SOLUTIONS

WEEKLY TEST-2

RBA

(JEE ADVANCED PATTERN)

Test Date: 22-07-2017

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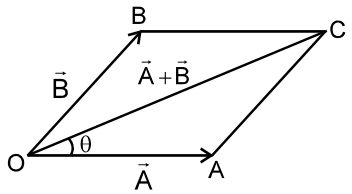
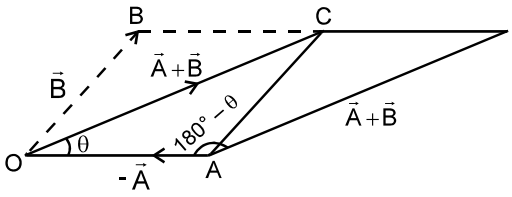
PHYSICS

1. (A)





From diagram $(\vec{A})\cdot(\vec{A}+\vec{B}) = |\vec{A}|\cdot|\vec{A}+\vec{B}|\cos\theta = 20$

or $\cos\theta = \frac{20}{4\times 10} = \frac{1}{2}$

Hence $\theta = \frac{\pi}{3}$

$\therefore \quad |\vec{B}| = AC = \sqrt{|\vec{A}|^2 + |\vec{A}+\vec{B}|^2 + 2|\vec{A}||\vec{A}+\vec{B}|\cos(\pi-\theta)} = \sqrt{16+100-80\times\frac{1}{2}} = \sqrt{76}$ units

2. (A)

$$\frac{\bar{a}\,.\,(\hat{i}+\hat{j})}{\sqrt{2}} = \frac{\bar{b}\,.\,(\hat{i}+\hat{j})}{\sqrt{2}}$$

Let $\bar{b} = x\hat{i}+y\hat{j}$

$$\frac{(3\hat{i}+4\hat{j})(\hat{i}+\hat{j})}{\sqrt{2}} = \frac{(x\hat{i}+y\hat{j})\,.\,(\hat{i}+\hat{j})}{\sqrt{2}}$$

$x + y = 7$ and $|\vec{a}| = |\vec{b}| \Rightarrow x^2 + y^2 = 3^2 + 4^2 = 25$

$y = 3, 4$ for $\vec{b}$ $y \neq 4$ so $y = 3 \Rightarrow x = 4$

$\therefore \quad \vec{b} = 4\hat{i}+3\hat{j}$

3. (C)

Given $|\vec{a}_1+\vec{a}_2| = \sqrt{3}$, $|\vec{a}_1| = 1$, $|\vec{a}_2| = 1$

squaring both sides

$\therefore \quad |\vec{a}_1|^2 + |\vec{a}_2|^2 + 2\vec{a}_1\bullet\vec{a}_2 = 3$

$\Rightarrow 1 + 1 + 2 \times 1 \times 1 \cos\theta = 3$

$\Rightarrow \cos\theta = 1/2$

$(\vec{a}_1-\vec{a}_2)\bullet(2\vec{a}_1+\vec{a}_2) = 2|\vec{a}_1|^2 - |\vec{a}_2|^2 - \vec{a}_1\bullet\vec{a}_2$

$= 2\,(1)^2 - (1)^2 - 1 \times 1 \times 1/2$

$= 2 - 1 - 1/2 = 1/2$

4. (B)

For refraction at water surface,

$u = -90$ cm

$\mu_2 = 1.2$

$\mu_1 = 4/3$

$v = \frac{u\mu_2}{\mu_1} = -81$ cm

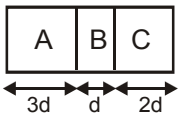
This image formed at 81 + 69 = 150 cm below the mirror, after reflection from the mirror will form its new image 150 cm above it.

5. (B)

Given that

Number of waves in A = Number of waves in B + Number of waves in C

$$\frac{3d}{\lambda_A} = \frac{d}{\lambda_B} + \frac{2d}{\lambda_C}$$



But $\mu = \frac{C}{f\lambda}$

Here C is constant (3×10^8 m/s) and frequency f does not depends on medium.

