SOLUTIONS

PROGRESS TEST-5

GZRM-1901-1902

GZR-1908-1909

(JEE MAIN PATTERN)

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PHYSICS

1. (A)

$\vec{A}\cdot(\vec{A}+\vec{B})$

$= A^2 + \vec{A}.\vec{B}$

$= A^2 + AB\cos 0^\circ$

$= A^2 + AB$

2. (C)

$R^2 = A^2 + B^2 + 2AB\cos\theta$ ...(i)

$R = A = B$

$A^2 = A^2 + A^2 + 2A^2\cos\theta$

$\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120^\circ$

3. (C)

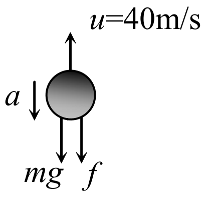
$W = \vec{F}\cdot\vec{S} = 12 + 30 = 42$ J

4. (D)

5. (B)

6. (D)

Let retardation of body is a and air resistance is f



$v = u + at$

$0 = 40 - 3a$

$a = \frac{40}{3}$ m/s^2

$ma = mg + f$

$f = ma - mg = 1.5\left(\frac{40}{3} - 10\right) = 5$ N

7. (D)

As weight = 0.3 × 10 = 3 N trying to slide the two block system,

but $f_{max} = 0.5 \times 1 \times 10 + 0.5 \times 1 \times 10 = 10$ N,

Hence the system is in equilibrium, and friction of block B is sufficient to balance the weight hence tension between A and B is zero.

8. (A)

During downward motion: $F = mg\sin\theta - \mu mg\cos\theta$

During upward motion: $2F = mg\sin\theta + \mu mg\cos\theta$

Solving above two equations: we get $\mu = \frac{1}{3}\tan\theta$

9. (A)

On cutting of string QR, the resultant force m_1 remains zero because its weight m_1g is balance by the tension in the spring but on block m_2 a resultant upward force $(m_1 - m_2)g$ is developed. Thus block m_1 will have no resultant acceleration whereas m_2 does have an upward acceleration given by $\frac{(m_1 - m_2)g}{m_2}$.

10. (D)

Surface between wall and A is smooth, so the system will fall with acceleration g.

11. (A)

$$f_s = \mu\, mg\cos\theta = 0.7 \times 2 \times 9.8 \times \frac{\sqrt{3}}{2} > 9.8$$

$$\therefore \quad f = mg\sin\theta = 9.8$$

12. (D)

T = 0, a = g

13. (A)

$$mv\frac{dv}{dx} = -Ax \quad \Rightarrow \quad \int_v^0 mv\,dv = -\int_0^x Ax\,dx \quad \Rightarrow \quad m\frac{v^2}{2} = A\frac{x^2}{2} \quad \Rightarrow \quad x = v\sqrt{\frac{m}{A}}$$

14. (C)

$$\vec{a} = \frac{\vec{F}}{m} = \frac{3\hat{i} + \hat{j}}{0.1} = 30\hat{i} + 10\hat{j}$$

$$\vec{r}(t) = \vec{r}(0) + \vec{u}t + \frac{1}{2}\vec{a}t^2$$

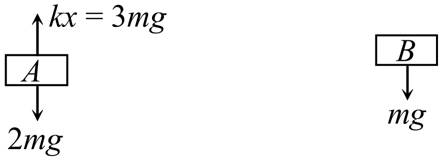
$$\vec{r}(t) = \hat{i}(5t + 15t^2) + \hat{j}(-2 - 2t + 5t^2)$$

$$x = 10 \quad \Rightarrow \quad 5t + 15t^2 = 10 \quad \Rightarrow \quad t = \frac{2}{3}s$$

$$y = -2 - 2t + 5t^2 = -\frac{10}{9}m$$

15. (C)

$$a_A = \frac{3mg - 2mg}{2m} = \frac{g}{2}$$



and $a_B = \frac{mg}{m} = g$

16. (B)

$$v = \alpha\sqrt{x} \Rightarrow \frac{dx}{dt} = \alpha x^{1/2}, \quad x^{-1/2}dx = \alpha\,dt,$$

$$\int_0^x x^{-1/2}dx = \alpha\int_0^t dt, \quad 2\sqrt{x} = \alpha t, \quad x \propto t^2$$

17. (A)

This is the situation similar to elastic collision of ball impinging on floor and bouncing back.

