SOLUTIONS

WEEKLY TEST-8

RBA

(JEE MAIN PATTERN)

Test Date: 23-09-2017

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PHYSICS

1. (B)

F sin 53° = 20

$\Rightarrow\ 5t \times \frac{4}{5} = 20$

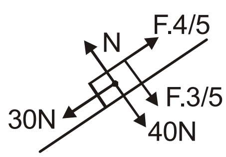
$\Rightarrow\ t = 5s$

$a = \frac{F\cos 53^\circ}{6} = \frac{5 \times 5 \times \frac{3}{5}}{6}$

$= \frac{5}{2} m/s^2$

$F' = 4 \times \frac{5}{2} = 10N$

2. (C)



$N = F.\frac{3}{5} + 40$

$\Rightarrow\ 160 = F.\frac{3}{5} + 40 \quad \Rightarrow\ F = 200N$

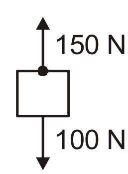
$\therefore\ a_{max} = \frac{200 \times \frac{4}{5} - 30}{5}$ = 26 m/s^2

3. (A)

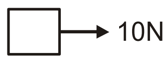
for $T = T_{max} = 300N$ in the upper string, tension in the string connected to the block = 150N

$\therefore\ a_{max} = \frac{150 - 100}{10}$

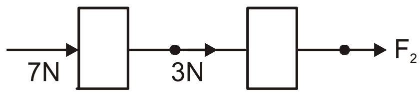
= 5m/s^2



4. (A)



$$\frac{2kg}{a} = \frac{10}{2} = 5\,m/s^2$$



$$F_2 + 7 + 3 = (3+1) \times 5$$

$$F_2 = 20 - 10 = 10N$$

$$F_2, max = 10N$$

5. (A)

$$T \cos 53° = 2 \times 1$$

$$\Rightarrow \; T \times \frac{3}{5} = 2 \quad \Rightarrow T = \frac{10}{3}$$

$$T \cos 37° = 3 \times a$$

$$\Rightarrow \; \frac{10}{3} \times \frac{4}{5} = 3a$$

$$= \; a = \frac{8}{9}\,m/s^2$$

6. (B)

$$f_1 = 0.5 \times 20 = 10N$$

$$f_2 = 0.3 \times 20 = 6N$$

$$f_{net} = 16N$$

$$F = 4t = 20 + 16$$

$$\Rightarrow \; t = 9s$$

at t = 9s, block will start accelerating

7. (C)

8. (C)

From constraint relation $v_B = \frac{v}{3}$

9. (A)

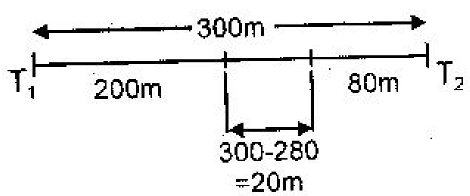
Applying Snell's law between the points *O* and *P*, we have

$$2\times\sin 60^0=\left(\sin 90^0\right)\times\frac{2}{\left(1+H^2\right)},\qquad 2\times\frac{\sqrt{3}}{2}=1\times\frac{2}{\left(1+H^2\right)}$$

$$\left(1+H^2\right)=\frac{2}{\sqrt{3}},\qquad H=\sqrt{\left(\frac{2}{\sqrt{3}}-1\right)}$$

10. (D)

Initial distance between trains is 300m. Displacement of 1st train is calculated by area under V-t. curve of train $1=\frac{1}{2}\text{ x }10\text{ x }40=200\text{ m}$



Displacement of train $2=\frac{1}{2}\text{ x }8\text{ x }(-20)=-80\text{ m}$.

Which means it moves towards left.

$\therefore$ Distance between the two is 20 m.

11. (C)

The retardation is given by $\frac{dv}{dt}=-av^2$

integrating between proper limits $\Rightarrow -\int_u^v\frac{dv}{v^2}=\int_0^t a\,dt$ or $\frac{1}{v}=at+\frac{1}{u}$

$$\Rightarrow\quad \frac{dt}{dx}=at+\frac{1}{u}\quad\Rightarrow\quad dx=\frac{u\,dt}{1+aut}$$

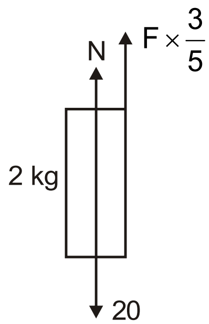
integrating between proper limits $\int_0^s dx=\int_0^t\frac{u\,dt}{1+aut}\quad\Rightarrow\quad S=\frac{1}{a}\ln\left(1+aut\right)$

12. (C)

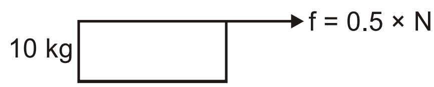
Fringe width (B) = $\frac{\lambda D}{d}$

