SOLUTIONS

PROGRESS TEST-2

RBPA

(JEE ADVANCED PATTERN)

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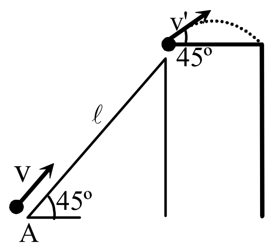
PHYSICS

1. **(A)**

2. **(D)**



$\frac{v'^2}{g} = 40$ [$\because$ $\theta = 45º$] $v'^2 = 40$ g but $v'^2 = v^2 - 2a\ell$

or $40g = v^2 - 2$ (g sin 45º). $20\sqrt{2}$ or $40\,g = v^2 - 40g$

or $v^2 = 80g$ or $v = 20\sqrt{2}$ m/s

3. **(B)**

$$\frac{\frac{1}{2}mu_1^2\cos^2\theta_1}{\frac{1}{2}mu_2^2\cos^2\theta_2} = \frac{4}{1}$$

$\Rightarrow \frac{u_1\cos\theta_1}{u_2\cos\theta_2} = 2$ ...(1)

and $\frac{u_1^2\sin^2\theta_1}{u_2^2\sin^2\theta_2} = \frac{4}{1}$

or $\frac{u_1\sin\theta_1}{u_2\sin\theta_2} = \frac{2}{1}$ ...(2)

from equation no. (1) and (2)

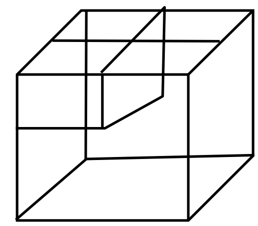
$$\frac{u_1\sin\theta_1.u_1\cos\theta_1}{u_2\sin\theta_2.u_2\cos\theta_2} = \frac{4}{1}$$

$$\frac{gR_1/2}{gR_2/2} = \frac{4}{1}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{4}{1}$$

4. **(B)**

ρ- charge density of the cube



V_ℓ^{corner} = potential at the corner of a cube of side ℓ.

V_ℓ^{centre} = potential at the centre of a cube of side ℓ.

$V_{\ell/2}^{centre}$ = potential at the centre of a cube of side $\frac{\ell}{2}$.

$V_{\ell/2}^{corner}$ = potential at the corner of a cube of side $\frac{\ell}{2}$.

By dimensional analysis $V_\ell^{corner} \propto \frac{Q}{\ell} = \rho\ell^2$

$V_\ell^{corner} = 4\,V_{\ell/2}^{corner}$

But by super position $V_\ell^{centre} = 8\,V_{\ell/2}^{corner}$ because the centre of the larger cube lies at a corner of the eight smaller cubes of which it is made

Therefore $\frac{V_\ell^{corner}}{V_\ell^{centre}} = \frac{4V_{\ell/2}^{coner}}{8V_{\ell/2}^{centre}} = \frac{1}{2}$

5. **(A)**

6. **(B, D)**

$a = -(g + kv^2) \Rightarrow \frac{vdv}{dh} = -(g + kv^2)$

$\Rightarrow \int_{v_0}^{0} \frac{vdv}{g + kv^2} = -\int_0^{h_{max}} dh$ on solving $h_{max} = 36\,m$

7. **(A.B,C,D)**

Area under a-t curve :

Δv = Area 1 = 2 × 4 = 8

v – u = 8

v = u + 8 = 0 + 8 = 8 m/s

$v' - v$ = Area 2 = $-\left(\frac{1}{2} \times 8 \times 2\right)$ = – 8 m/s

$v' = v - 8 = 8 - 8 = 0$

final velocity is zero at t = 10 sec

Displacement: Can be directly calculated from a–t cure without using v–t curve.