18. (B)

$$\frac{2u\sin(\alpha-\beta)}{g\cos\beta} = \frac{u\cos(\alpha-\beta)}{g\sin\beta}$$

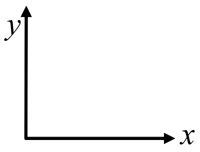
$$\therefore \quad \tan(\alpha-\beta) = \frac{1}{2}\cot\beta$$

19. (C)

$$\vec{V}_w = \frac{v}{\sqrt{2}}\hat{i} + \frac{v}{\sqrt{2}}\hat{j}$$

$$\vec{V}_m = (at)\hat{j}$$

$$\vec{V}_{wm} = \frac{v}{\sqrt{2}}\hat{i} + \left(\frac{v}{\sqrt{2}} - at\right)\hat{j}$$



It appears due east when, $\frac{v}{\sqrt{2}} - at = 0$

$$\therefore \quad t = \frac{v}{\sqrt{2}a}$$

20. (C)

$$\vec{r} = 4\sin 2\pi t\,\hat{i} + 4\cos 2\pi t\,\hat{j}$$

$$x = 4\sin 2\pi t$$

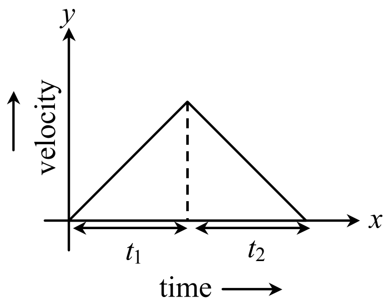
$$y = 4\cos 2\pi t$$

$$x^2 + y^2 = 16\sin^2 2\pi + 16\cos^2 2\pi t$$

$$x^2 + y^2 = 16$$

21. (B)

$$a_1 t_1 = a_2 t_2 \qquad \ldots \text{(i)}$$



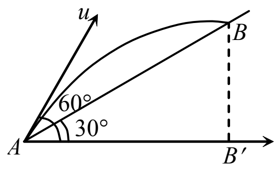
$$\frac{1}{2}\left(t_1 + t_2\right) a_1 t_1 = 4 \qquad \ldots \text{(ii)}$$

$$t_1 + t_2 = 4 \qquad \ldots \text{(iii)}$$

$$\frac{1}{a_1} + \frac{1}{a_2} = 2$$

22. (B)

$$AB' = u\cos 60^\circ \times t = \frac{ut}{2}$$



From $\Delta ABB'$ $\cos 30^\circ = \dfrac{AB'}{AB}$

or $AB = \dfrac{2AB'}{\sqrt{3}} = \dfrac{2 \times ut}{2 \times \sqrt{3}} = \dfrac{ut}{\sqrt{3}}$

23. (B)

$$a = -s, \qquad v\frac{dv}{ds} = -s, \qquad \int_{v_0}^{0} vdv = -\int_{0}^{s} sds, \qquad \frac{v_0^2}{2} = \frac{s^2}{2} \qquad \Rightarrow s = v_0$$

24. (B)

At any instant of time, let the length of the string BP = l_1 and the length PA = l_2. In a further time t, let B move to the right by x and A move down by y, while P moves to the right by ut. As the length of the string must remain constant.

$l_1 + l_2 = (l_1 - x + ut) + (l_2 + y)$

or $x = ut + y$

or $\dot{x} = u + \dot{y}$

$\dot{x}$ = speed of B to the right $= v_B$, $\dot{y}$ = downward speed of A = v_A

$\therefore \quad v_B = u + v_A$.