$\Rightarrow \beta \propto \frac{1}{d}$

13. (A)



$N = 20 - F \times \frac{3}{5}$



$a = \frac{f}{10} = \frac{0.5 \times \left(20 - f \times \frac{3}{5}\right)}{10}$ ....(1)

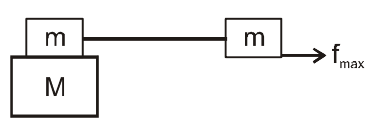
$F + F\cos 37° = (2 + 10) \times a$ ...(2)

from (1) & (2) F = $\frac{50}{9}$N

14. (A)

As the value of F is increased from zero, the frictional force between right upper and lower block is more in comparision to that between left upper and lower block. Hence friction will reach its maximum value between the right two blocks first. Based on the maximum value of friction f_{max} and the mass that needs to be accelerated by this force (see figure), the maximum acceleration of these blocks is

$a_{max} = \frac{f_{max}}{(m + m + M)} = \frac{\mu mg}{2m + M}$



Now, looking at the entire system as whole

$F = (2m + 2M)\, a_{max} = \frac{2\mu mg(m + M)}{2m + M}$

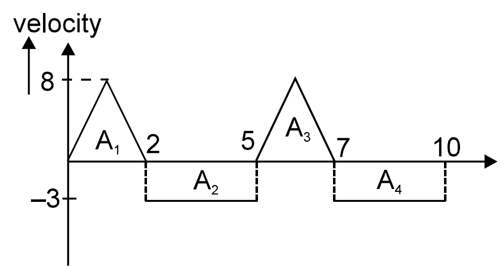
15. (D)

$A_1 = A_3 = 8$ (area)

$A_2 = A_4 = 9$

Position of the particle at any time t is given by

$X = X_0 + \int_0^t Vdt$ $\quad X_0$ = Initial position



$\int_0^t Vdt$ = Area under the curve

Now at $t = 0 \; X = X_0 = -1$

at $t = 2 \; X = X_0 + A_1 = -1 + 8 = 7$

at $t = 5 \; X = X_0 + A_1 - A_2 = -1 + 8 - 9 = -2$

$t = 7 \quad X = X_0 + A_1 - A_2 + A_3 = -1 + 8 - 9 + 8 = 6$

$t = 10 \quad X = X_0 + A_1 - A_2 + A_3 - A_4 = -1 + 8 - 9 + 8 - 7 = 3$

As during 10 seconds four times the position of the particle changed in sign. Particles passes 4 times the origin.

16. (C)

$$v = \cos\left(\frac{\pi}{3}t\right) \Rightarrow \frac{dx}{dt} = \cos\left(\frac{\pi}{3}t\right)$$

$$\Rightarrow x = \int_0^2 \cos\left(\frac{\pi}{3}t\right)dt = \int_0^{\frac{3}{2}} \cos\left(\frac{\pi}{3}t\right)dt + \left|\int_{\frac{3}{2}}^{2} \cos\left(\frac{\pi}{3}t\right)dt\right| \Rightarrow x = \frac{3}{\pi}\left[2 - \frac{\sqrt{3}}{2}\right]$$

17. (A)

$$4I = I_0 \Rightarrow I = \frac{I_0}{4}$$ = intarsity of one slit.

$$8x = \sqrt{D^2 + d^2} - D \simeq \frac{1}{2} \times \frac{d^2}{D} = \frac{1}{2} \times \frac{25\lambda^2}{50\lambda} = \frac{\lambda}{4}$$

$$\therefore \quad \phi = \frac{2\pi}{\lambda} \times \Delta x \; \phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{\pi}{2}$$

$$\therefore \quad I_{net} = I + I + 2\sqrt{I\,I}\cos\frac{\pi}{2}$$

$$= 2I = \frac{I_0}{2}$$

18. (D)

$$\frac{f_0}{fe} = 10 \Rightarrow f_0 = 200$$

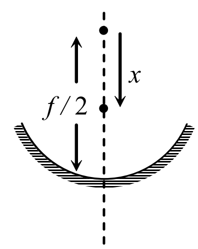
$\therefore$ **(D)**

19. (A)

$$V_I = -m^2 v_0$$

$$m = \frac{f}{u - f}$$

$$V_I = \left[\frac{-f}{-\left(\frac{f}{2} - x\right) + f}\right]^2 \sqrt{2gx}$$



For V_I to be max $\frac{dv_I}{dx} = 0$

We get $x = \frac{f}{6}$

$$V_I = \frac{3}{4}\sqrt{3gf}$$

But, $f = \frac{3}{40}\text{m}$;

$\therefore$ $V_I = \frac{9}{8}$ m/s.

