin\alpha+\sin\beta=-\frac{21}{65}$ and $\cos\alpha+\cos\beta=-\frac{27}{65}$

squaring and adding, we get

$\sin^2\alpha+\sin^2\beta+2\sin\alpha\sin\beta+\cos^2\alpha+\cos^2\beta+2\cos\alpha\,.\,\cos\beta$

$$=\left(-\frac{21}{65}\right)^2+\left(-\frac{27}{65}\right)^2$$

$$\Rightarrow 2+2\cos(\alpha-\beta)=\frac{1170}{4225}$$

$$\Rightarrow \cos^2\left(\frac{\alpha-\beta}{2}\right)=\frac{1170}{4\times 4225}=\frac{9}{130}$$

$$\Rightarrow \cos\left(\frac{\alpha-\beta}{2}\right)=\frac{-3}{\sqrt{130}}\quad \left(\because \pi<\alpha-\beta<3\pi \Rightarrow \frac{\pi}{2}<\left(\frac{\alpha-\beta}{2}\right)<\frac{3\pi}{2}\right)$$

89. (A)

tan 50 – tan 40 = k tan 10

$$\frac{\sin 50}{\cos 50}-\frac{\sin 40}{\cos 40}=k\tan 10$$

$$\Rightarrow \frac{\sin 50\cos 40-\sin 40\cos 50}{\cos 50\cos 40}=k\tan 10$$

$$\Rightarrow \frac{\sin 10}{\cos 50\cos 40}=\frac{k\sin 10}{\cos 10}$$

$$k=\frac{\cos 10}{\cos 50\sin 50}=\frac{2\cos 10}{\sin 100}=2$$

90. (B)

Clearly, $f(x) \geq 0$ for any $x \in R$. Moreover, $(x^2 - 1)^2 = x^4 - 2x^2 + 1 \geq 0$ for any $x \in R$,

so $\frac{x^2}{x^4+1} \leq 1/2$. Hence the range of f is [0, 1/2].