Also $\dot{v}_B = \dot{v}_A$ or $a_B = a_A$

25. (B)

$x = 4(t-2) + a(t-2)^2$

At t = 0, x = –8 + 4a = 4a – 8,

$v = \frac{dx}{dt} = 4 + 2a(t-2)$

At t = 0, v = 4 – 4a = 4(1 – a)

But acceleration, $a = \frac{d^2x}{dt^2}$ = 2a

26. (B)

Distance = $\int_0^2 v\,dt = \int_0^2 2t\,dt$ = 4 m

Average speed = $\frac{4}{2}$ = 2 m/s

$\omega = \frac{v}{R} = (2t)$ rad/s, $\theta = \int_0^2 \omega\,dt$ = 4 rad

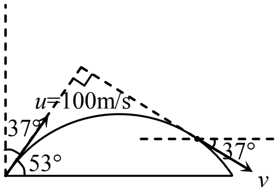
$\therefore$ Displacement = $2R\sin\frac{\theta}{2}$ = (2 sin 2) m

Average velocity = $\sin 2$ m/s

27. (B)

$u\cos 53° = v\cos 37°$

$\Rightarrow \quad 100 \times \frac{3}{5} = v \times \frac{4}{5} \Rightarrow v = 75\,m/s$



$v_y = -v\sin 37° = -45\,m/s$

$u_y = u\sin 53° = 80\,m/s$

$v_y = u_y + gt \;\Rightarrow\; -45 = 80 - 10t$

$t = 12.5\,s$

28. (C)

$v^2 - u^2 = 2al$ and $v'^2 - u^2 = 2a\frac{l}{2} = al$ or $2(v'^2 - u^2) = 2al$

Equating, $2(v'^2 - u^2) = v^2 - u^2$ or $v'^2 = u^2 + \frac{v^2 - u^2}{2} = \frac{v^2 + u^2}{2}$ or $v' = \sqrt{\frac{v^2 + u^2}{2}}$

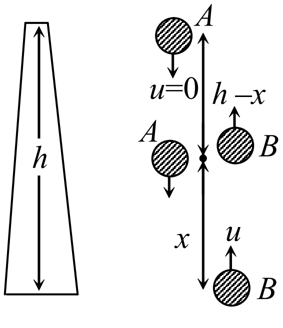
29. (B)

If u is the initial speed of the second stone, then

$0 = u^2 - 2g(4h)$

or $u = \sqrt{8gh}$

If they meet at the height x from ground,



For A, $h - x = \frac{1}{2}gt^2$

For B, $x = (\sqrt{8gh})\,t - \frac{1}{2}gt^2$

$\therefore \quad h = \sqrt{8gh}\; t$

or $t = \sqrt{\frac{h}{8g}}$

30. (D)

$h = \frac{1}{2}gt_1^2$; $2h = \frac{1}{2}g(t_1 + t_2)^2$ and $3h = \frac{1}{2}g(t_1 + t_2 + t_3)^2$

i.e., $t_1 : (t_1 + t_2) : (t_1 + t_2 + t_3) = 1 : \sqrt{2} : \sqrt{3}$

or $t_1 : t_2 + t_3 = 1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$

CHEMISTRY

31. (B)

Since, PV = nRT

$$PV = \frac{W}{M} RT \qquad \therefore \quad W = \frac{PVM}{RT} = \frac{730}{760} \times \frac{100 \times 4}{0.082 \times 298} \text{ g}$$

i.e., Wt. of He = 15.72 g

Wt. of air displaced = 100 × 1.25 g/L = 125 g

∴ Lifting power of the balloon = 125 g – 15.72 g = **109.28 g Ans.**

32. (B)

Moles of N_2 = 7/28 = 1/4

Moles of CO_2 = 22/44 = 1/2

Moles of CO = 5.6 / 22.4 = 1/4

Mean molar mass = $M_{min} = \frac{n_1 M_1 + n_2 M_2 + n_3 M_3}{n_1 + n_2 + n_3}$ = (7 + 7 + 22) / 1 = 36