20. (C)

Torque of electrostatic force is zero.

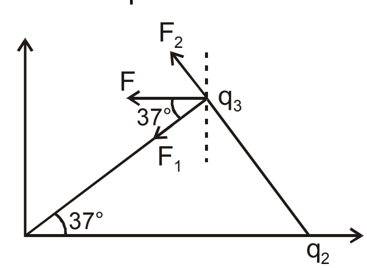
21. (C)

For F to be along negative x-axis, q_1 has to be negative while q_2 has to be positive.

also $F_1 \cos 53 = F_2 \cos 37°$

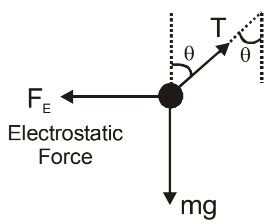
where $F_1 = \frac{K.q_1 q_3}{(4cm)^2}$ and $F_2 = \frac{K.q_2 q_3}{(3cm)^2}$

on putting values $q_2 = \frac{27}{32}\,\mu C$



22. (A)

Since both the small spheres are at same horizontal level, the electrostatic forces on both spheres are in horizontal direction. The FBD of left sphere is shown in figure



∵ The sphere is in equilibrium

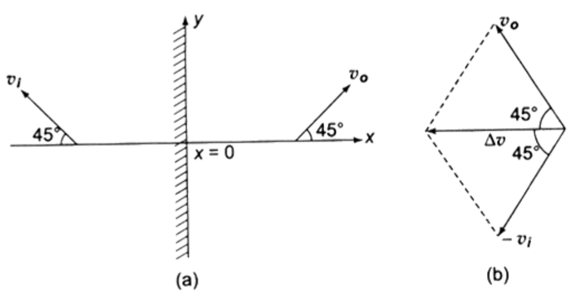
⇒ $T \cos\theta = mg$ and $T \sin\theta = F_E$

∴ $\tan\theta = \frac{F_E}{mg}$

The magnitude of electrostatic force on each sphere is same irrespective of its charge

∴ for $\theta_1 = \theta_2$ the necessary condition is $m_1 = m_2$

23. (C)



Velocity of the object is $v_0 = \left(2\,\hat{i} + 2\hat{j}\right)\, ms^{-1}$

∴ Speed of obejct is $v_0 = \sqrt{2^2 + 2^2} = 2\sqrt{2}\ ms^{-1}$

⇒ speed of the image (v_i). The velocity v_i of the image will beas shown in Fig. (a) The relative velocity of the image with respect to the object is

$$\Delta v = v_i - v_0 = v_1 + (-v_0)$$

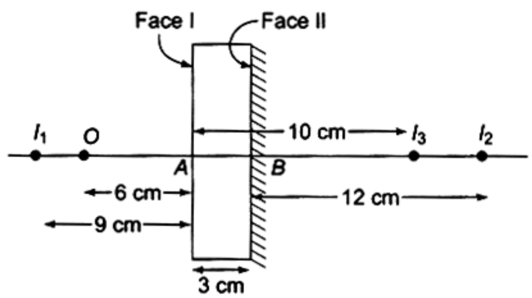
The magnitude of Δv is given by [see Fig. (b)]

$$\Delta v = [v_0^2 + v_1^2 - 2v_0\ v_i\ \cos 90°]^{1/2}$$

$$= \left[(2\sqrt{2})^2 + \left(2\sqrt{2}\right)^2\right]^{1/2}$$

= 4 ms^{-1} along $-x$ axis.

24. (B)



I_1 is the image O due to refraction at face I

$AI_1 = \mu(OA) = 1.5 \times 6 = 9$ cm

I_2 is the image of I_1 due to reflection at face II. Since

$I_1B = 9 + 3 = 12$ cm, $I_2B = 12$ cm

I_3 is the image of I_2 due to refraction at Face I

again,

$$\therefore \; AI_3 = \frac{I_2A}{\mu} = \frac{15}{1.5} = 10 \text{ cm}$$

$\therefore$ Distance of I_3 from B = 10 – 3 = 7 cm.

