SOLUTIONS

PROGRESS TEST-4(A)

RBS-1801 & 1802

(JEE ADVANCED PATTERN)

Test Date: 23-09-2017

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PHYSICS

1. Given $E = \dfrac{q}{4\pi\varepsilon_0 x^2}$.

Hence the magnitude of the electric intensity at a distance 2*x* from charge *q* is

$$E' = \frac{q}{4\pi\varepsilon_0 (2x)^2} = \frac{q}{4\pi\varepsilon_0 x^2} \times \frac{1}{4} = \frac{E}{4}$$

Therefore, the force experienced by a similar charge *q* at a distance 2*x* is

$$F = qE' = \frac{qE}{4}$$

$\therefore$ **(D)**

2. Net force on any charge = 0.Force on any charge *Q* at end

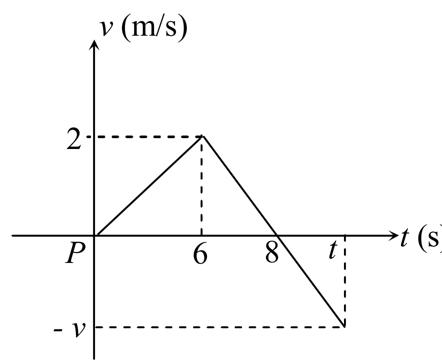
$F = K\dfrac{Q^2}{4x^2} + \dfrac{KqQ}{x^2} = 0$. Hence, $q = \dfrac{-Q}{4}$

$\therefore$ **(A)**

3. $T_1 = \dfrac{2u\sin\theta}{g}$, $T_2 = \dfrac{2u\cos\theta}{g}$ and $R = \dfrac{u^2 \sin 2\theta}{g}$ $\Rightarrow$ $R = \dfrac{T_1 T_2 g}{2}$

$\therefore$ **(B)**

4. Let after *t* second particle will reach at P again,



$\therefore$ area of $v - t$ curve = 0

$$\frac{1}{2} \times 2 \times 8 - \frac{1}{2} \times (t-8) \times (t-8) \times 1 = 0$$

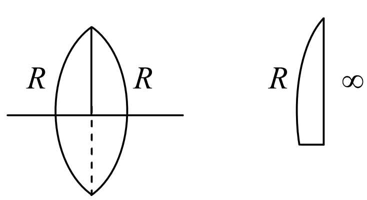
$$(t-8)^2 = 16$$

$$t - 8 = 4$$

$$t = 12s$$

$\therefore$ **(C)**

5.



$$\frac{1}{20} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$= \frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

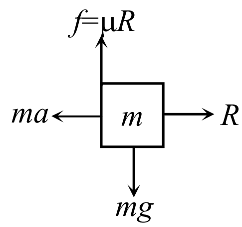
$$f = 40cm$$

$\therefore$ **(C)**

6. $\Sigma F_y = 0$, $R = ma$

$mg = \mu R = \mu ma$

$\mu = \frac{g}{a} = 0.5$



$\therefore$ **(C)**

7. $E = \dfrac{Qz}{4\pi\varepsilon_0\left(R^2 + z^2\right)^{\frac{3}{2}}}$, for $z << R$ $F \propto x$ also force is towards centre of ring.

$\therefore$ **(A) (C)**

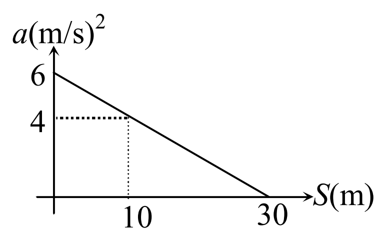
8. In equilibrium position, if x_0 is stretch of spring then $kx_0 = qE$ or $x_0 = \dfrac{qE}{k}$

If x_m is maximum stretch of spring , $\dfrac{1}{2}kx_m^2 = qEx_m$ or $x_m = \dfrac{2qE}{k}$

