SOLUTIONS

WEEKLY TEST-8

GZRA-1901 & 1902

(JEE MAIN PATTERN)

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PHYSICS

1. (A)

Here a = 4, b = – 3. So, the equation of the line is

$$\frac{x}{a}+\frac{y}{b}=1 \text{ or}, \frac{x}{4}+\frac{y}{-3}=1 \text{ or}, 3x-4y=12.$$

2. (D)

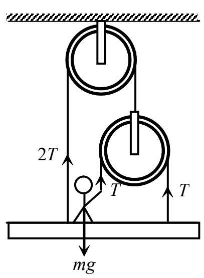
$$\frac{dy}{dx}=4-4x=0$$

$$x=1$$

$$\frac{d^2y}{dx^2}=-4<0$$

So, maxima will occur at x = 1

3. (A)



$$4T=mg$$

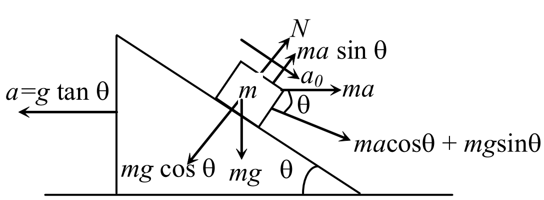
$$\therefore \quad T=\frac{60\times 10}{4} = 150 \text{ N}$$

4. (A)

$$a=\frac{F}{m_{total}}; \; N_{2kg}=F-3a; \; N_{1kg}=1a$$

5. (B)

Drawing *FBD* of block *m* from the frame of wedge,



let a_0 is acceleration of block with respect to wedge,

$ma_0 = ma\cos\theta + mg\sin\theta$

$a_0 = g\tan\theta\cos\theta + g\sin\theta$

$a_0 = 2g\sin\theta$

6. (B)

From figure (1)

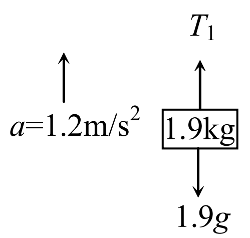


Figure (1)

$$T_1 - 1.9g = 1.9\times 1.2$$

$\Rightarrow\ T_1 = 20.9$ N

From figure (2)

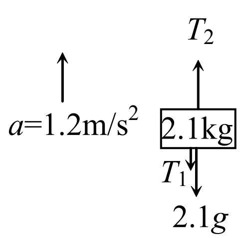
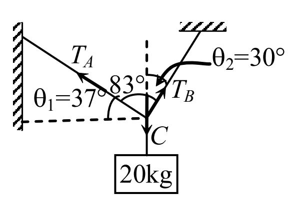


Figure (2)

$$T_2 - T_1 - 2.1g = 2.1\times 1.2$$

$T_2 = 44$ N

7. (A)



$T_A\cos\theta_1 = T_B\sin\theta_2$

$T_A\cos 37° = T_B\sin 30°$

$T_A\times\frac{4}{5} = T_B\times\frac{1}{2};\quad \frac{T_A}{T_B} = \frac{5}{8}$

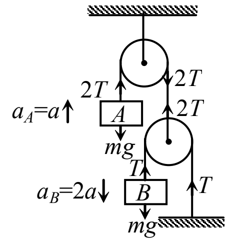
8. (D)

$2T - mg = ma$ …(i)

$mg - T = 2ma$ …(ii)

(i) and (ii) $\Rightarrow \quad a = \dfrac{g}{5}$

$\therefore \quad a_B = \dfrac{2g}{5}$



9. (C)

From constraint relation, $a_B = 8a_A$

10. (D)

$a_1 > 0$ when $\dfrac{F}{4} > 50$, $F > 200$

$a_2 > 0$ when $\dfrac{F}{4} > 100$, $F > 400$

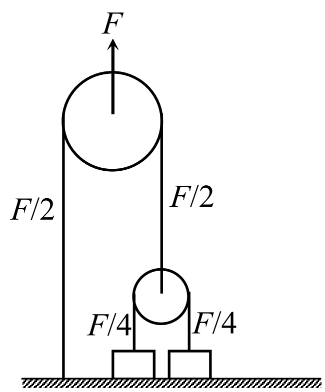
F = 300 N

$a_1 = \dfrac{F/4 - 50}{5} = \dfrac{300/4 - 50}{5} =$ 5 m/s²

$a_2 = 0$

If $F = 500$ N

$a_1 = 15$ m/s², $a_2 = 2.5$ m/s²



11. (A)

