SOLUTIONS

PROGRESS TEST-4(A)

RB-1801 To 1807

RBK-1801 To 1803

(JEE ADVANCED PATTERN)

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PHYSICS

1. Given $E = \dfrac{q}{4\pi\varepsilon_0 x^2}$.

Hence the magnitude of the electric intensity at a distance 2*x* from charge *q* is

$$E' = \frac{q}{4\pi\varepsilon_0 (2x)^2} = \frac{q}{4\pi\varepsilon_0 x^2} \times \frac{1}{4} = \frac{E}{4}$$

Therefore, the force experienced by a similar charge *q* at a distance 2*x* is

$$F = qE' = \frac{qE}{4}$$

$\therefore$ **(D)**

2. Net force on any charge = 0. Force on any charge *Q* at end

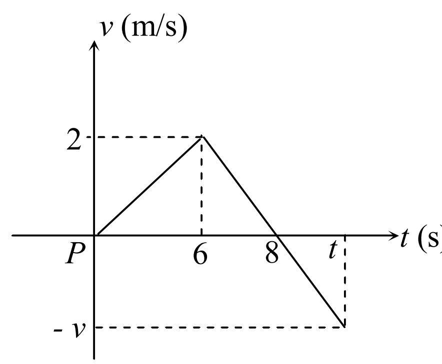
$$F = K\frac{Q^2}{4x^2} + \frac{KqQ}{x^2} = 0. \quad \text{Hence, } q = \frac{-Q}{4}$$

$\therefore$ **(A)**

3. $T_1 = \dfrac{2u\sin\theta}{g}$, $T_2 = \dfrac{2u\cos\theta}{g}$ and $R = \dfrac{u^2\sin 2\theta}{g}$ $\Rightarrow$ $R = \dfrac{T_1T_2g}{2}$

$\therefore$ **(B)**

4. Let after *t* second particle will reach at P again,



$\therefore$ area of $v - t$ curve = 0

$$\frac{1}{2} \times 2 \times 8 - \frac{1}{2} \times (t-8) \times (t-8) \times 1 = 0$$

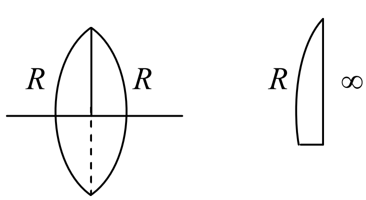
$$(t-8)^2 = 16$$

$$t - 8 = 4$$

$$t = 12s$$

$\therefore$ **(C)**

5. **(C)**



$$\frac{1}{20} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$= \frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

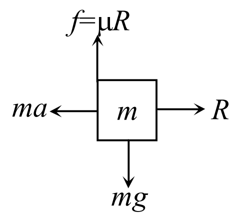
$$f = 40cm$$

6. $\Sigma F_y = 0$, $R = ma$

$mg = \mu R = \mu ma$

$\mu = \frac{g}{a} = 0.5$

$\therefore$ **(C)**



7. $E = \dfrac{Qz}{4\pi\varepsilon_0\left(R^2 + z^2\right)^{\frac{3}{2}}}$, for $z << R$ $F \propto x$ also force is towards centre of ring.

$\therefore$ **(A) (C)**

8. In equilibrium position, if x_0 is stretch of spring then $kx_0 = qE$ or $x_0 = \dfrac{qE}{k}$

If x_m is maximum stretch of spring , $\dfrac{1}{2}kx_m^2 = qEx_m$ or $x_m = \dfrac{2qE}{k}$