Amplitude will be $x_m - x_0 = \dfrac{2qE}{k} - \dfrac{qE}{k} = \dfrac{qE}{k}$

$\therefore$ **(A,B, C)**

9. Area $= \dfrac{1}{2} \times 10 \times (6 + 4) = \dfrac{v^2}{2}$



$v = 10$ m/s

Area upto 30 m $= \dfrac{1}{2} \times 30 \times 6 = \dfrac{v^2}{2}$

$v^2 = 180$

$v_{max} = \sqrt{80} < 14$

$\therefore$ **(B) and (C)**

10. v_x= 3 m/s

a_x = – 1.0 m/s^2

$\therefore \quad v_x^2 = u_x^2 + 2a_x.x$

or $0 = (3)^2 + 2(-1)(x)$ or x = 4.5 m

Also $v_x = u_x + a_x t$

$0 = 3 - (1.0)t$ or t = 3s

$y = u_y t + \frac{1}{2} a_y t^2 = 0 + \frac{1}{2}(-0.5)(3)^2 =$ -2.25 m

and $v_y = a_y t = (-0.5)(3) = -$ 1.5 m/s

$\therefore \quad \vec{v} = v_x \hat{i} + v_y \hat{j} = 0 - 1.5\hat{j} = (-1.5\hat{j})$m/s

and $\vec{r} = x\hat{i} + y\hat{j} = (4.5\hat{i} - 2.25\hat{j})$m

$\therefore$ **(B) and (C)**

11. For maxima path difference = $n\,\lambda$

If d = path difference between waves reaching point O = 7 λ

O will be maxima.

For d = λ only one maxima at O is possible, the screen being finite.

$\therefore$ **(C) and (D)**

12. If the image is real and magnified means object is between f and 2f.

When lens immersed in water focal length, $f_1 = \frac{(\mu - 1)}{\left(\frac{\mu}{\mu_r} - 1\right)} f = 4f$

Now object is between pole and focus so image is virtual and magnified.

$\therefore$ **(A) and (C)**

13. Friction maximum = 24 N

So net applied force on P is less than f_{max}.

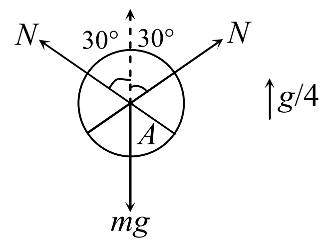
Hence acceleration is zero and T_A = 20 N, T_B = 40 N

Contact force $= \sqrt{N^2 + (f)^2} = \sqrt{(40)^2 + (20)^2} = 20\sqrt{5}$ N

$\therefore$ **(A) (B) (C) and (D)**

14. Net upward force on three spheres applied by bottom

$= 3mg + \frac{3}{4}mg = \frac{15mg}{4}$



For sphere A, $N\sqrt{3} = mg + \frac{mg}{4}$, $N = \frac{5mg}{4\sqrt{3}}$

$\therefore$ **(B) and (D)**

15. **(D)**

16. **(C)**

17. As displacement along y-axis is zero, $k = -V_y = -10$

$\therefore$ **(B)**

18. $\tan\alpha = \frac{u_y}{u_x} = 1$

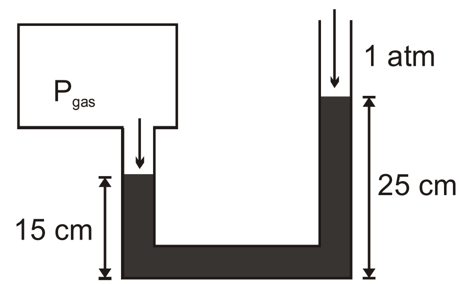
$\therefore$ **(A)**

CHEMISTRY

19. (C)

$P_{gas f} + 15 = 76 + 25$

$P_{gas f} = 76 + 25 - 15 = 86$ cm of Hg



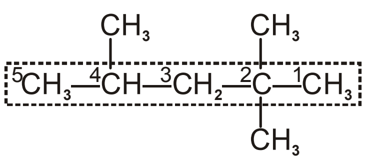
From Boyle's Law

$P_i V_i = P_f V_f$

$P_i V = 86 \times V \times \frac{110}{100}$

$P_i = 86 \times 1.1 = 94.6$ cm of Hg

20. (B)



iso-octane (common name)

IUPAC name : 2,2,4-Trimethylpentane.

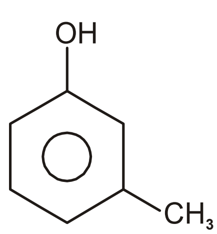
21. (A)

Homolytic fission :The cleavage of a bond is homolytic fission, so that each of the molecular fragments between which the bond is broken retains one of the bonding electrons.