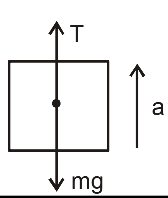
Given that $\vec{p} = p_x\hat{i} + p_y\hat{j} = 2\cos t\,\hat{i} + 2\sin t\,\hat{j}$

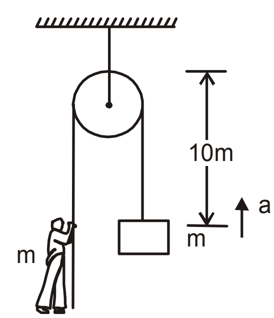
$\therefore \quad \vec{F} = \dfrac{d\vec{p}}{dt} = -2\sin t\,\hat{i} + 2\cos t\,\hat{j}$

Now, $\vec{F}\cdot\vec{p} = 0$ i.e. angle between $\vec{F}$ and $\vec{p}$ is 90°.

12. (B)

Let us suppose, block goes up with acceleration a; then from F.B.D. of block ;





T – mg = ma ....(i)

As man is going up with 2.5 ms^{-2} acceleration with respect to string. And string goes down with respect to ground. So net acceleration of main w.r.t. ground = $(2.5-a)\uparrow$

From FB.D. of man ;

$(2.5-a)\uparrow$

T – mg = m(2.5 – a) ....(ii)

from (i) & (ii), we get ;

ma = m (2.5 – ma)

ma + ma = 2.5 m

$\Rightarrow \ 2\cancel{m}a = 2.5\cancel{m}$

$$\therefore \ a = \frac{2.5}{2} = 1.25 \text{ m/s}^2$$

$\therefore$ Time taken by block to reach the pulley ;

$$t = \sqrt{\frac{2S}{a}} = \sqrt{\frac{2\times \cancel{10}}{\cancel{1.25}}} = \sqrt{16}$$

13. (C)

$$\vec{A}-\vec{B} = -\hat{i}+5\hat{j}+\hat{k}; \qquad \theta = \cos^{-1}\left(\frac{(\vec{A}-\vec{B})\cdot\vec{C}}{|\vec{A}-\vec{B}||\vec{C}|}\right)$$

$$= \cos^{-1}\left(\frac{(-1+5+2)\cancel{P}}{3\sqrt{3}\sqrt{6}\cancel{P}}\right) \quad = \cos^{-1}\left(\frac{\cancel{6}\,2}{3\cancel{\sqrt{3}}\ \cancel{\sqrt{3}}\sqrt{2}}\right) = \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

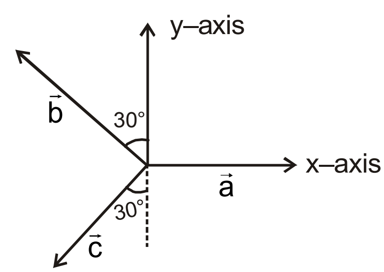
14. (C)

$$\vec{a} = 2\hat{i}, \ \vec{b} = 3\cos 30^\circ\hat{j} - 3\sin 30\,\hat{i}$$

$$= \frac{3\sqrt{3}}{2}\hat{j} - \frac{3}{2}\hat{i}$$

$$\vec{c} = -6\cos 30\,\hat{j} - 6\sin 30\,\hat{i}$$

$$= -3\sqrt{3}\,\hat{j} - 3\hat{i}$$



$\therefore \quad \vec{a}+\vec{b}+\vec{c} = 2\hat{i}-3\hat{i}-\frac{3}{2}\hat{i}+\frac{3\sqrt{3}}{2}\hat{j}-3\sqrt{3}\,\hat{j}$