Amplitude will be $x_m - x_0 = \dfrac{2qE}{k} - \dfrac{qE}{k} = \dfrac{qE}{k}$

$\therefore$ **(A,B, C)**

9. Area $= \dfrac{1}{2} \times 10 \times (6 + 4) = \dfrac{v^2}{2}$

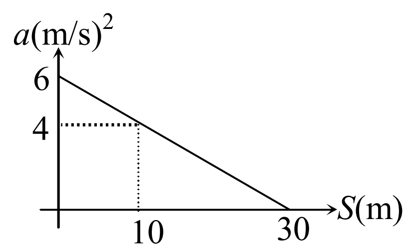
$v =$ 10 m/s

Area upto 30 m $= \dfrac{1}{2} \times 30 \times 6 = \dfrac{v^2}{2}$

$v^2 = 180$

$v_{\max} = \sqrt{80} < 14$

$\therefore$ **(B) and (C)**



10. $v_x = 3$ m/s

$a_x = -1.0$ m/s^2

$\therefore \quad v_x^2 = u_x^2 + 2a_x.x$

or $\quad 0 = (3)^2 + 2(-1)(x)$ or $x = 4.5$ m

Also $v_x = u_x + a_x t$

$0 = 3 - (1.0)t$ or t = 3s

$y = u_y t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-0.5)(3)^2 = -2.25$ m

and $v_y = a_y t = (-0.5)(3) = -1.5$ m/s

$\therefore \quad \vec{v} = v_x\hat{i} + v_y\hat{j} = 0 - 1.5\hat{j} = (-1.5\hat{j})$ m/s

and $\quad \vec{r} = x\hat{i} + y\hat{j} = (4.5\hat{i} - 2.25\hat{j})$ m

$\therefore$ **(B) and (C)**

11. For maxima path difference = $n\lambda$

If d = path difference between waves reaching point O = 7 λ

O will be maxima.

For $d = \lambda$ only one maxima at O is possible, the screen being finite.

$\therefore$ **(C) and (D)**

12. If the image is real and magnified means object is between f and $2f$.

When lens immersed in water focal length, $\quad f_1 = \dfrac{(\mu - 1)}{\left(\dfrac{\mu}{\mu_r} - 1\right)} f = 4f$

Now object is between pole and focus so image is virtual and magnified.

$\therefore$ **(A) and (C)**

13. Friction maximum = 24 N

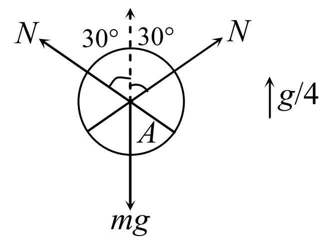
So net applied force on P is less than f_{max}.

Hence acceleration is zero and T_A = 20 N, T_B = 40 N

Contact force $=\sqrt{N^2+(f)^2}=\sqrt{(40)^2+(20)^2}=20\sqrt{5}$ N

$\therefore$ **(A) (B) (C) and (D)**

14. Net upward force on three spheres applied by bottom



= $3mg+\frac{3}{4}mg=\frac{15mg}{4}$

For sphere A, $N\sqrt{3}=mg+\frac{mg}{4}$, $N=\frac{5mg}{4\sqrt{3}}$

$\therefore$ **(B) and (D)**

15. **(D)**

16. **(C)**

17. As displacement along y-axis is zero, $k=-V_y=-10$

$\therefore$ **(B)**

18. $\tan\alpha=\frac{u_y}{u_x}=1$

$\therefore$ **(A)**

CHEMISTRY

19. (B)

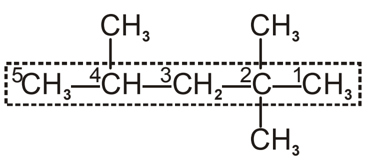
$$\frac{P_r V_r}{T_r} = 2.21$$

$$\frac{P}{P_C} \times \frac{V}{V_C} \times \frac{T_C}{T} = 2.21$$

$$\frac{PV}{T} \times \frac{T_C}{P_C V_C} = 2.21$$

v = 0.1243 L = 124.31 mL

20. (B)



iso-octane (common name)

IUPAC name : 2,2,4-Trimethylpentane.