33. (C)

Here, $n_{H_2} : n_{O_2} = 4 : 1$ & $\frac{r_{H_2}}{r_{O_2}} = \frac{n_{H_2}}{n_{O_2}} \sqrt{\frac{M_{O_2}}{M_{H_2}}}$

$$\Rightarrow \frac{r_{H_2}}{r_{O_2}} = \frac{4}{1} \times \sqrt{\frac{32}{2}} = \frac{16}{1} \Rightarrow \frac{(\text{no. of moles of } H_2 \text{ coming out})/\Delta t}{(\text{no. of moles of } O_2 \text{ coming out})/\Delta t} = \frac{16}{1}$$

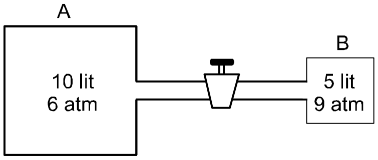
∴ Required composition of $H_2 : O_2$ coming out = **16 : 1 Ans.**

34. (D)

After the opening of the stop cock the pressure of the each bulb will remain same.

At the beginning, the no. of moles of gas in A = $\frac{10 \text{ x } 6}{RT}$

At the beginning, the no. of moles of gas in B = $\frac{5 \text{ x } 9}{RT}$



∴ total no. of mole at the beginning = $\frac{105}{RT}$

Total no. of mole of gas before opening the stop cock

= total no. of moles of gas after opening stop cock = $\frac{105}{RT}$

∴ pressure after the opening of the stop cock

$$P = \frac{105}{RT} \text{ x } \frac{RT}{V_{total}} = \frac{105}{10+5} = 7 \text{ atm}$$

35. (C)

$$K.E_{max} = E_{photon} - \phi$$

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi$$

on calculation

$$V = 2.18\sqrt{2} \times 10^6 \text{ m / s}$$

36. (B)

(Pressure of He)$_A$ = (Pressure of He)$_B$

$$\Rightarrow \frac{2 \times \cancel{R} \times 400}{\cancel{8.21}} = \frac{n \times \cancel{R} \times 500}{\cancel{8.21}}$$

$$\Rightarrow n = \frac{8}{5} \text{ moles}$$

$$(P_{He})_B = \frac{8}{5} \times \frac{R \times 500}{8.21} = 8 \text{atm}$$

37. (B)

ratio by mass = $\frac{CO_2(g)}{H_2(g)} = \frac{2.2}{1}$

ratio by mole = $\frac{\frac{2.2}{44}}{\frac{1}{2}} = \frac{1}{20} \times \frac{2}{1} = \frac{1}{10}$

So, ratio by molecules $= \frac{1 \times N_A}{10 \times N_A} = \frac{1}{10}$

38. (C)

$$\frac{w}{M} = \frac{1 \times 22.4 \times 10^{-3}}{22.4} \qquad M = 78 \text{ g}$$

$$n = \frac{78}{13} = 6$$

$\therefore$ MF = $(CH)_6$ or C_6H_6

39. (C)

C is the limiting reagent.

40. (C) **41. (D)**

42. (A)

$_{37}Rb$: [Kr] $5s^1$

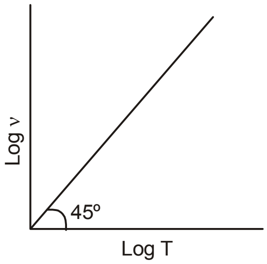
43. (B)

Angular momentum = $\frac{nh}{2\pi}$

44. (A)

$$Pv = nRT \Rightarrow \cancel{0.821} \times v = 10 \times \cancel{0.0821} \times T$$

$$v = T \Rightarrow \text{Log } v = \text{Log}\,T$$



45. (C)

$$\frac{r_{H_e}}{r_{H_2}} = \frac{P_{H_e}}{P_{H_2}}\sqrt{\frac{M_{H_2}}{M_{He}}}$$

$$\frac{10}{r_{H_2}} = \frac{\cancel{1000}}{\cancel{2000}}\sqrt{\frac{\cancel{2}}{\underset{2}{\cancel{4}}}}$$

$$r_{H_2} = 20\sqrt{2} \text{ torr min}^{-1}$$

46. (B)

Due to smaller size and half filled atomic orbital.