So, $\mu \propto \frac{1}{\lambda}$

$3\mu_A = \mu_B + 2\mu_C$

$3(1.5) = \mu_B + 2(1.6)$

$\mu_B = 4.5 - 3.2 = 1.3$.

6. (B)

Let the critical angle of interface between media 1 and 2 is C_1 and between 1 and 3 is C_2.

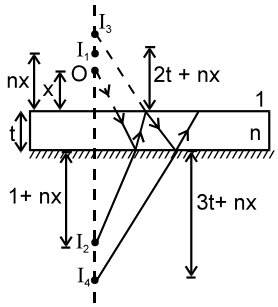
Then $\sin C_1 = \frac{n_2}{n_1}$ and $\sin C_2 = \frac{n_3}{n_1}$.

From TIR at second interface $90 - C_1 > C_2$. taking sin of both side we get

$\cos C_1 > \sin C_2$ or $\sqrt{1-\left(\frac{n_2}{n_1}\right)^2} > \frac{n_3}{n_1}$ or $n_1^2 - n_3^2 > n_2^2$

7. (B)

The figure shows the image formation



in different steps. Given 2 t = 4 $\Rightarrow$ t = 2 cm

8. (D)

The image is erect hence, mirror must be between object & image. Virtual image of real object is diminshed, hence mirror is convex.

9. (C)

10. Due to M_1, an image is formed at a distance x from M_1, i.e. at a distance $(x-y)$ behind M_2. Thus, for M_2

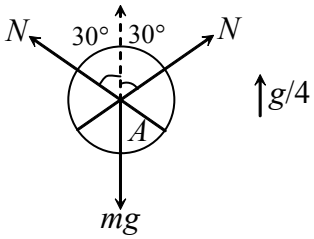
$u = -(x+y),\ v = x-y$

Use $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\therefore$ **(A)**

11. Net upward force on three spheres applied by bottom = $3mg+\frac{3}{4}mg=\frac{15mg}{4}$

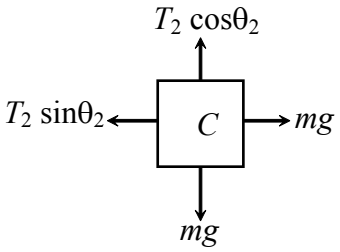
For sphere A, $N\sqrt{3}=mg+\frac{mg}{4}$, $N=\frac{5mg}{4\sqrt{3}}$

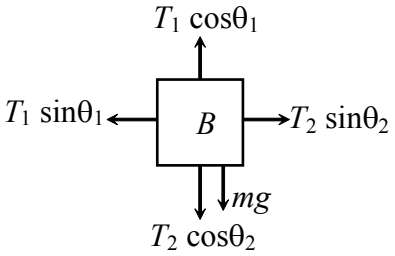


$\therefore$ **(B) and (D)**

12. $T_2 \sin\theta_2 = mg$ … (i) $T_1 \cos\theta_1 = mg + T_2 \cos\theta_2$

$T_2 \cos\theta_2 = mg$ …(ii) $T_1 \sin\theta_1 = T_2 \sin\theta_2$





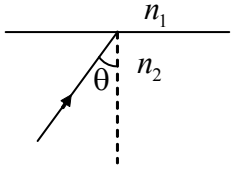
$\therefore$ **(A) (C) and (D)**

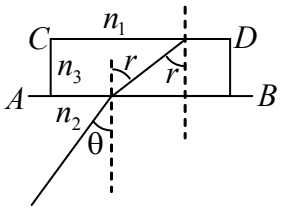
13. $\theta > \theta_{C_1} = \sin^{-1}\frac{n_1}{n_2}$

when slab is placed, $\theta_{C_2} = \sin^{-1}\frac{n_3}{n_2}$ if $n_3 < n_1$, $\theta_{C_2} < \theta_{C1} < \theta$

Hence ray will totally reflect at surface *AB*.