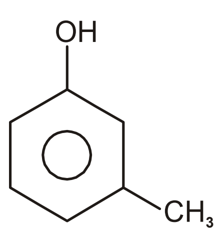
21. (A)

Homolytic fission :The cleavage of a bond is homolytic fission, so that each of the molecular fragments between which the bond is broken retains one of the bonding electrons.

$$C_2H_6 : \; H_3C - CH_3 \longrightarrow \underbrace{\dot{C}H_3 + \dot{C}H_3}_{\text{Free-redicals}}$$

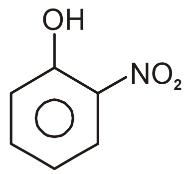
$$\underset{Sp^2}{\dot{C}H_3} + \underset{Sp^2}{\dot{C}H_3}$$

22. (A)



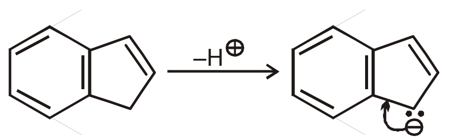
IUPAC Name : 3-Methylphenol

23. (B)



—NO_2(–M, –I) Increases acidic strength as compared to phenol

24. (D)



Conjugate Base

4n + 2 = 10
n = 2
(Aromatic)

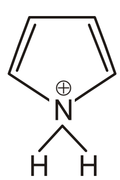
25. (B, C, D)

$$xSn^{2+} + yCr_2O_7^{2-} + zH^{+} \longrightarrow aSn^{4+} + bCr^{3+} + cH_2O$$

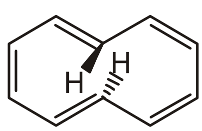
Upon balancing the redox–reaction

$$3Sn^{2+} + Cr_2O_7^{2-} + 14H^{+} \longrightarrow 3Sn^{4+} + 2Cr^{3+} + 7H_2O$$

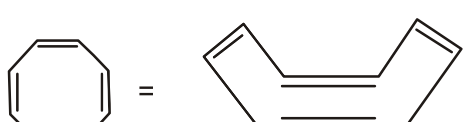
26. (A,B,D)



Non-aromatic



Non-planer (i.e., Non-aromatic)



Tube shaped structure
↓
Non-planer
↓
Non-aromatic

27. (A, C, D)

28. (A), (B)

Nitrogen molecule has 14 electrons. So the electron distributed in molecular orbital as below:
[($\star$) star indicated for anti bonding]

$$N_2 = \sigma 1s^2, \sigma_1^{\star} s^2, \sigma 2s^2, \sigma_2^{\star} s^2 \left[\pi 2p_x^2, \pi 2p_y^2\right] \sigma 2p_z^2$$

Where :

Nb = Nos of bonding electrons

Na = Nos. non-bonding electrons

N_2 – bond order = $\dfrac{10-4}{2} = 3$

N_2^+ – bond order = $\dfrac{9-4}{2} = 2.5$

Oxygen molecule has 16 electron, so the electron distributed in molecules orbital as below.

$$O_2 = \sigma 1s^2, \sigma_1^{\star} s^2, \sigma 2s^2 \sigma_2^{\star} s^2, \sigma 2p_z^2, \left[\pi 2p_x^2 = \pi 2p_y^2\right]\left[\pi_2^{\star} p_x^1 = \pi_2^{\star} p_y^1\right]$$

O_2 – Bond order $= \dfrac{10-6}{2} = 2$

O_2^+ – Bond order $= \dfrac{10-5}{2} = 2.5$

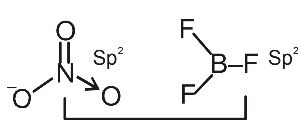
Bond dissociation α bond order

Bond length $\alpha \dfrac{1}{\text{Bond order}}$

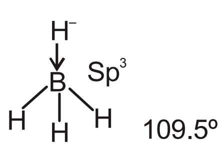
29. (A), (B), (C)

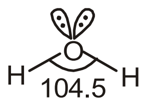
$CsBr_3$ contains Cs^+ and Br_3^- ions. due to energy effect (large cation stablise by large anion)

(A) Cs Br_3= Cs^+ + 3 Br^-

(B) 

isostructural

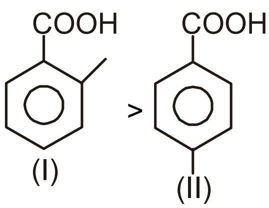
(C) 

(D) H_2O : 

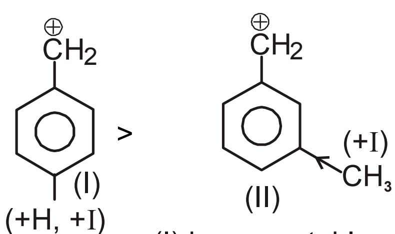
30. (A), (B), (C), (D)

31. (A,B,C)

32. (C,D)

(C) 

(Acidic strength due to ortho effect in (I).