47. (C)

In $[\Delta H_{ion}]_{III}$ & $[\Delta H_{ion}]_{IV}$ has sudden jump so after removal of three electrons element acheived inert gas configuration.

48. (B)

$Pb > Pb^{2+} > Pb^{4+}$

49. (A)

$E_n = \frac{I.E. + E_A}{2}$ (Here I.E. & E_A in ev/atom)

$E_n = \frac{I.E. + E_A}{540}$ KJ/mol

50. (B)

(ΔHeg)I is exothermic whereas (ΔHeg)II is endothermic.

51. (B)

(A) Mg (B) Na (C) Ne (D) F

$$\begin{array}{lcccc} 1P_1 & Na & < Mg & < F & < Ne \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ & \textcircled{$2p^6$} & 2p^6 3s^2 & 2p^4 & 2p^5 \end{array}$$

After loss of one electron

$1P_2$ = Mg < F < Ne < Na

52. (B)

(i) Na_2CO_3 form by option V and it is not dicomposed due to high lattic energy.

(ii) D-block elements are most likely to forms coloured ionic compound option – X

(iii) As per option above given mention electronic configuration element is (Kr) krypton and this is belong from.inert gases Group VIII-A and has vender wall forces. So it has largest atomic radius.

(iv) Option (W) can forms only oxides.

Like - MgO

53. (A)

Ni = 3d

Pd = 4d

Pt = 5d

$1P_1$ = 3d < 4d < 5d

54. (B)

Conceptual.

55. (D)

B < S < P < F (Data base)

56. (B)

All ions has 10 electrons. (iso-electronic species)

57. (A)

Cl > F > Br > I

58. (B)

x – c = ?

dx – x = 1.0 Å

dc – c = 1.54 Å

En of x = 3.00

En of c = 2.00

$dA - B = r_A + r_B - 0.09\ (x_A - x_B)$

where x_A = En of A

x_B = En of B

dx – c = 0.50Å + 0.77Å – 0.09 (3 – 2.00) = 1.18Å

59. **(B)**

$$\text{Size} \propto \frac{1}{\oplus\text{oxidation state}}$$

60. **(B)**

Factual

MATHEMATICS

61. **(D)**

For internal point p(2, 8) $4 + 64 - 4 + 32 - p < 0 \Rightarrow p > 96$ for which x intercept = $2\sqrt{1+p}$ also

y intercept = $2\sqrt{4+p}$

circle cuts x and y axis. Hence no such value of p is possible

62. **(B)**

Let $(\alpha, 3, -\alpha)$ be any point on $x + y = 3$

$\therefore$ equation of chord of contact is $ax + (3 - \alpha)\,y = 8$

i.e. $\alpha\,(x - y) + 3y - 9 = 0$

$\therefore$ the chord passes through the point (3, 3) for all values of α.

63. **(C)**

The equation of the tangent at (α, α) to $x^2 + y^2 = 2\alpha^2$ is $\alpha x + \alpha y = 2\alpha^2$.

Therefore slope of the tangent = $-\alpha/\alpha = -1$

Hence (C) is the correct answer.