$\sin\theta_{C_1} = \frac{n_1}{n_2}$ $\theta > \theta_{C_1}$





at surface *AB*

$\frac{\sin r}{\sin\theta} = \frac{n_2}{n_3}$

$\sin r = \sin\theta\frac{n_2}{n_3}$

$\sin r > \sin\theta_{C_1}\frac{n_2}{n_3}$

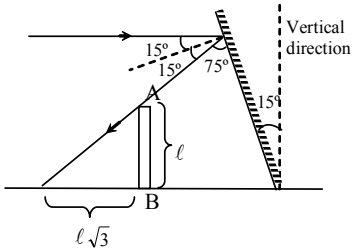
$$\sin r > \frac{n_1}{n_2} \times \frac{n_2}{n_3}$$

$$\sin r > \frac{n_1}{n_3}$$

r is incident angle at surface *CD* which is greater the critical angle at that surface so any value of n_3 say will reflect at surface.

∴ **(A), (C) And (D)**

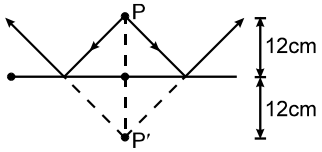
14. [A,B,C]

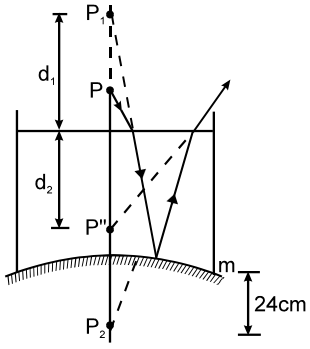


15. [A,C]

16. (2)

Due to reflection at plane water surface and other image formation is shown in the figure.





due to refraction at water $d_1 = \frac{12}{\frac{1}{4/3}}$ = 16 cm. for m, P_1 is an object.

for this $\frac{1}{v} + \frac{1}{-40} = \frac{1}{+60} \Rightarrow$ v = 24 cm this is at P_2.

It will act as object for water surface which makes image at P″. $d_2 = \frac{24+24}{4/3}$ = 36 cm

final images are P′ and P″

distance P′P″ = 36 – 12 = 24 cm.

$\therefore \quad n = 2$

17. (6)

I_1 is the image of object O formed by the lens.

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f}$$

$u_1 = -15 \qquad f_1 = 10$

Solving we get

$v_1 = 30$ cm

I_1 acts as source for mirror

$\therefore \quad u_2 = -(45 - v_1) = -15$ cm

I_2 is the image formed by the mirror

$$\therefore \quad \frac{1}{v_2} = \frac{1}{f_m} - \frac{1}{u_2} = -\frac{1}{10} - \frac{1}{15} \quad \therefore v_2 = -30\,\text{cm}$$

The height of I_2 above principal axis of lens is = $\frac{v_2}{u_2} \times 1 + 1 = 3\,\text{cm}$

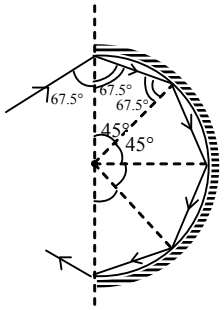
I_2 acts a source for lens $\quad \therefore u_3 = -(45 - v_2) = -15$ cm

Hence the lens forms an image I_3 at a distance $v_3 = 30$ cm to the left of lens and at a distance

The height of I_2 above principal axis of lens is = $\frac{v_2}{u_2} \times 1 + 1 = 3\,\text{cm}$

$\therefore \quad$ required distance = $\sqrt{30^2 + 6^2} = 6\sqrt{26}\,\text{cm}$

18. (5)



19. (3)

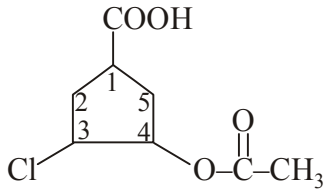
$t = \frac{\ell\sqrt{2}}{v}$

20. (6)