(D) 

(I) is more stable than (II) due to +H-effect)

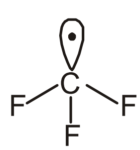
33. (C)

34. (D)

35. (C)

Only (C) has unpaired electron.

36. (A)



sp^3-C(non bonding electron in sp^3 hybrid orbital) that why pyramidal in shape

MATHEMATICS

37. (C)

$$f(x)=[x]^2-[x^2]=\begin{cases}0, & 0\le x<\sqrt{2}\\ -1, & \sqrt{2}\le x<\sqrt{3}\\ -2, & \sqrt{3}\le x<2\\ 0, & x=2\end{cases}$$

$\therefore$ Range of f(x) is {–2, –1, 0}.

38. (C)

$ex+y=a$

$e=\frac{2}{3},\ a=\frac{10}{3}$

39. (B)

(i) When $0\le x<1$

$f(x)$ doesn't exist as [x] = 0 here.

(ii) Also $\lim\limits_{x\to I+} f(x)$ and $\lim\limits_{x\to I-} f(x)$ does not exist.

Hence $f(x)$ is discontinuous at all integers and also in (0, 1)

40. (D)

$$f'(0)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}=\lim_{h\to 0}\frac{he^{-\left(\frac{1}{h}+\frac{1}{|h|}\right)}-a}{h}$$

$$f'(0+0)=\lim_{h\to 0}\frac{he^{-\left(\frac{1}{h}+\frac{1}{h}\right)}-a}{h}=\lim_{h\to 0}\frac{h.e^{-2/h}-a}{h}=0-\lim_{h\to 0}\frac{a}{h}=0$$

provided a = 0

Also, $f'(0-0)=\lim\limits_{h\to 0}\frac{-he^{-\left(-\frac{1}{h}+\frac{1}{h}\right)}-a}{-h}=1+\lim\limits_{h\to 0}\frac{a}{h}=1$, provided a = 0

Thus $f(x)$ can't be made differentiable at x = 0.

41. (B)

$$\lim_{x\to 0}\frac{\tan([-\pi^2]x^2)-\tan([-\pi^2]).x^2}{\sin^2 x}$$

Since $9<\pi^2<10$

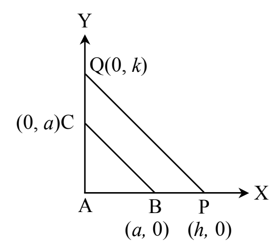
$\therefore$ $-10<-\pi^2<-9$; $\therefore$ $[-\pi^2]=-10$

$$=\lim_{x\to 0}\frac{\tan(-10x^2)-\tan(-10).x^2}{\sin^2 x}$$

$$=\lim_{x\to 0}\frac{-\tan(10x^2)+\tan(10).x^2}{x^2.\frac{\sin^2 x}{x^2}}=\lim_{x\to 0}\left[\tan 10-\left(\frac{\tan(10x^2)}{10x^2}\right).10\right]=\tan 10-10$$

42. (C)

We take A as the origin and AB and AC as x-axis and y-axis respectively.



Let AP = h, AQ = k

Equation of the line PQ is

$\frac{x}{h}+\frac{y}{k}=1$ …(i)

Given, BP . CQ = AB²

$\Rightarrow$ $(h-a)(k-a)=a^2$ $\Rightarrow$ $hk-ak-ah+a^2=a^2$ or $ak+ha=hk$

or $\frac{a}{h}+\frac{a}{k}=1$ ….(ii)

From (ii), it follows that line (i) i.e. PQ always passes through a fixed point (a, a).