64. **(C)**

Equation of altitudes are $y = 5x + 13$ and $2x - 3y = 13$

$$\text{Let } \frac{1}{H_{i+1}} - \frac{1}{H_i} = k \Rightarrow \frac{H_i - H_{i+1}}{H_i\,H_{i+1}} = k \Rightarrow (H_i - H_{i+1}) = k\,H_i\,H_{i+1}$$

(where k is the common difference of corresponding A.P.)

$$\therefore \sum_{i=1}^{100} (-1)^i \left(\frac{H_i + H_{i+1}}{H_i - H_{i+1}}\right) = \sum_{i=1}^{100} \frac{(-1)^i}{k} \frac{(H_i + H_{i+1})}{H_i\,H_{i+1}} = \sum_{i=1}^{100} \frac{(-1)^i}{k} \left(\frac{1}{H_{i+1}} + \frac{1}{H_i}\right)$$

$$= \frac{1}{k}\left[\left(\frac{-1}{H_2}-\frac{1}{H_1}\right)+\left(\frac{1}{H_3}+\frac{1}{H_2}\right)+\left(-\frac{1}{H_4}-\frac{1}{H_3}\right)+\left(\frac{1}{H_5}+\frac{1}{H_4}\right)\ldots\ldots\ldots+\left(\frac{1}{H_{101}}+\frac{1}{H_{100}}\right)\right]$$

$$= \frac{1}{k}\left(\frac{1}{H_{101}}-\frac{1}{H_1}\right) = \frac{100k}{k} = 100 \text{ Ans.}$$

$$\begin{array}{l} \frac{1}{H_2} - \frac{1}{H_1} = k \\ \frac{1}{H_3} - \frac{1}{H_2} \\ \vdots \qquad \vdots \\ \frac{1}{H_{100}} - \frac{1}{H_{99}} \\ \frac{1}{H_{101}} - \frac{1}{H_{100}} \\ \hline \frac{1}{H_{101}} - \frac{1}{H_1} = 100\,k \end{array}$$

65. (B)

66. (C)

67. (B)

If d is the common difference of the arithmetic progression and r is the common ratio of geometric progression, then

$2 + 9d = 3$ and $2r^9 = 3 \Rightarrow d = \frac{1}{9}$ and $r = \left(\frac{3}{2}\right)^{1/9}$

So, $a_7 g_{19} = \left(2+6\times\frac{1}{9}\right)(2r^{18}) = \frac{8}{3}\cdot 2\cdot\frac{9}{4} = 12$

Now$a_{19}\, g_{28} = \left(2+18\times\frac{1}{9}\right)(2r^{27}) = 27$

Hence,$(a_7\, a_{19} + a_{19}\, g_{28}) = 12 + 27 = 39$. **Ans.**]

68. (D)

69. (A)

70. **(C)**

$$S = \sum_{n=1}^{10}(n+10)^2 - n^2 = \sum_{n=1}^{10}(100+20n) = 1000 + 20\left(\frac{10\cdot 11}{2}\right) = 100\,(10+11) = 2100 \textbf{ Ans.}$$

Aliter:

$$10[(11+1)+(12+2)+(13+3)+....+(20+10)] = 10\,(1+2+3+.....+20)$$

$$= 10 \times \frac{20}{2} \times (1+20) = 10 \times 10 \times 21 = 2100 \textbf{ Ans.}$$

71. **(D)**

Let d_1 be the common difference,

so, $S_1 = a = T_1$

$S_2 = 4a + \frac{d}{2} \Rightarrow T_1 + T_2 = 4a + \frac{d}{2}$

$\Rightarrow T_2 = 3a + \frac{d}{2} = a + d_1$

$\Rightarrow d_1 = T_2 - T_1 = 2a + \frac{d}{2}$ **Ans.**]