25. (C)

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) = \frac{1}{R} \; (\because \; R_1 = -R_2 = R)$$

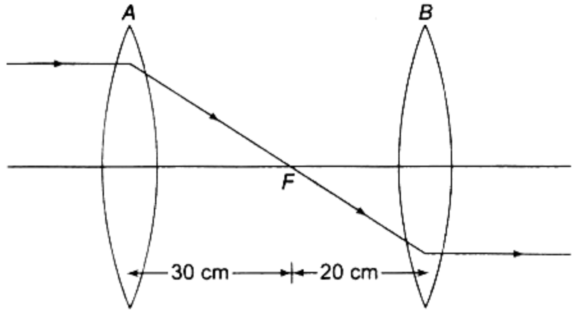
which gives f = R. When the space between the lenses is filled with water, we have a concave water lens of $\mu_\omega = 4/3$ surrounded by a medium of $\mu_g = 3/2$. Therefore, for the watern lens.

$$\frac{1}{f'} = \left(\frac{4}{3} - 1\right)\left(\frac{-2}{f}\right) \; \Rightarrow \; f' = -\frac{3f}{2}$$

The focal length of the combination of the three lenses is given by

$$\frac{1}{f_{eq}} = \frac{1}{f} + \frac{1}{f} - \frac{2}{3f} \quad \Rightarrow \; f_{eq} = \frac{3f}{4}$$

26. (A)



For no deviation, the ray must emerge from lens B parallel to the principal axis. For this to happen, point F must be at the second focus of lens A and at the the first focus of lens B. Hence d = 30 + 20 = 50 cm, which is choice (a).

27. (B)

The value of the effective focal length F is given by

$$\left|\frac{1}{F}\right| = \frac{1}{f_1} + \frac{1}{f_m} + \frac{1}{f_1} = \frac{2}{f_1} + \frac{1}{f_m} = \frac{2}{20} + \frac{2}{22}$$

or $|F| = \frac{110}{21}$ cm

Since the convex lens with a silvered surface behaves as a concave mirror of effective focal length F, we have

$$F = -\frac{110}{21} \text{ cm and } u = -10 \text{ cm} \qquad \frac{1}{v} - \frac{1}{-10} = -\frac{21}{110} \Rightarrow v = -11$$

28. (D)

The focal length of a lens does not change if a part of it is blcoked. If the central part of the aperture upto d/2 is blocked, the exposed area of the aperture reduces by one-fourth the earlier area because

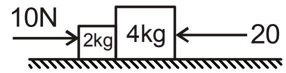
$$\frac{\pi\left(\frac{d}{2}\right)^2}{nd^2} = \frac{1}{4}$$

29. (D)

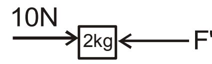
$$\frac{10-2t}{6} = a = \frac{dv}{dt}$$

$$\Rightarrow \int_0^0 dv = \int_0^t \left(\frac{10-2t}{6}\right) dt$$

$\Rightarrow\ t = 10\,\text{sec}$



$$a = \frac{20-10}{6} = \frac{10}{6} m/s^2$$



$$F' - 10 = 2 \times \frac{10}{6}$$

$$F' = 10 + \frac{10}{3} = \frac{40}{3} N$$

30. (B)

The system will be in equilibrium if the net force on charge q at one vertex due to charges q at the other two vertices is equal and opposite to the force due to charge Q at the centroid, i.e. (here a is the side of the triangle)

$$\frac{\sqrt{3}q^2}{4\pi\varepsilon_0 a^2} = -\frac{Qq}{4\pi\varepsilon_0\left(\frac{a}{\sqrt{3}}\right)^2}$$

which gives Q = $-\frac{q}{\sqrt{3}}$. Hence the correct choice is (b).

CHEMISTRY

31. (D)

C_2H_6O (g) + $3O_2$(g) $\rightarrow$ $2CO_2$(g) + $3H_2O$ (ℓ)

20 80 0

0 80 – 60 40

V_i = 20 + 80 = 100, V_f = 20 + 40 = 60

Reduction in volume = $V_i - V_f$ = 100 – 60 = 40

% Reduction in volume = $\frac{40}{100} \times 100$ = 40%

32. (C)

RMS speed does not depend upon the pressure.

$\bar{C}_2 = \sqrt{\frac{3R \times 3 \times 27}{M}}$ $\therefore$ $\frac{\bar{C}_2}{\bar{C}_1} = \sqrt{3}$; $\bar{C}_1 = \sqrt{\frac{3R \times 27}{M}}$ or $\bar{C}_2 = \sqrt{3}\,\bar{C}_1 = \sqrt{3} \times 10^4$ cm/sec

33. (D)

At the same temperature, oxygen and hydrogen molecules will have the same average energy; weight of H_2 molecules is

1/16 of O_2 molecules. So statements 2 and 4 are wrong.

34. (A)

$V_c = 3b$, assuming the gas to obey van der walls' equation.

$\therefore$ b (the covolume) = $\frac{0.072}{3}$ = 0.024 litre mol^{-1}.

$b = \frac{24cm^3}{6\times10^{23}}$ per molecule, where $N_A \approx 6 \times 10^{23}$

$b = 4 \times 10^{-23}$ cm^3 per molecule $= 4 \times \frac{4}{3}\pi r^3$.