$$=-\frac{5}{2}\hat{i}-\frac{3\sqrt{3}}{2}\hat{j}$$

$\therefore \quad |\vec{a}+\vec{b}+\vec{c}| = \left[\left(-\frac{5}{2}\right)^2+\left(-\frac{3\sqrt{2}}{2}\right)^2\right]^{\frac{1}{2}} = \left(\frac{25}{4}+\frac{27}{4}\right)^{\frac{1}{2}}$

$$\left(\frac{52}{4}\right)^{\frac{1}{2}} = \sqrt{13}$$

15. (A)

$$\theta = \cos^{-1}\left(\frac{2+\cancel{2}-\cancel{2}}{\sqrt{3}\,2\sqrt{3}}\right) = \cos^{-1}\left(\frac{\cancel{2}}{\sqrt{3}\,\cancel{2}\sqrt{3}}\right)$$

$$\boxed{\therefore\ \theta = \cos^{-1}(1/3)}$$

16. (B)

$|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2 \qquad \text{...(i)}$

$= a^2+b^2-2ab\cos\theta_1+b^2+c^2-2bc\cos\theta_2+c^2+a^2-2ca\cos\theta_3$

$= 6-2(\cos\theta_1+\cos\theta_2+\cos\theta_3)$

for max. value of (i) $2(\cos\theta_1+\cos\theta_2+\cos\theta_3)$ should be minimum.

Now, $(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c}) = |\vec{a}+\vec{b}+\vec{c}|^2$

$\Rightarrow a^2+b^2+c^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = |\vec{a}+\vec{b}+\vec{c}|^2$

$\Rightarrow 3+2(\cos\theta_1+\cos\theta_2+\cos\theta_3) = |\vec{a}+\vec{b}+\vec{c}|^2$

$\therefore\ 2(\cos\theta_1+\cos\theta_2+\cos\theta_3) = \underbrace{|\vec{a}+\vec{b}+\vec{c}|^2}_{\geq 0}-3$

So, its minimum value will be only when $|\vec{a}+\vec{b}+\vec{c}|^2=0$. i.e. 120° angle setup with each vector $\vec{a},\vec{b}$ and $\vec{c}$

So, $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=6-\left(2\times\frac{-3}{2}\right)=9$

Hence Maximum value of equation (i) is 9.

17. (A)

Let $\vec{b}=b_x\hat{i}+b_y\hat{j}+b_z\hat{k}$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ b_x & b_y & b_z\end{vmatrix}=\hat{i}(b_z-b_y)+\hat{j}(b_x-b_z)+\hat{k}(b_y-b_x)$$

given : $\vec{a}\times\vec{b}=\hat{j}-\hat{k}\Rightarrow$ $b_x-b_z=1$ .... (i) & $b_y-b_x=-1$ ...... (ii) and $b_z-b_y=0$

and also, $\vec{a}.\vec{b}=1$

$b_x+b_y+b_z=1$ .... (iii)

from equation (i), (ii), (iii) ; $b_x=1$, $b_y=0$, $b_z=0$

$\vec{b}=\hat{i}$

18. (C)

$\vec{a}=\hat{i}+2\hat{j}+\hat{k},\ \ \vec{b}=\hat{i}-\hat{j}+\hat{k}$

Let $\vec{d}$ is a vector in the plane of $\vec{a}$ & $\vec{b}$. then ,

$\vec{d}=\vec{a}+\lambda\vec{b}=(1+\lambda)\hat{i}+(2-\lambda)\hat{j}+(1+\lambda)\hat{k}$

Given projection of $\vec{d}\,\text{on}\,\vec{c}=1/\sqrt{3}$

$\hat{c}.\vec{d}=1/\sqrt{3}$

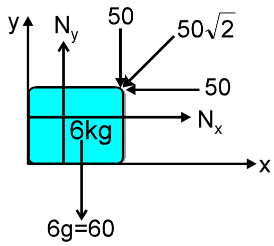
$$\Rightarrow\ \frac{(\hat{i}+\hat{j}-\hat{k})}{\sqrt{\cancel{3}}}.\left[(1+\lambda)\hat{i}+(2-\lambda)\hat{j}+(1+\lambda)\hat{k}\right]=\frac{1}{\sqrt{\cancel{3}}}$$

$\Rightarrow\ \cancel{1}+\cancel{\lambda}+2-\cancel{\lambda}-\cancel{1}-\lambda=1$