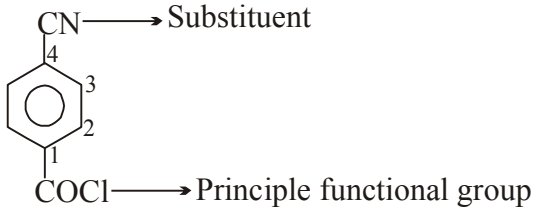
Draw & count

CHEMISTRY

21. (C)



22. (B)



23. (B)

24. (B)

$4Al + 3O_2 \longrightarrow 2Al_2O_3$

12 mole 12 mole

(LR) –9 mole

Excess remening = 3 % excess $= \frac{3}{9} \times 100 = 33.33\%.$

25. (A)

26. (A)

Lets 100 g of each

Total weight = 100 + 100 + 100 = 300

Water added = 30 g

$\% = \frac{340}{330} \times 100 = 103.03\%.$

27. (B)

28. (B)

$$2NO + O_2 \longrightarrow 2NO_2$$

$\leftarrow$ 2V 2V 0 Total = 4V

0 V 2V Total = 3V

Charge = 4V – 3V = V

$$\% = \frac{V}{4V} \times 100 = 25\%.$$

29. (A)

Allred-Roschew's scale

$$En = 0.359\frac{Zeff}{r^2} + 0.744$$

Zeff = $Z - \sigma$

r = 1.175 Å

30. (B)

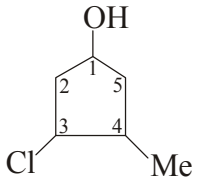
$$3Cu + 8HNO_3 \longrightarrow 3Cu(NO_3)_2 + 2NO + 4H_2O$$

Out of 8 nitrogen only 2 nitrogen is reduced.

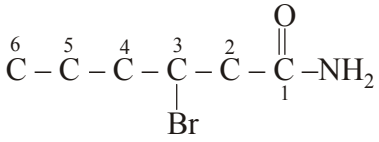
$$\% \text{ of Unreduced } = \frac{6}{8} \times 100 = 75\%.$$

31. (A, B, C, D)

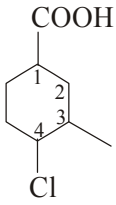
32. (A, B, D)

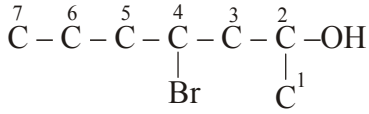
(A) 

3-Chloro-4-methylcyclopentanol-1

(B) 

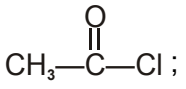
3-Bromo hexanamide

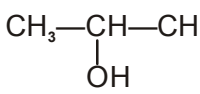
(C)  4-chloro-3-methylcyclohexanecarboxylic acid.

(D) 

4-Bromoheptan-2-ol

33. (A), (B), (C)

(A)  ; Ethanoyl chloride

(B)  ; Propan-2-ol

(C) $Cl—CH_2—CH=CH_2$; 3-Chloro-1-propene

34. (A,C,D)

$H_2O = a$ mole

$\mathbf{X} = b$ mole

$$\frac{18a + 40b}{a + b} = 20$$

2 a = 20 b

$$\frac{a}{b} = 10$$

H_2O = 10 mole **X** = 1

$$\text{Molefraction} = \frac{1}{11}$$

$$\text{Weight ratio} = \frac{1 \times 40}{10 \times 18} = \frac{2}{9}$$

$$M = \frac{1}{10 \times 18} \times 1000 = \frac{100}{18} = \frac{50}{9}.$$

35. (B,D)

$$K_4\left[Fe(CN)_6\right] \longrightarrow K^+ + Fe^{3+} + N^{+5} + C^{+4}$$

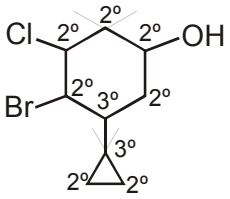
$Fe + 2 \longrightarrow +3$

$N \quad -3 \longrightarrow +5$

$C \quad -2 \longrightarrow +4$

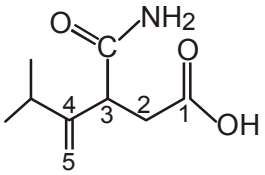
No. of electrons = 1 + 6 × 2 + 6 × 8 = 61.