43. (B, D)

$$\sum_{k=1}^{n}\frac{6^k}{(3^k-2^k)(3^{k+1}-2^{k+1})}=\sum_{k=1}^{n}\left[\left(\frac{3^k}{3^k-2^k}\right)-\left(\frac{3^{k+1}}{3^{k+1}-2^{k+1}}\right)\right]$$

$$=\left(\frac{3}{3-2}-\frac{3^2}{3^2-2^2}\right)+\left(\frac{3^2}{3^2-2^2}-\frac{3^3}{3^3-2^3}\right)+\left(\frac{3^3}{3^3-2^3}-\frac{3^4}{3^4-2^4}\right)+$$

$$......+\left(\frac{3^n}{3^n-2^n}-\frac{3^{n+1}}{3^{n+1}-2^{n+1}}\right)$$

$$= 3 - \frac{3^{n+1}}{3^{n+1} - 2^{n+1}}$$

Required limit = $\lim_{n\to\infty}\left(3 - \frac{3^{n+1}}{3^{n+1} - 2^{n+1}}\right) = 3 - 1 = 2$

44. (A, B, C)

45. (A, B)

Required locus is tangent to $y^2 = 4x$ at (1, 2) $\quad$ y = x + 1

46. (A, B, C)

47. (A, B, C)

$$f\left(\frac{3}{2}\right) = \frac{9}{4};\ f\left(\frac{9}{4}\right) = 2 \times \frac{9}{4} - 3 = \frac{3}{2}$$

$$f\left(\frac{5}{2}\right) = 2;\ f(2) = 1;\quad f(1) = 1$$

48. (A, B, D)

$y^2 = 4(x - h)$ pass through (0, 2) and (0, –2)

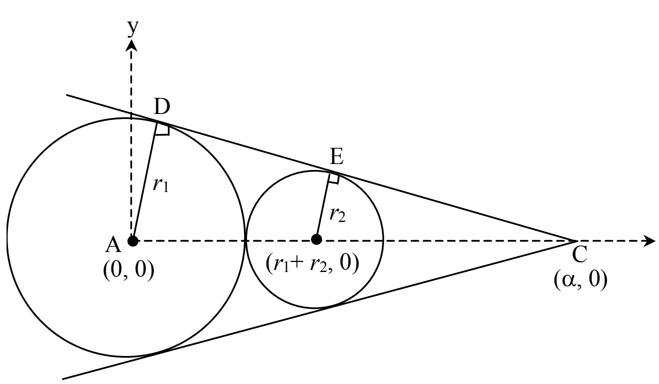
49. (B)

Let $A \equiv (0, 0)$ and $B \equiv (r_1 + r_2, 0)$ be the centres of the two given fixed circles.

Let $C \equiv (\alpha, 0)$ be the point of intersection of direct common tangents.

Now, $\frac{r_2}{r_1} = \frac{\alpha - (r_1 + r_2)}{\alpha} \Rightarrow r_2\alpha = r_1\alpha - r_1^2 - r_1 r_2 \Rightarrow \alpha = \frac{r_1^2 + r_1 r_2}{r_1 - r_2}$

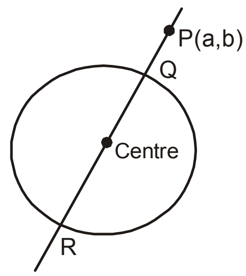
$\therefore$ Locus of C is $x = \frac{r_1^2 + r_1 r_2}{r_1 - r_2} = a$ constant, which is a straight line.



50. (A)

The given circle is $(x+1)^2+(y+2)^2=9$ has radius = 3

The points on the circle which are nearest and farthest to the point P (a, b) are Q and R respectively (see fig.)



Thus, the circle centred at Q having radius PQ will be the smallest required circle while the circle centred at Q having radius PR be the largest required circle. Hence, difference between their radii = PR - PQ = QR = 6.

51. (B)

52. (A)

Centre is (h, k)

$(x-h)^2+(y-k)^2=5$ pass through (0, 0)

(x_1, y_1) and (x_2, y_2) are foci

$x_1 x_2 = y_1 y_2 = 1$

53. (A)

54. (D)

$$f(x)=\begin{cases} 0\ ; & 0\le x<1 \\ x\ ; & 1\le x<2 \\ 2(x-1)\ ; & 2\le x<3 \\ 6\ ; & x=3 \end{cases}$$