$\lambda=1$

$\therefore\ \vec{d}\ =2\hat{i}+\hat{j}+2\hat{k}$

19. (A)



$N_y = 110$

$N_x = 50$

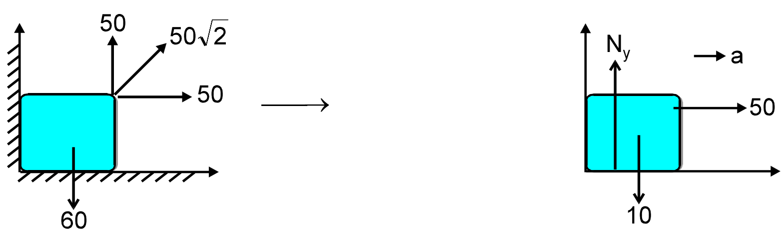
$\vec{N}_{block} = 50\hat{i} + 110\hat{j}$

So A is correct

$\vec{N}_{wall}$ will be opposite of N_{block} = $-50\hat{i} - 110\hat{j}$

So B is incorrct.

If $\vec{F}$ were reversed



it will loose contact with verticle wall and $N_y = 10\hat{j}$

So option C is incorrect.

Net force = $50\hat{i} = 6\vec{a} \Rightarrow$ $\boxed{\vec{a} = \frac{50}{6}\hat{i}}$

20. (B)

21. (A)

Tension = m (g + a), when lift moving up, putting the values, we get

$$175 = 25 (9.8 + a)$$

$$\Rightarrow \quad a = 2.8 \text{ m/s}^2$$

[negative sign shows that lift is moving downward]

22. (B)

Apparent tension,

$T = 2T_0$

So $T = 2T_0 = T_0\left(1+\frac{a_0}{g}\right)$

or $2 = 1+\frac{a_0}{g}$

$\Rightarrow a_0 = g = 9.8\,m/s^2$

23. (A)

As net force on the rod = $F_1 - F_2$ and its mass is M so acceleration of the rod will be

$a = (F_1 - F_2)/M$ ...(i)

Now considering the motion of part AB of the rod, which has mass (M/L)y,

Acceleration a given by

(i) Assuming that tension at B is T

$F_1 - T = \frac{M}{L}y \times a$ (from F = ma)

$\Rightarrow F_1 - T = \frac{M}{L}y\frac{F_1 - F_2}{M}$ (using eqn. (i))

$\Rightarrow T = F_1\left(1-\frac{y}{L}\right)+F_2\left(\frac{y}{L}\right)$

24. (A)

25. (A)

26. (A)

27. (D)

$T = 0$, $a = g$

28. (A)

29. (A)

In condition (i), $20g - T = 20a$, $N = 20a$

$T - N = 40a$ $\Rightarrow a = \frac{20g}{80} = \frac{g}{4}$

Net acceleration = $a_1 = a\sqrt{2}$, $\sqrt{2}\,a = \frac{\sqrt{2}\,g}{4} = \frac{g}{2\sqrt{2}}$

In condition (ii) $20g - T = 20a$, $T = 40a$, $a = \frac{g}{3}$, $a_2 = \frac{g}{3}$

$$\frac{a_1}{a_2} = \frac{g/2\sqrt{2}}{g/3} = \frac{3}{2\sqrt{2}}$$

$\therefore$ **(A)**

30. **(C)**

CHEMISTRY

31. **(C)**

$\phi = \frac{\pi}{2}$ is yz plane.

32. **(C)**

$$R\left(1 - \frac{1}{4}\right) = Rz^2\left(\frac{1}{4} - \frac{1}{16}\right)$$

or, z = 2

33. **(B)**

For 4d, n = 4, l = 2

34. **(A)**

l = 2

$$\sqrt{l(l+1)}\hbar = \sqrt{6}\hbar$$

35. **(D)**

36. **(D)**

Two electrons in an orbital have opposite spin.

37. **(A)**

l = 0 to (n – 1)

38. **(B)**

39. **(C)**

40. **(B)**

ψ^2 represents the probability density.