$\frac{4}{3}\pi r^3 = 10^{-23}$; $r^3 = \frac{3}{4\pi} \times 10^{-23}$; $r = \left(\frac{3}{4\pi}\times10^{-23}\right)^{\frac{1}{3}}$ cm

35. (D)

n_1 moles $= \frac{770\times10^6}{760\times300\times0.0821} = 41.1 \times 10^3$ moles,

n_2 moles $= \frac{125\times10^6}{760\times260\times0.0821} = 7.705 \times 10^3$ moles

$\therefore$ the weight of helium to be released = $(41.1 - 7.705) \times 10^3$ moles $= 33.395 \times 10^3$

Moles which corresponds to heliun = $(33.395 \times 10^3 \times 4)$g $= 1.3358 \times 10^5$ g = 133.58 kg.

36. (B)

For ideal gases,

$PV = nRT = \frac{m}{M}RT$: $P = \frac{RT}{M}\left(\frac{m}{V}\right) = \frac{RT}{M}d$

(or) $M = RT\left(\frac{d}{P}\right)$.

Given : $d = 2.00\ P + 0.020\ P^2$ (for a real gas).

$\frac{d}{P} = 2.00 + 0.040\ P$: $\underset{P\to0}{Lt}\ \frac{d}{P} = 2.00$, which is $\frac{d}{P}$ for an ideal gas.

Thus M = RT x 2 = 25 x 2 = 50 g Mol^{-1}.

37. (A)

P (V – b) = RT as Isochoric process (V = constant)

$$P = \frac{R}{(V - b)}T$$

Slope = $\frac{R}{(V - b)}$ in P-T graph

38. (A)

$P_{atm} = P_{gas} + P_{column} + P_{vapour}$

$\Rightarrow P_{gas} = 760 - \frac{240 \times 3.4}{13.6} - 50$ = 650mm of Hg = 65 cm of Hg

39. (C)

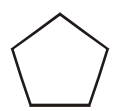
Work is path function.

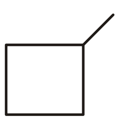
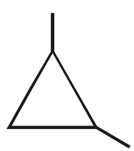
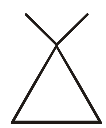
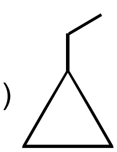
40. (C)

Surface tension does not depends on quantity taken.

41. (A)

(1) C = C – C – C – C (2) C – C = C – C – C (3) C = C(C) – C – C

(4) C = C – C(C) – C (5) C – C(C) = C – C (6) 

(7)  (8)  (9)  (10) 

42. (A)

Different number of carbon atoms in the parent chain.

43. (B)

Different number of carbon atoms in the parent chain.

44. (D)

$CH_3—CH_2—CH_2—NH_2$ (1°) ; $CH_3—CH(NH_2)—CH_3$ (1°) ; $CH_3—N(H)—CH_2CH_3$ (2°) ; $CH_3—N(CH_3)—CH_3$ (3°)

45. (D)

2,4,6-Trinitrophenol is the most acidic isomer.

46. (C)

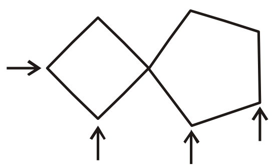
o-Toluic acid is the most acidic structural isomer of the given compounds.

47. (D)

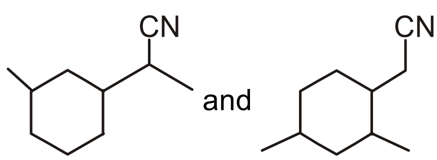
(A) (I), (II) & (IV) have different number of carbon atoms in the parent chain.

(B) (I) and (III) have same number of carbon in the parent chain but differ in the position of methyl group.

48. (B)



49. (C)



have different number of carbon atoms in the parent chain.

50. (D)

Alcohol and Phenol are functional isomers.

51. (D)

52. (D)

Electrons in orbitals bearing a lower 'n' value are more attracted to the nucleus then electrons in orbitals bearing a higher 'n' value. Hence, the removal of electrons from orbitals bearing a higher 'n' value is easier than the removal of electrons from orbitals having a lower 'n' value.

53. (C)

Of the absence of 'd' orbitals in the valence shell of fluorine.

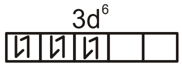
54. (A)

(A) After loss of one electron K^+ becomes as noble gas (Argon) and not colour ion.

(B) $_{1}Na = 1s^2, 2s^2, 2p^6, 3s^1$

(C) $_{29}Cu = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^1, 3d^{10}$

(D) $_{24}Cr = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^6\ 4s^0$



55. (D) **56. (B)**

57. (A)

Ionic compound has high melting point and non-directional bonds.

58. (C)

When cation and anion combined to form 1 mole ionic solid is called lattice energy.

59. (B)

F_2 is the most reactive due to its low bond energy and high hydration energy of F^- ion.