$C_2H_6 : \ H_3C - CH_3 \longrightarrow \dot{C}H_3 + \dot{C}H_3$ (Free-redicals)

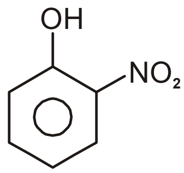
$\underset{Sp^2}{\dot{C}H_3} + \underset{Sp^2}{\dot{C}H_3}$

22. (A)



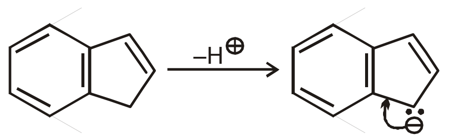
IUPAC Name : 3-Methylphenol

23. (B)



—NO_2(–M, –I) Increases acidic strength as compared to phenol

24. (D)



4n + 2 = 10
n = 2
(Aromatic)

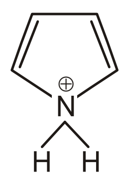
25. (B, C, D)

$$xSn^{2+} + yCr_2O_7^{2-} + zH^+ \longrightarrow aSn^{4+} + bCr^{3+} + cH_2O$$

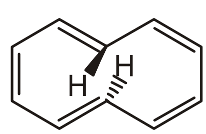
Upon balancing the redox–reaction

$$3Sn^{2+} + Cr_2O_7^{2-} + 14H^+ \longrightarrow 3Sn^{4+} + 2Cr^{3+} + 7H_2O$$

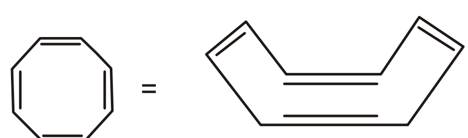
26. (A,B,D)



Non-aromatic



Non-planer (i.e., Non-aromatic)



Tube shaped structure
↓
Non-planer
↓
Non-aromatic

27. (A,C)

m.g.eq of I^- = 10

m.g. eq of $IO_3^- = 50$

So, I^- is limiting

n-factor of I_2 = 5/3

So, m.mole of I_2 $= \frac{10}{\frac{5}{3}} = 6$

m.mole of $MnO_4^- = \frac{6 \times 14}{5} = 16.8$

m.mole of Mn^{+2} = 16.8

28. (A), (B)

Nitrogen molecule has 14 electrons. So the electron distributed in molecular orbital as below: [(★) star indicated for anti bonding]

$N_2 = \sigma 1s^2, \sigma_1^{\star} s^2, \sigma 2s^2, \sigma_2^{\star} s^2 \left[\pi 2p_x^2, \pi 2p_y^2\right] \sigma 2p_z^2$

Where :

Nb = Nos of bonding electrons

Na = Nos. non-bonding electrons

N_2 – bond order = $\frac{10-4}{2} = 3$

N_2^+ – bond order = $\frac{9-4}{2} = 2.5$

Oxygen molecule has 16 electron, so the electron distributed in molecules orbital as below.

$O_2 = \sigma 1s^2, \sigma_1^{\star} s^2, \sigma 2s^2 \sigma_2^{\star} s^2, \sigma 2p_z^2, \left[\pi 2p_x^2 = \pi 2p_y^2\right]\left[\pi_2^{\star} p_x^1 = \pi_2^{\star} p_y^1\right]$

O_2 – Bond order $= \frac{10-6}{2} = 2$

$O_2^+ -$ Bond order $= \frac{10-5}{2} = 2.5$

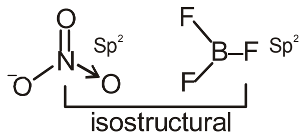
Bond dissociation α bond order

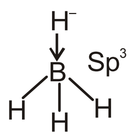
Bond length $\alpha \frac{1}{\text{Bond order}}$

29. (A), (B), (C)

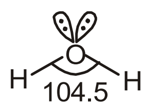
$CsBr_3$ contains Cs^+ and Br_3^- ions. due to energy effect (large cation stablise by large anion)

(A) Cs Br_3= Cs^+ + 3 Br^-

(B) 

(C) 

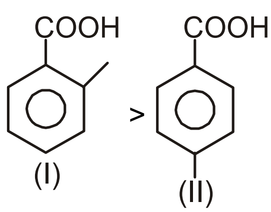
109.5º

(D) H_2O : 

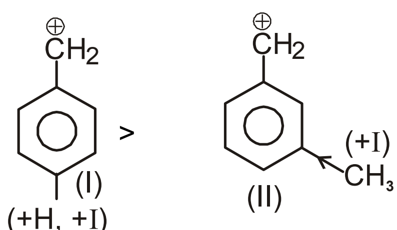
30. (A), (B), (C), (D)

31. (A,B,C)

32. (C,D)

(C) 

(Acidic strength due to ortho effect in (I).

(D) 

(I) is more stable than (II) due to +H-effect)

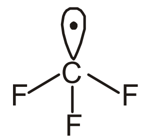
33. (C)

34. (D)

35. (C)

Only (C) has unpaired electron.