41. **(C)**

42. **(A)**

$$mv = \frac{h}{\lambda}, m = \frac{h}{\lambda c} \ (v = c) \text{ or } m \propto \frac{1}{\lambda}$$

43. **(C)**

Limiting line comes from ∞

44. **(B)**

$$R4\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

equating, $n_1 = 4$, $n_2 = 6$

45. **(B)**

$E_{Green} > E_{Yellow} > E_{Red}$.

46. **(C)**

Photoelectric current depends only on intensity

47. **(A)**

48. **(B)**

$\lambda \propto \frac{1}{\sqrt{K \cdot E}}$ and K.E decreases as n increases

49. **(A)**

$\lambda = v$

$$mv = \frac{h}{\lambda} = \frac{h}{v}$$

or, $v = \sqrt{\frac{h}{m}}$

50. **(B)**

51. **(D)**

$2\pi r = n\lambda$

52. **(B)**

$$P \cdot E = \frac{-2 \times 13.6 \times 1}{4} = -6.8 \text{ e.V}$$

53. **(A)**

$\Delta E = h\upsilon$

54. **(A)**

$$54.4 = \frac{E^0 \times 4}{1} \text{ or } E^0 = 13.6 \text{ e.V}$$

$$I.E(H) = 13.6 \text{ e.V}$$

$$I.E(Li^{+2}) = 13.6 \times 9 \text{ e.V} = 122.4 \text{ e.V}$$

55. **(C)**

For H energy depends only on n

56. **(A)**

$$K \cdot E = \frac{+13.6z^2}{n^2}$$

57. **(B)**

$$r = \frac{a_0 n^2}{z}$$

58. **(A)**

$$E = \frac{-13.6z^2}{n^2}$$

59. **(C)**

$$\frac{n_1}{n_2} = \sqrt{\frac{R}{4R}} = \frac{1}{2}$$

$$T \propto n^3$$

So, $\frac{T_1}{T_2} = \frac{1}{8}$

60. **(A)**

K.E = P.E = QV = ex1= 1e.V.

MATHEMATICS

61. **(B)**

$$\sqrt{12 - \sqrt{68 + 48\sqrt{2}}}$$

$= \sqrt{12 - \sqrt{(6 + 4\sqrt{2})^2}} = \sqrt{12 - 6 - 4\sqrt{2}} = \sqrt{6 - 4\sqrt{2}} = \sqrt{(2 - \sqrt{2})^2} = 2 - \sqrt{2}$

62. **(B)**

x = $\sqrt{3 - \sqrt{5}}$ y = $\sqrt{3 + \sqrt{5}}$

xy = 2

x + y = $\sqrt{x^2 + y^2 + 2 \times 2}$

= $\sqrt{6 + 4}$ = $\sqrt{10}$

x – y = $\sqrt{6 - 4}$ = $\sqrt{2}$

Put the value we get the ans.

$(x - y) + 2xy\,(x + y) - xy(x - y)\,(x^2 + y^2 + xy)$

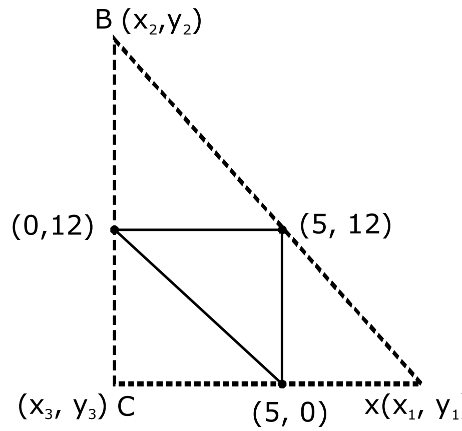
$-\sqrt{450} + \sqrt{160}$

63. (A)

$$\begin{cases} x_1 + x_3 = 10 \quad , \quad y_1 + y_3 = 0 \\ x_2 + x_3 = 0 \quad , \quad y_2 + y_3 = 24 \\ x_1 + x_2 = 10 \quad , \quad y_2 + y_2 = -24 \end{cases}$$

$x_1 = x_2 = 10$, $y_1 - y_2 = -24$
$x_1 = 10$, $y_1 = 0$
$x_2 = 0$, $y_2 = 24$
$x_3 = 0$, $y_3 = 0$