36. (4)



11 – 7 = 4

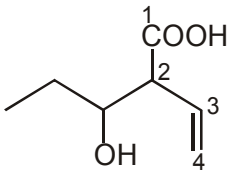
37. (5)



38. (2)

(a) & (d)

39. (4)



40. (3)

1000 ml of 1 M = 1000 milimole

If 750 ml. is taken out remaining milimole is 1000 – 750 = 250 milimole

New molarity = .25 M $= \frac{1}{4}$

After n step $\Rightarrow 1 \times \left(\frac{1}{4}\right)^n = \frac{1}{64} = n = 3.$

MATHEMATICS

41. (A)

$\left[\frac{x}{99}\right]=\left[\frac{x}{101}\right]=0$ iff $x \in \{1,2,...,98\}$. 98 such numbers

$\left[\frac{x}{99}\right]=\left[\frac{x}{101}\right]=1$ iff $x \in \{101,102,...,197\}$. 97 such numbers

In general; If

$\left[\frac{x}{99}\right]=\left[\frac{x}{101}\right]=k$ where $k \geq 1$, then $x \in \{101k, 101k+1, ..., 99(k+1)-1\}$

(99 – 2k) such numbers.

$\therefore \quad 99(k+1)-1 \geq 101k$

$98 \geq 2k$, $k \leq 49$

so, $98+\sum_{k=1}^{49}(99-2k)$

$=98+2401=2499$

42. (D)

$|\sin x| = \cos x \geq 0$

$\therefore$ x lies in either first quadrant or fourth quadrant & they are equal for

$x=\frac{\pi}{4}$ or $\frac{7\pi}{4}$

$\therefore$ General solution is $2n\pi \pm \frac{\pi}{4}$

43. (B)

$\because \log_{\cos x}\left(\frac{\sqrt{3}}{2}\sin x\right)-\log_{\cos x}(\tan x)=2$

or, $\log_{\cos x}\left(\frac{\sqrt{3}}{2}\cos x\right)=2$

$\Rightarrow \frac{\sqrt{3}}{2}\cos x=\cos^2 x$

$\Rightarrow \cos x(\cos x-\sqrt{3}/2)=0$

$\therefore x=\frac{\pi}{6}$ $\left(\because x=\frac{\pi}{2} \text{ is rejected}\right)$

44. (B)

$\sin 41x + \sin 29x = \sin 71x - \sin x$

$\Rightarrow$ $2 \sin 35x \cos 6x = 2 \cos 36x \sin 35x$

$\Rightarrow$ $2 \sin 35x (\cos 6x - \cos 36x) = 0$

$\Rightarrow$ $4 \sin 35x \sin 21x \sin 15x = 0$

$\Rightarrow$ $x = \frac{k_1\pi}{35}$ or $\frac{k_2\pi}{21}$ or $\frac{k_3\pi}{15}$, where $k_1, k_2, k_3 \in I$

$\because$ $x \in [0, 2\pi]$

$\therefore$ The solution set is $A \cup B \cup C$, where

$A = \left\{0, \frac{\pi}{35}, \frac{2\pi}{35}, ..., \frac{70\pi}{35}\right\}$, $B = \left\{0, \frac{\pi}{21}, \frac{2\pi}{21}, ..., \frac{42\pi}{21}\right\}$,

and $C = \left\{0, \frac{\pi}{15}, \frac{2\pi}{15}, ..., \frac{30\pi}{15}\right\}$

$\therefore$ Number of solutions in $[0,2\pi]$ = $n(A \cup B \cup C)$

= $n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 71 + 43 + 31 - 15 - 7 - 11 + 2 = 114$

45. (C)