36. (A)



sp^3-C(non bonding electron in sp^3 hybrid orbital) that why pyramidal in shape

MATHEMATICS

37. (C)

$$f(x) = [x]^2 - [x^2] = \begin{cases} 0, & 0 \le x < \sqrt{2} \\ -1, & \sqrt{2} \le x < \sqrt{3} \\ -2, & \sqrt{3} \le x < 2 \\ 0, & x = 2 \end{cases}$$

$\therefore$ Range of f(x) is {–2, –1, 0}.

38. (C)

$ex + y = a$

$e = \frac{2}{3}, \; a = \frac{10}{3}$

39. (B)

(i) When $0 \le x < 1$

$f(x)$ doesn't exist as [x] = 0 here.

(ii) Also $\lim_{x \to I+} f(x)$ and $\lim_{x \to I-} f(x)$ does not exist.

Hence $f(x)$ is discontinuous at all integers and also in (0, 1)

40. (D)

$$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \frac{he^{-\left(\frac{1}{h} + \frac{1}{|h|}\right)} - a}{h}$$

$$f'(0+0)=\lim_{h\to 0}\frac{he^{-\left(\frac{1}{h}+\frac{1}{h}\right)}-a}{h}=\lim_{h\to 0}\frac{h.e^{-2/h}-a}{h}=0-\lim_{h\to 0}\frac{a}{h}=0$$

provided a = 0

Also, $f'(0-0)=\lim_{h\to 0}\frac{-he^{-\left(-\frac{1}{h}+\frac{1}{h}\right)}-a}{-h}=1+\lim_{h\to 0}\frac{a}{h}=1$, provided a = 0

Thus $f(x)$ can't be made differentiable at x = 0.

41. (B)

$$\lim_{x\to 0}\frac{\tan\left([-\pi^2]x^2\right)-\tan\left([-\pi^2]\right).x^2}{\sin^2 x}$$

Since $9<\pi^2<10$

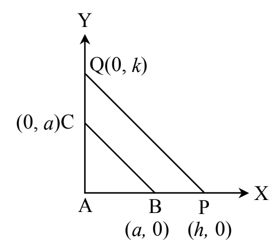
$\therefore \quad -10<-\pi^2<-9$; $\therefore \quad [-\pi^2]=-10$

$$=\lim_{x\to 0}\frac{\tan\left(-10x^2\right)-\tan(-10).x^2}{\sin^2 x}$$

$$=\lim_{x\to 0}\frac{-\tan\left(10x^2\right)+\tan(10).x^2}{x^2.\frac{\sin^2 x}{x^2}}=\lim_{x\to 0}\left[\tan 10-\left(\frac{\tan\left(10x^2\right)}{10x^2}\right).10\right]=\tan 10-10$$

42. (C)

We take A as the origin and AB and AC as x-axis and y-axis respectively.



Let AP = h, AQ = k

Equation of the line PQ is

$$\frac{x}{h}+\frac{y}{k}=1 \qquad \ldots(i)$$

Given, BP . CQ = AB²

$\Rightarrow \quad (h-a)(k-a)=a^2 \quad \Rightarrow \quad hk-ak-ah+a^2=a^2$ or $ak+ha=hk$

or $$\frac{a}{h}+\frac{a}{k}=1 \qquad \ldots.(ii)$$

From (ii), it follows that line (i) i.e. PQ always passes through a fixed point (a, a).

43. (B, D)

$$\sum_{k=1}^{n} \frac{6^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}=\sum_{k=1}^{n}\left[\left(\frac{3^k}{3^k-2^k}\right)-\left(\frac{3^{k+1}}{3^{k+1}-2^{k+1}}\right)\right]$$

$$=\left(\frac{3}{3-2}-\frac{3^2}{3^2-2^2}\right)+\left(\frac{3^2}{3^2-2^2}-\frac{3^3}{3^3-2^3}\right)+\left(\frac{3^3}{3^3-2^3}-\frac{3^4}{3^4-2^4}\right)+$$

$$......+\left(\frac{3^n}{3^n-2^n}-\frac{3^{n+1}}{3^{n+1}-2^{n+1}}\right)$$

$$=3-\frac{3^{n+1}}{3^{n+1}-2^{n+1}}$$

Required limit = $\lim_{n\to\infty}\left(3-\frac{3^{n+1}}{3^{n+1}-2^{n+1}}\right)=3-1=2$

44. (A, B, C)

45. (A, B)

Required locus is tangent to $y^2 = 4x$ at (1, 2) $\qquad y = x + 1$

46. (A, B, C)

47. (A, B, C)