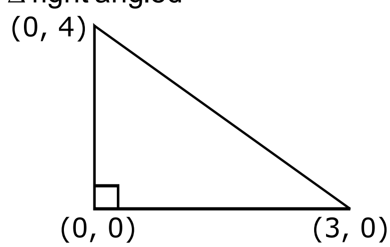
$$\left.\begin{matrix} x_1 = 10 \quad , \quad y_1 = 0 \\ x_2 = 0 \quad , \quad y_2 = 24 \\ x_3 = 0 \quad , \quad y_3 = 0 \end{matrix}\right\} \Rightarrow \begin{matrix} A(10,0) & \text{on} & x-\text{axis} \\ B(0,24) & \text{on} & y-\text{axis} \\ C(0,0) & \text{is} & \text{origin} \end{matrix}$$



ΔABC is right angled $\Rightarrow$ orthocentre is (0, 0)

64. (C)

Δ right angled

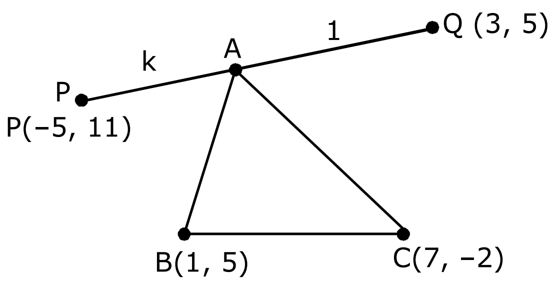


$\Rightarrow$ circum centre

= mid point of hypotaneous = $\left(\frac{3}{2}, 2\right)$

65. (C)

$$\left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$$



$$\frac{1}{2}\begin{vmatrix} \frac{3k-5}{k+1} & \frac{5k+1}{k+1} & 1 \\ 1 & 5 & 1 \\ 7 & -2 & 1 \end{vmatrix} = |2|$$

$\Rightarrow$ $1.(-2-3)-1.\left(\frac{-6k+10}{k+1}-\frac{35k+7}{k+1}\right)+\left(\frac{15k-25}{k+1}-\frac{5k+1}{k+1}\right)=\pm 4$

$\Rightarrow$ 6k – 10 + 35 k + 7 + 15k – 25 – 5k – 1

= ± 4 + 37 (k + 1)

$\Rightarrow$ 51 k – 29 = 41 k + 41 or 51 k – 29

= 33k + 33

$\Rightarrow$ 10 k = 70or 18 k = 62

k = 7 $\quad\quad$ $k = \frac{31}{9}$

66. (A)

$x = 2^{\log_2 8 \log_{11}^{1331}}$

$x = 2^9$ $\quad\quad$ $y = 2^{\frac{1}{4}}$

$\text{Log}_B N = \frac{\log 4}{\log 5}\frac{\log 5}{\log 6}\text{------}\frac{\log 36}{\log 37}$

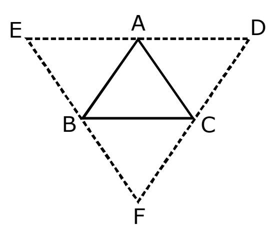
$= \frac{\log 4}{\log 37} = \log_{37}{}^{4}$

$Z = \frac{4}{37}$

$(x\,y)^z = (2^9.2^{1/4})\frac{4}{37} = \left(2^{\frac{37}{4}}\right)^{\frac{4}{37}} = 2$

67. (C)

$A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$



only three sides can be made parallel to corresponding sides of triangle passing through vertex of triangle respectively

$\Rightarrow$ So no. of llgrams is 3.

68. **(A)**

Let A(–a, –b); B (0, 0); C (a, b); D (a^2, ab)

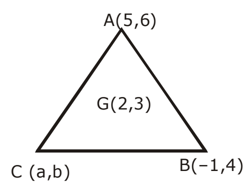
$$\therefore \text{Area of quadrilateral } \Delta ACD = \frac{1}{2}\begin{vmatrix} -a & -b \\ 0 & 0 \\ a & b \\ a^2 & ab \\ -a & -b \end{vmatrix}$$

$= \frac{1}{2}$ [(0 + 0 + a^2 b – a^2 b) +

(0 + 0 + a^2 b – a^2 b)]

Hence, A, B, C, D are collinear.