60. (D)

Solubility of fluorides of IIA

$BeF_2 > BaF_2 > SrF_2 > CaF_2 > MgF_2$

(BeF_2 is most soluble)

MATHEMATICS

61. (D)

$$f(x+y+1) = \left(\sqrt{f(x)} + \sqrt{f(y)}\right)^2$$

Put $x = 0, y = 0 \Rightarrow f(1) = 2^2$

put x= 1, y = 0 $\Rightarrow f(2) = 3^2 \ldots$

$$f(x) = (x+1)^2$$

62. (B)

$$g(h(x)) = x$$

$$g'(h(x))h'(x) = 1$$

$$h'(x) = \frac{1}{g'(h(x))} = 1 + (h(x))^2 + (h(x))^3$$

63. (C)

Put x = 1+h

$$f(b) = \lim_{h\to 0}\left(\frac{(1+h)^b - b(1+h) + b - 1}{(1+h-1)^2}\right)$$

$$= \lim_{h\to 0}\frac{\left(1 + hb + \frac{b(b-1)}{2!}h^2 + \ldots\right) - b - bh + b - 1}{h^2}$$

$f(b)=\frac{b(b-1)}{2}$

$f(5)=10$

64. (B)

$$f(g(x))=\frac{1}{\left(\frac{1}{x^2}-1\right)\left(\frac{1}{x^2}-2\right)}=\frac{x^4}{\left(1-x^2\right)\left(1-2x^2\right)}$$

$\Rightarrow$ f(g(x)) is discontinuous at $x=\pm1, x=\pm\frac{1}{\sqrt{2}}$ and x = 0 since, g(x) is discontinuous at x = 0.

65. (A)

$$f'(x)=\frac{f(b)-f(a)}{b-a}=\frac{f(8)-f(2)}{8-2}\geq 1$$

$f(8)-f(2)\geq 6$

$f(8)\geq 6+f(2)$

$f(8)\geq 5$

66. (C)

$f(x)=(x-2)(x+2)|x+2||x-3|+\sin|x-1|$

$(x+2)|x+2|$ is differentiable at x = –2

$|x-3|$ is not differentiable at $x=3$

$\sin(|x-1|)$ is non-differentiable at x = 1

Hence f(x) is differentiable at x = –2 but not at x = 1 & 3.

67. (C)

y = $e^{\lim_{x\to x_2}\frac{\sin\left(a(x-x_1)(x-x_2)(x-x_3)\right)}{a(x-x_1)(x-x_3)}\times\frac{1}{(x-x_2)}\times a(x-x_1)(x-x_3)}$

$=e^{a(x_2-x_1)(x_2-x_3)}$

68. (B)

69. (B)

Taking $x=y=1$, we get

$f(1)f(1) - f(1) = 6$

$\Rightarrow (f(1) + 2)\,(f(1) - 3) = 0 \;\; (f(1) > 0)$

$\Rightarrow f(1) = 3$

put y = 1

$f(x)f(1) - f(x) = 4x + 2$

$2f(x) = 4x + 2$

$f(x) = 2x + 1$

$f^{-1}(x) = \dfrac{x-1}{2}$

$f(x)f^{-1}(x) = \dfrac{(2x+1)(x-1)}{2}$

$= \dfrac{2x^2 - x - 1}{2}$

70. (C)

Let $y = \lim_{t\to\infty}\left(\left(\dfrac{f\left(5+\frac{1}{t}\right)}{f(5)}\right)^t\right)$

$\log y = \lim_{t\to\infty} t\left(\log f(5+\frac{1}{t}) - \log f(5)\right)$

$= \lim_{t\to\infty}\dfrac{\log f(5+\frac{1}{t}) - \log f(5)}{\frac{1}{t}} = \lim_{t\to\infty}\dfrac{1}{f(5+\frac{1}{t})}\dfrac{f'\left(5+\frac{1}{t}\right)\left(-\frac{1}{t^2}\right)}{-\frac{1}{t^2}}$

$= \dfrac{5}{10} = \dfrac{1}{2}$

$y = e^{\frac{1}{2}}$

71. (B)

$g'(0^+) = \lim_{h\to 0}\dfrac{f(|h|) - |\sinh| - f(0)}{h}$

$$= \lim_{h\to 0} \frac{f(|h|) - f(0)}{h} - \lim_{h\to 0} \frac{\sinh}{h}$$

$$= 1 - 1 = 0$$

$$g'(0^-) = \lim_{h\to 0} \frac{f(|-h|) - |\sin(-h)| - f(0)}{-h}$$

$$= \lim_{h\to 0} \frac{f(h) - f(0)}{-h} + \lim_{h\to 0} \frac{\sinh}{h}$$

$$= -1 + 1 = 0$$

72. (C)

$$0 \le f(x) < \frac{\pi}{2}$$

$$0 \le \tan^{-1}\left(x^2 + x + \lambda\right) < \frac{\pi}{2}$$

$$0 \le x^2 + x + \lambda < \infty$$

$$\Rightarrow D \le 0$$

$$1 - 4\lambda \le 0$$

$$4\lambda \ge 1$$

$$\lambda \ge \frac{1}{4}$$

73. (D)