$f\left(\frac{3}{2}\right)=\frac{9}{4};\ f\left(\frac{9}{4}\right)=2\times\frac{9}{4}-3=\frac{3}{2}$

$f\left(\frac{5}{2}\right)=2;\ f(2)=1;\ \ f(1)=1$

48. (A, B, D)

$y^2=4(x-h)$ pass through (0, 2) and (0, –2)

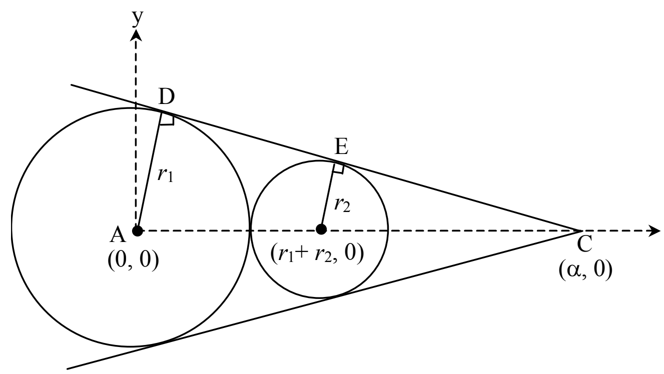
49. (B)

Let $A\equiv(0,\,0)$ and $B\equiv(r_1+r_2,\,0)$ be the centres of the two given fixed circles.

Let $C\equiv(\alpha,0)$ be the point of intersection of direct common tangents.

Now, $\frac{r_2}{r_1}=\frac{\alpha-(r_1+r_2)}{\alpha}\ \Rightarrow\ r_2\alpha=r_1\alpha-r_1^2-r_1r_2\ \Rightarrow\ \alpha=\frac{r_1^2+r_1r_2}{r_1-r_2}$

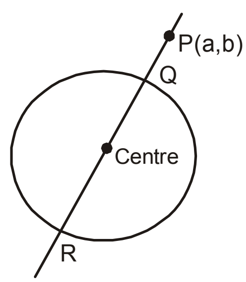
$\therefore$ Locus of C is $x = \dfrac{r_1^2 + r_1 r_2}{r_1 - r_2} = a$ constant, which is a straight line.



50. (A)

The given circle is $(x+1)^2 + (y+2)^2 = 9$ has radius = 3

The points on the circle which are nearest and farthest to the point P (a, b) are Q and R respectively (see fig.)



Thus, the circle centred at Q having radius PQ will be the smallest required circle while the circle centred at Q having radius PR be the largest required circle. Hence, difference between their radii = PR - PQ = QR = 6.

51. (B)

$A = (\tan^{-1}x + \cot^{-1}x)^3 - 3\tan^{-1}x \cot^{-1}x (\tan^{-1}x + \cot^{-1}x)$

$\Rightarrow A = \left(\frac{\pi}{2}\right)^3 - \frac{3\pi}{2}(\tan^{-1}x \cot^{-1}x) \quad \Rightarrow A = \frac{\pi^3}{8} - \frac{3\pi}{2}(\tan^{-1}x)\left(\frac{\pi}{2} - \tan^{-1}x\right) \quad \Rightarrow A = \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\tan^{-1}x - \frac{\pi}{4}\right)^2$

$\therefore \quad \frac{\pi^3}{32} \le A < \frac{\pi^3}{8}$

52. (A)

$B = (\sin^{-1} t + \cos^{-1} t)^2 - 2\sin^{-1} t \cos^{-1} t$

$\Rightarrow B = \frac{\pi^2}{4} - 2\sin^{-1} t\left(\frac{\pi}{2} - \sin^{-1} t\right) \Rightarrow B = \frac{\pi^2}{8} + 2\left(\sin^{-1} t - \frac{\pi}{4}\right)^2$

$\therefore$ maximum value of B = $\frac{\pi^2}{8} + \frac{2\pi^2}{16} = \frac{\pi^2}{4}$

Now $\lambda = \frac{\pi^3}{32}$ and $\mu = \frac{\pi^2}{4}$

$\therefore \frac{\lambda - \mu\pi}{\mu} = -\frac{7\pi}{8}$ $\quad\therefore \cot^{-1}\cot\left(\frac{\lambda - \mu\pi}{\mu}\right) = \frac{\pi}{8}$

53. (A)

54. (D)

Sol.

$$f(x) = \begin{cases} 0\ ; & 0 \le x < 1 \\ x\ ; & 1 \le x < 2 \\ 2(x-1)\ ; & 2 \le x < 3 \\ 6\ ; & x = 3 \end{cases}$$