69. **(B)**



$\frac{5-1+a}{3} = 2 \Rightarrow a = 2$

& $\frac{6+4+b}{3} = 3 \Rightarrow b = (-1)$

70. **(B)**

Let ratio be $\lambda : 1 \Rightarrow \frac{6\lambda - 3}{\lambda + 1} = 0,\ \lambda = \frac{1}{2}$

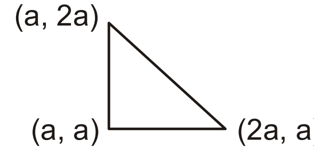
71. **(C)**

$x = \frac{711}{13+11m} = \frac{9 \times 79}{13+11m}$

if x is an integer, then m = 6

72. **(D)**

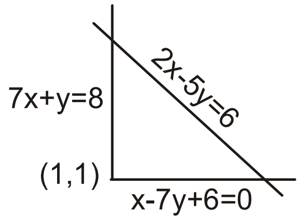
$\frac{1}{2}a^2 = 72$



$a = \pm 12$

Centroid = (16, 16) or (-16, -16)

73. (C)



74. (A)

$$y = \frac{3}{4}(x-9)+6$$

75. (C)

$$S_\infty = \frac{a}{1-r} = 2$$

$$\Rightarrow \frac{(\alpha / r)}{1-r} = 2 \Rightarrow \frac{\alpha}{2} = r - r^2$$

If $-1 < r < 1$, then, $-2 < r - r^2 < \frac{1}{4}$

$$\therefore \quad -2 < \frac{\alpha}{2} < \frac{1}{4} \Rightarrow \alpha \in \left(-4, \frac{1}{2}\right)$$

76. (C)

$(A \cap B) \cup C = \{1,3,5,7,8,9\}$

$A' \cap B' = \{10\}$

$(A \cup B)' = \{10\}$

$(A \cap B) \cap (A \cap C) = \{8\}$

77. (C)

We have,

$(2x-3y)^2 + (3y-4z)^2 + (4z-2x)^2 = 0 \Rightarrow 2x = 3y = 4z$

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in AP $\Rightarrow$ x, y, z are in HP

78. (C)

$$\frac{H_1+2}{H_1-2}+\frac{H_{20}+3}{H_{20}-3} = \frac{\frac{1}{2}+\frac{1}{H_1}}{\frac{1}{2}-\frac{1}{H_1}}+\frac{\frac{1}{3}+\frac{1}{H_{20}}}{\frac{1}{3}-\frac{1}{H_{20}}}$$

$$=\frac{\frac{1}{2}+\frac{1}{2}+d}{\frac{1}{2}-d-\frac{1}{2}}+\frac{\frac{1}{3}+\frac{1}{3}-d}{\frac{1}{3}+d-\frac{1}{3}} = \frac{1+d}{-d}+\frac{\frac{2}{3}-d}{d} = \frac{\frac{2}{3}-1}{d}-2 = 2\times 21-2 = 40$$

79. (A)

$\sin\alpha+\cos\alpha=-\frac{b}{a}$ and $\sin\alpha\cos\alpha=\frac{c}{a}$

$$\Rightarrow 1+2\sin\alpha\cos\alpha=\frac{b^2}{a^2} \Rightarrow 1+\frac{2c}{a}=\frac{b^2}{a^2} \Rightarrow a^2+2ac-b^2=0$$

80. (C)

sec 40°, sec 80°, sec 160° are the roots of $\frac{8}{t^3}-\frac{6}{t}+1=0$

or $t^3-6t^2+8=0$

$\therefore$ Sum of roots = 6.

81. (B)

We have, $\sin\theta+\cos\theta=m$

and $\sec\theta+\operatorname{cosec}\theta=n$

$$\Rightarrow \frac{1}{\cos\theta}+\frac{1}{\sin\theta}=n \Rightarrow \frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}=n$$

$$\Rightarrow \frac{m}{\cos\theta\sin\theta}=n$$

$$\Rightarrow \cos\theta\sin\theta=\frac{m}{n}$$

Squaring (i), we get

$$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=m^2 \Rightarrow 1+2\cdot\frac{m}{n}=m^2$$

$$\Rightarrow \frac{2m}{n}=m^2-1 \Rightarrow 2m=n(m^2-1).$$

82. **(C)**

a_1, a_2, a_3, a_4, a_5 are in H.P.