74. (C)

$g(x)$ must be linear $\Rightarrow$ g(x) = ax + b

$g(1) = a + b = 3$ ... (I)

$g(7) = 7a + b = 15$ ... (II)

From (I) & (II)

a = 2

b = 1

g(x) = 2x+1

$g(6) = 13$

75. (C)

Equation of normal is $y = mx - 2am - am^3, \quad 4a = \frac{1}{4}$

$$a = \frac{1}{16}$$

$y = mx - \frac{1}{8}m - \frac{1}{16}m^3$ it passes through (a, 0)

$0 = 16ma - 2m - m^3$

$m\left(16a - 2 - m^2\right) = 0$

$\Rightarrow 16a - 2 > 0$

$a > \frac{1}{8}$

76. (A)

4A = 2a

$2A = a = \frac{2 \times 3 \times 2}{3 + 2} = \frac{12}{5}$

$4A = \frac{24}{5}$

77. (D)

$y = mx + \frac{8}{m}$

$x\left(mx + \frac{8}{m}\right) = -2$

$m^2x^2 + 8x = -2m$

$m^2x^2 + 8x + 2m = 0$

$D = 0 \Rightarrow 64 - 8m^3 = 0$

$m = 2$

$y = 2x + 4$

78. **(B)**

The parabola $y = x^2 + px + q$ cuts the straight line $y = 2x - 3$ at a point with abscissa 1. Therefore, y = 2 – 3 = –1.

So, the point is (1, –1). This point lies on the parabola.

Therefore,

$-1 = 1 + p + q$ or $p + q = -2$ (i)

Distance between vertex and x-axis $= \frac{p^2}{4} - q$

$$= \frac{p^2}{4} + p + 2$$

$$= \frac{1}{4}\left\{(p+2)^2 + 4\right\}$$

For minimum, p = –2.

Hence, q = 0.

79. **(C)**

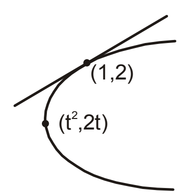
Equation of the tangent

$2y = 2x + 2$

$y - x - 1 = 0$

Let (h, k) be image of the point $(t^2, 2t)$ in $x - y + 1 = 0$ is given by

$$\frac{h - t^2}{1} = \frac{k - 2t}{-1} = \frac{-2\left(t^2 - 2t + 1\right)}{1 + 1}$$



$h = t^2 - 2t^2 + 2t - 1$

$h = 2t - 1$

$k = 2t + t^2 - 2t + 1$

$k = t^2 + 1$

$4(k-1) = (h+1)^2$

80. **(A)**

Let $A\left(at_1^2, 2at_1\right) = (x_1, y_1), B\left(at_2^2, 2at_2\right) = (x_2, y_2), C\left(at_3^2, 2at_3\right) = (x_3, y_3)$

Normals at A, B, C meet at point (h, k).

t_1, t_2, t_3 are roots of the equation

$$at^3 + (2a-h)t - k = 0;\ t_1 + t_2 + t_3 = 0$$

$$t_1t_2 + t_2t_3 + t_3t_1 = \frac{2a-h}{a};\ t_1t_2t_3 = \frac{k}{a}$$

$$\frac{x_1 - x_2}{y_3} + \frac{x_2 - x_3}{y_1} + \frac{x_3 - x_1}{y_2} = 0$$

81. (A)

$$y^2 = 8x\left(\frac{y - mx}{c}\right)$$

$$cy^2 = 8xy - 8mx^2$$

$$8mx^2 + cy^2 - 8xy = 0$$

$$8m + c = 0 \quad \Rightarrow \ c = -8m$$

82. (D)

Let $y = mx + \frac{2a}{m}$ be the tangent to the parabola

$$ym = m^2x + 2a$$

$m^2x - ym + 2a = 0$ it passes through (2a, 2b)

$$2am^2 - 2bm + 2a = 0$$

$$am^2 - bm + a = 0$$

$$b^2 - 4a^2 > 0$$

$$b^2 > 4a^2$$

83. (D)

Equation of circle

$$x^2 + (y-h)^2 + \lambda x = 0$$

it passes through A(2, 0) and B(6, 0)

$4 + h^2 + 2\lambda = 0$ .... (1)

$36 + h^2 + 6\lambda = 0$ .... (2)

$$\Rightarrow h = 2\sqrt{3}$$

84. (D)