46. (B)

$$\frac{|1+\{x\}|}{1+\{x\}} = 1$$

47. (B)

$$\left|\left[\frac{x}{4}\right] + \left[\frac{x+2}{4}\right]\right| \le 3 \quad \Rightarrow \left|\left[\frac{x}{2}\right]\right| \le 3 \Rightarrow -3 \le \left[\frac{x}{2}\right] \le 3 \Rightarrow -3 \le \frac{x}{2} < 4 \Rightarrow -6 \le x < 8$$

48. (B)

Point I and I_1 divide AD in the same ratio internally & externally respectively

49. (D)

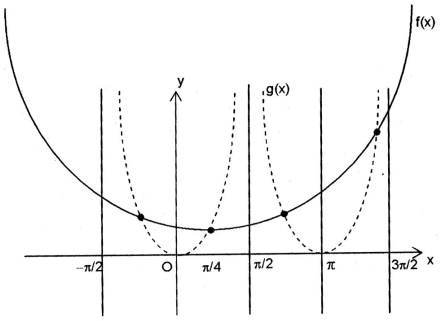
Given equation is

$16x^2 - 8\pi x + \pi^2 + 16 = |\tan x|$

Let $f(x) = 16x^2 - 8\pi x + \pi^2 + 16$

and $g(x) = |\tan x|$

$\Rightarrow$ $f(x) = 16\left[\left(x - \frac{\pi}{4}\right)^2 + 1\right]$



or y = f(x) is an upward parabola with vertex $\left(\frac{\pi}{4}, 16\right)$

Now, from graph it is clear that f(x) and g(x) intersect at infinitely many points.

50. (C)

$$\frac{1}{z} = 5 - x, y = 29 - \frac{1}{x}$$

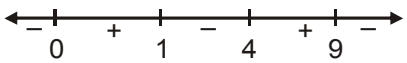
$$\therefore \quad x\left(29 - \frac{1}{x}\right) = 5 - x$$

$$\therefore \quad x = \frac{1}{5}, y = 24, z = \frac{5}{24}$$

51. (A, B, C, D)

$|x^2 - 9x| + |x^2 - 5x + 4| > 4|x + 1|$ is true if

$$(9x - x^2)(x^2 - 5x + 4) < 0$$



$$\therefore x \in (-\infty, 0) \cup (1, 4) \cup (9, \infty)$$

52. (A, B, C)

(B) $\log_{\cos\frac{7\pi}{4}}\left(\sin\frac{5\pi}{6}\right) = \log_{\frac{1}{\sqrt{2}}}\left(\frac{1}{2}\right) > 0$

(C) $\log_{\tan\frac{4\pi}{3}}\left(\cot\frac{7\pi}{6}\right) = \log_{\sqrt{3}}\sqrt{3} = 1 > 0$

53. (A, B)

$4(\cos\theta - \sin\theta)(\cos^2\theta + \sin\theta\cos\theta + \sin^2\theta - \sin\theta\cos\theta)$

$$= -4\sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right) > 0$$

$$\Rightarrow \quad \sin\left(\theta - \frac{\pi}{4}\right) \text{ is – ve}$$

$$\Rightarrow \quad (2n - 1)\pi < \theta - \frac{\pi}{4} < 2n\pi, \; n \in I$$

54. (A,B)

$\sum \tan x_1 = 0 \qquad \Rightarrow \tan x_3 = -p$

$\Rightarrow ap^3 + (2a - k_1)p - k_2 = 0$

Also $\tan x_1.\tan x_2.\tan x_3 = \dfrac{-k_2}{a} \Rightarrow \tan x_3 = \dfrac{-k_2}{aq} \Rightarrow k_2^2 + (2a - k_1)aq^2 - a^2q^3 = 0$

55. (B, C)