$\Rightarrow a_2 = \dfrac{2a_1 a_3}{a_1 + a_3} \quad \Rightarrow 2a_1a_3 = a_2a_1 + a_3a_2$

$a_4 = \dfrac{2a_3a_5}{a_3 + a_5} \quad \Rightarrow 2a_3a_5 = a_3a_4 + a_5a_4$

$\Rightarrow a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5 = 2a_1a_3 + 2a_3a_5$ ... (i)

$a_3 = \dfrac{2(a_1a_5)}{a_1 + a_5} \quad \Rightarrow a_1a_3 + a_5a_3 = 2a_1a_5$ ... (ii)

using (i) and (ii)

$a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5 = 2(2a_1a_5) = 4a_1a_5$

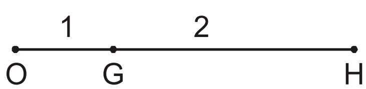
83. **(D)**

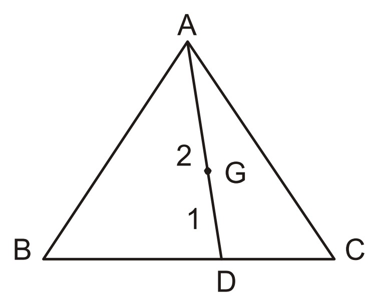
$\text{Area} = \frac{1}{2}\left|P^3 - P + 1\right|$

Cubic in 'P' so it must vanish

84. **(A)**

Circum-centre O, ortho-centre H centroid G





$\therefore G \equiv \left(1, \frac{8}{9}\right)$

$AG : GD = 2 : 1$

85. **(A)**

P lies on perpendicular bisector of BC and at a distance of $\frac{\sqrt{3}}{2}BC = \sqrt{15}$ units from the mid-point of BC.

$\therefore$ P can be $\left(\sqrt{3}, 2-\sqrt{3}\right)$ or $\left(-\sqrt{3}, 2+2\sqrt{3}\right)$ but A and P should lie on the same side of BC.

$\therefore$ P is $\left(-\sqrt{3}, 2+2\sqrt{3}\right)$

86. (A)

$$\frac{1}{2}ab = 11 \quad \Rightarrow ab = 22$$

Also $\frac{2}{a} + \frac{3}{b} = 1 \Rightarrow 2b + 3a = ab$ $\Rightarrow 4b^2 + 9a^2 + 12ab = a^2b^2$

$\therefore \quad 4b^2 + 9a^2 = 220$

87. (D)

$2^{n+1}(n - 1) + 2 = 2^{n+10} + 2.$

$\therefore \quad n = 513.$

Sum of digits = 9.

88. (B)

$$\frac{(a^2 + 3a + 1)(b^2 + 3b + 1)(c^2 + 3c + 1)}{abc}$$
$$= \left(a + \frac{1}{a} + 3\right)\left(b + \frac{1}{b} + 3\right)\left(c + \frac{1}{c} + 3\right)$$
$$\geq (2 + 3)(2 + 3)(2 + 3) = 125$$

89. (C)

$$6\left\{\left(\sin^2\theta + \cos^2\theta\right)^3 - 3\sin^2\theta.\cos^2\theta\left(\sin^2\theta + \cos^2\theta\right)\right\} - 9\left\{\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta.\cos^2\theta\right\}$$

$$= 6\left\{1 - 3\sin^2\theta.\cos^2\theta\right\} - 9\left\{1 - 2\sin^2\theta\cos^2\theta\right\} = -3$$

90. (A)

$a_{2r} = a_{2r-1} + d$

$$\sum_{r=1}^{10^{99}} a_{2r} = \sum_{r=1}^{10^{99}} (a_{2r-1} + d)$$

$10^{100} = 10^{99} + 10^{99}\, d$

$d = 9.$