9+16-12-24+c <0

$\Rightarrow c < 11$

& $\sqrt{13-c} < 2$

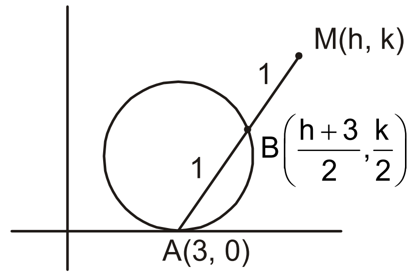
$13 - c < 4$

c > 9

$\Rightarrow c \in (9,11)$

85. (C)

B lies on circles



$$\left(\frac{h+3}{2}-3\right)^2+\frac{k^2}{4}+\frac{4k}{2}=0$$

$$(h-3)^2+k^2+8k=0$$

$$(x-3)^2+y^2+8y=0$$

86. (C)

Equation of chord of contact

$$x\alpha + y\left(\frac{12-3\alpha}{4}\right)=4 \quad \text{... (i)}$$

Let mid point b (h, k)

$$xh+yk=h^2+k^2 \quad \text{... (ii)}$$

From (i) & (ii)

$$\frac{\alpha}{h}=\frac{12-3\alpha}{4k}=\frac{4}{h^2+k^2}$$

$$\alpha=\frac{4h}{h^2+k^2}$$

$$12-3\alpha=\frac{16k}{h^2+k^2}$$

$$12\left(h^2+k^2\right)-12h=16k$$

$h^2 + k^2 - h - \frac{4}{3}k = 0$

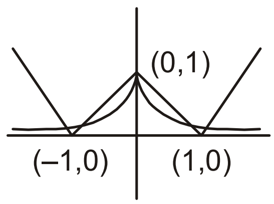
$OP = \sqrt{\frac{1}{4} + \frac{4}{9}}$

$= \frac{5}{6}$

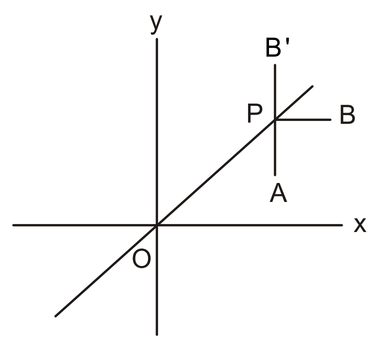
87. (B)

$|1 - |x|| = 7^{-|x|}$

from the graph



88. (A)



PA + PB = PA + PB' will be

minimum if the points A, P, B' are collinear.

Equation of AB'

$y - 3 = \frac{13 - 3}{7 - 4}\ (x - 4)$

$y - 3 = \frac{10}{3}(x - 4)$

$3y - 9 = 10x - 40$ .... (i)

P is point of intersection of y = x and (i)

7x = 31

$x = \frac{31}{7}$, $y = \frac{31}{7}$

89. (A)

The line y = $\sqrt{3}$ x can be written as $x = \frac{r}{2}, y = \frac{r\sqrt{3}}{2}.$ If this line cuts the given curve, then

$$\frac{r^4}{16}+\frac{ar^3\sqrt{3}}{8}+\frac{br^2\sqrt{3}}{4}+\frac{cr}{2}+\frac{dr\sqrt{3}}{2}+6=0.$$

Therefore, $\frac{1}{12}OA\cdot OB\cdot OC\cdot OD=\frac{1}{12}|r_1r_2r_3r_4|=8$

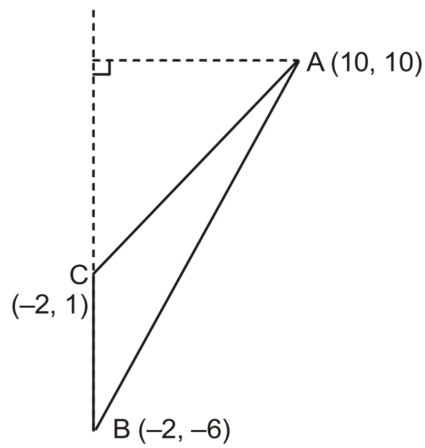
90. (C)

Let perpendicular bisector of AB is 3x + 4y – 20 = 0

and perpendicular bisector of AC is 8x + 6y – 65 = 0.

Image of A w.r.t. 3x + 4y – 20 = 0 is B

and image of A w.r.t. 8x + 6y – 65 = 0 is C.



For B, $\frac{x-10}{3}=\frac{y-10}{4}=-2\left(\frac{30+40-20}{25}\right)$

$\Rightarrow$ B = (–2, –6)

For C, $\frac{x-10}{8}=\frac{y-10}{6}=-2\left(\frac{80+60-65}{100}\right)$

$\Rightarrow$ C = (–2, 1)

Area of ΔABC = $\frac{1}{2}$ (10 + 2)(1 + 6) = 42.