72. **(C)**

equation of any line through (2, 3) is $y - 3 = m(x - 2)$

$y = mx - 2m + 3$

area of Δ OAB = 12

i.e. $\frac{1}{2}\left(\frac{2m-3}{m}\right)(3-2m) = \pm 12$

taking + sign, we get $(2m+3)^2 = 0$

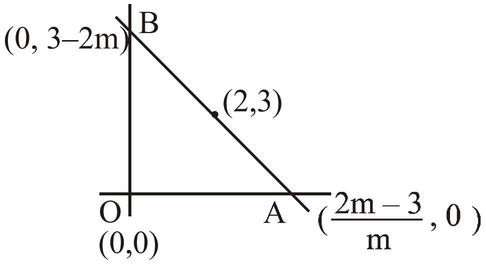
this gives one value of $m = -3/2$

taking negative sign, we get

$4m^2 - 36m + 9 = 0 (D > 0)$

quadratic in m gives 2 values of m

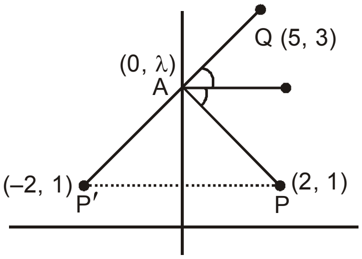
$\therefore$ 3 st. lines are possible.



73. **(A)**

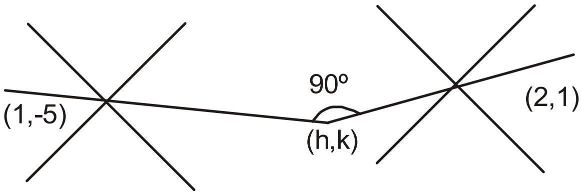
Equating slopes of P ′A and P ′Q are equal

$$\frac{\lambda - 1}{2} = \frac{3-1}{7}$$



$\Rightarrow \quad \lambda = \dfrac{11}{7} \quad \Rightarrow A = \left(0, \dfrac{11}{7}\right)$

74. (D)



$\dfrac{k+5}{h-1} \cdot \dfrac{k-1}{h-2} = -1$

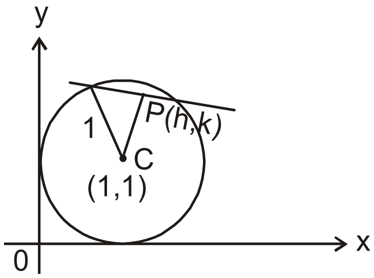
$x^2 + y^2 - 3x + 4y - 3 = 0$

75. (B)

We have, $\log_3 y = x$ and $\log_2 z = x \Rightarrow \quad y = 3^x$ and $z = 2^x$

$\therefore \quad 72^x = (2^3 \times 3^2)^x = (2^x)^3 (3^x)^2 = (z)^3 (y)^2 = y^2 z^3.$

76. (A)

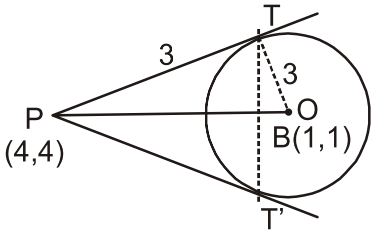


$CP = \dfrac{\sqrt{3}}{2} = \sqrt{(h-1)^2 + (k-1)^2}$

Locus of point P(h, k) is

$(x-1)^2 + (y-1)^2 = \dfrac{3}{4}$

77. (B)



$PT = \sqrt{16+16-8-8-7} = 3$

$\Rightarrow TT' = 2BT = 2 \cdot 3 \cos 45^\circ = 3\sqrt{2}$

78. (D)

Triangle is right angled

79. (C)

$h^2 = 5$

80. (A)

Centroid is (h,k)

81. (C)

Lies on director circle

82. (A)

tan β = 2 sin α. sin γ. cosec (α + γ) = θ cot β = $\frac{\sin(\alpha+\gamma)}{2\sin\alpha\sin\gamma}$

i.e. 2 cot β = $\frac{\sin\alpha\cos\gamma+\cos\alpha\sin\gamma}{\sin\alpha\sin\gamma} = \cot\alpha+\cot y$

83. (A)

The given condition can be written

$(\cos^2\alpha + \sin^2\alpha)^3 - 3\sin^2\alpha\cos^2\alpha(\cos^2\alpha + \sin^2\alpha) + k\sin^2 2\alpha = 1.$

$\Rightarrow$ $(-3/4)\sin^2\alpha + k\sin^2 2\alpha = 0$

showing that k = 3/4.