$|\cos x + \sin x| = |1 - a\sin x\cos x|, \sin x \neq 0 \Rightarrow 1 + \sin 2x = 1 + a^2\sin^2 x\cos^2 x - 2a\sin x\cos x$

either $\sin 2x = 0, \forall a \in R$ or $\sin 2x = \dfrac{4(a+1)}{a^2}$

$\Rightarrow x = (2n+1)\dfrac{\pi}{2}$ or $x = \dfrac{1}{2}\sin^{-1}\dfrac{4(a+1)}{a^2}$ for $-1 \le \dfrac{4(a+1)}{a^2} \le 1$

56. (4)

$\dfrac{\sqrt{3}-1}{\sin x} + \dfrac{\sqrt{3}+1}{\cos x} = 4\sqrt{2}$

$\Rightarrow \left(\dfrac{\sqrt{3}+1}{2\sqrt{2}}\right)\sin x + \left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right)\cos x = 2\sin x\cos x$

$\Rightarrow \sin(x+\alpha) = \sin 2x$, where $\alpha = \dfrac{\pi}{12}$

$\Rightarrow 2x = 2k\pi + x + \alpha$ or $2x = (2k+1)\pi - (x+\alpha), k \in I$

$\therefore$ solutions in $(0, 2\pi)$ are $\alpha, \dfrac{\pi}{3} - \dfrac{\alpha}{3}, \pi - \dfrac{\alpha}{3}$ and $\dfrac{5\pi}{3} - \dfrac{\alpha}{3}$

57. (8)

$3\sin^2 A + 2\sin^2 B = 1 \Rightarrow 3\cos 2A + 2\cos 2B = 3$

$\Rightarrow \cos 2A + \dfrac{2}{3}\cos 2B = 1$ .....(i)

Also 3sin2A = 2sin2B $\Rightarrow \dfrac{2}{3} = \dfrac{\sin 2A}{\sin 2B}$

putting this value in (i), we get

$\cos 2A + \dfrac{\sin 2A}{\sin 2B}\cdot\cos 2B = 1 \Rightarrow \sin 2A\cdot\cos 2B + \sin 2B\cdot\cos 2A = \sin 2B \Rightarrow 2\sin A\cdot\cos(A+2B) = 0$

$\because 0 < \sin A < 1$, as $0 < A < \pi/2$ $\quad\therefore \cos(A + 2B) = 0 \Rightarrow A + 2B = \pi/2$

From $3\sin^2 A + 2\sin^2 B = 1$, A & B both lie between 0 to $\frac{\pi}{4}$. $\quad\therefore \lambda = 8$

58. (4)

From $\log_{100} |x + y| = \frac{1}{2}$, we get $|x + y| = 100^{1/2} = 10$.

From $\log_{10} y - \log_{10} |x| = \log_{100} 4 = \log_{10} 2$, we have

$\frac{y}{|x|} = 2 \Rightarrow y^2 = 4x^2 \Rightarrow x^2 + y^2 + 2xy = 100$ or $5x^2 + 4x|x| = 100$

$\Rightarrow x = \frac{10}{3}$ for $x > 0 \Rightarrow y = \frac{20}{3}$

and $x = -10$ for $x < 0 \Rightarrow y = 20$.

59. (4)

$$\frac{1}{\sqrt{6}+\sqrt{2}} = \frac{1}{\sqrt{2}(\sqrt{3}+1)} = \frac{(\sqrt{3}-1)}{2\sqrt{2}} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\cdot\frac{1}{2}$$

$$= \sin(45° - 30°) = \sin 15° = \sin\frac{\pi}{12}$$

$\therefore \left[\frac{[x]}{4}\right] = 12 \qquad \because \left[\frac{[x]}{4}\right] = \left[\frac{x}{4}\right]$

$\Rightarrow \quad 12 \le \frac{x}{4} < 13$

$\Rightarrow \quad 48 \le x < 52$

$\therefore \quad x = 48, 49, 50, 51$

60. (3)

Let [x] = n

then $x = \frac{10n - 14}{n + 1}$

Now, $n \le x < n + 1$

$n \le \frac{10n - 14}{n + 1} < n + 1 \Rightarrow n = 2, 6, 7$

$\Rightarrow \quad x = 2, \frac{46}{7}, 7.$