84. (B)

$$\frac{\cos x\tan x}{k^2}+\frac{1}{\tan x}+\frac{\sin x}{1+\cos x} = \frac{\sin x}{k^2}+\frac{\cos x(1+\cos x)+\sin^2 x}{\sin x(1+\cos x)} = \frac{a}{k}+\frac{1}{\sin x}=\frac{1}{\sin x}+\frac{1}{ak}$$

85. (B)

$\therefore\ -\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$ $\Rightarrow$ $-1\le\sin\theta\le 1$

Here $0 < \sin\theta < 1$ $\Rightarrow$ $\log_{\sin\theta}\cos2\theta = 2$

$\cos2\theta = \sin^2\theta$ $\Rightarrow$ $1-2\sin^2\theta = \sin^2\theta$

$3\sin^2\theta \Rightarrow = 1$ $\Rightarrow$ $\sin^2\theta = \frac{1}{3}$

$\therefore$ $\sin\theta = \frac{1}{\sqrt{3}}$ $\{\therefore\ 0 < \sin\theta < 1\}$ a unique solution

86. (A)

$\sin 2\beta = \sqrt{\sin\alpha.\cos\alpha}$

$\cos4\beta = 1 - 2\sin^2 2\beta = 1 - 2\sin\alpha.\cos\alpha = (\sin\alpha - \cos\alpha)^2 = 2\sin^2\left(\alpha-\frac{\pi}{4}\right)$

or = $2\sin^2\left(\frac{\pi}{4}-\alpha\right)$

87. **(C)**

Slope of OQ = $\frac{4}{3}$

Slope of OR = $-\frac{3}{4}$

$\therefore \angle ROQ = 90°$ $\quad\therefore\quad \angle QPR = \frac{\pi}{4}$

88. **(B)**

y = mx be chord

the points of intersection are given by $x^2(1 + m^2) - x(3 + 4m) - 4 = 0$

$\therefore x_1 + x_2 = \frac{3+4m}{1+m^2}$ and $x_1 x_2 = \frac{-4}{1+m^2}$

Since (0, 0) divides the point of (x_1, y_1) and (x_2, y_2) in the ratio 1 : 4

$\therefore x_2 = -4x_1$

then $-3x_1 = \frac{3+4m}{1+m^2}$ and $-4x_1^2 = -\frac{-4}{1+m^2}$

$\therefore 9 + 9m^2 = 9 + 16m^2 + 24m$

i.e. m = 0, $\quad -\frac{24}{7}$

$\therefore$ the lines are y = 0 and 7y + 24x = 0

89. **(D)**

$\therefore x^2 + y^2 - 2x - 2y - 6 = 0$

$\therefore$ radius = $\sqrt{8}$

$\therefore PT = \sqrt{2a^2 - 4a - 6}$ $\qquad \therefore \tan\theta = \frac{2\sqrt{2}}{\sqrt{2}\sqrt{a^2-2a-3}}$

$\therefore \frac{\pi}{3} < 2\theta < \pi \quad \Rightarrow \frac{\pi}{6} < \theta < \frac{\pi}{2}$

$\Rightarrow \frac{1}{\sqrt{3}} < \tan\theta < \infty \Rightarrow \frac{1}{\sqrt{3}} < \frac{2}{\sqrt{a^2-2a-3}} < \infty$

$\Rightarrow \frac{4}{a^2-2a-3} - \frac{1}{3} > 0 \Rightarrow \frac{(a-5)(a+3)}{(a-3)(a+1)} \quad \Rightarrow a \in (-3, -1) \cup (3, 5)$

90. **(B)**

$\therefore$ reflection of (5, 8) in BC will be circumcircle.

$\therefore$ (8,5) will lie on circicircle

$\therefore$ equation of circumcircle is

$(x - 2)^2 + (y - 3)^2 = (8 - 2)^2 + (3 - 5)^2$

$\Rightarrow x^2 + y^2 - 4x - 6y - 27 = 0$