$\Delta S = u_0 t_0$ + (area under a–t curve) $(t_0 - t_c)$

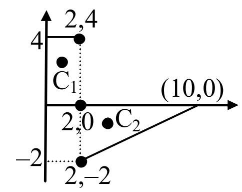
Where ΔS = displacement

u_0 = initial velocity

t_0 = total time

t_c = Abscissa of centroid

of corresponding area



Centroid of area 1: $C_1 = (1,2)$

Centroid of area 2 : $C_2 = \left(\frac{14}{3}, \frac{-2}{3}\right)$

$\Delta S = 0 + 8\,[10-1] + \left[(-8)\left(10-\frac{14}{3}\right)\right]$

$= 8 \times 9 + \left[-8\times\frac{16}{3}\right]$

$= 8\left[9-\frac{16}{3}\right]$

$= 8 \times \left[\frac{11}{3}\right]$

$= 8 \times 3.666$

$= 8 \times 3.67$

$\Delta S = 29.36$ m

8. **(A,B)**

Area under graph gives change in velocity

Train starts from rest and comes to rest

Therefore,

Positive area = negative area

$8 \times 1 + 6 \times 2 = 2 \times \Delta t$

$8 + 12 = 2\Delta t$

$\Delta t = \frac{20}{2} = 10$ sec

Displacement & distance are equal in this case displacement can be directly calculated from at curve without using v-t curve as follows

$\Delta S = ut_0 + (\text{area under a-t curve})\,(t_0 - t_c)$

Where $u \rightarrow$ initial velocity

$t_0 \rightarrow$ total time

$t_c \rightarrow$ abscissa of centroid of corresponding area

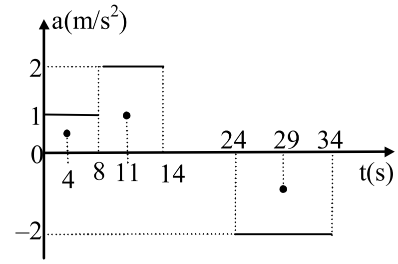
$\Delta s = 8\,[34 - 4] + 12\,[34-11] + (-20)\,[34-29]$

$= 8 \times 30 + 12 \times 23 - 20 \times 5$

$= 240 + 276 - 100$

$= 516 - 100$

$\Delta s = 416$ m



9. **(A, C)**

10. **(B,C)**

11. **(A,C)**

Near positive charge net potential is positive and near negative charge potential is negative. Q_1 is positive & Q_2 is negative, at 1 potential is zero and this point is near to Q_2

$\therefore$ Magnitude of Q_1 is more than magnitude of Q_2.

12. **(B, C)**

$$v^2 = v_0{}^2 + 2\left(\frac{qB_0}{m}\right)x_0$$

$$v = \sqrt{2}v_0$$

$$a_n = \frac{qE_0}{m}\frac{v_0}{\sqrt{v_x^2 + v_0^2}}$$

$$R = \frac{v^2}{a_n} = \frac{[m^2v_0^2 + 2qE_0mx_0]^{\frac{3}{2}}}{qE_0v_0m^2} = 4\sqrt{2}\,x_0$$

$\therefore$ **(B) and (C)**

13. **(B, D)**

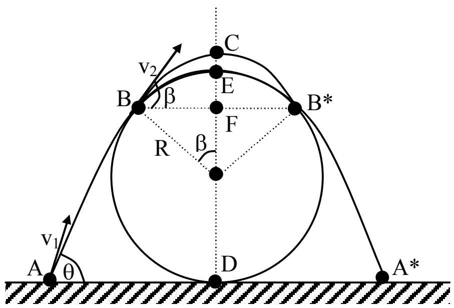
As, *A* and *C* are equidistance from the wire and $\vec{E}$ is perpendicular to wire so both *A* and *C* are at same potential.

$W_{ABC} = 0$ also from symmetry, $W_{AB} = -W_{BC}$

$\therefore$ **(B, D)**

14. **(2)**

The trajectory of the grasshopper is a parabola, which touches the trunk at two symmetrically placed points, B and B* on the two sides of the trunk (at the moment we don't know anything about these points-they may or may not coincide at the topmost point E. of the trunk). The grasshopper takes off from point A with an initial speed v_1 and at an angle θ with the horizontal, as shown in the figure. At the tangential points B and B* the grasshopper's velocity is v_2, making an angle β with the horizontal.



For the sake of simplicity we choose β as the independent variable of the problem. At point B the vertical component of velocity is

$v_2 \sin\beta = gt_2$,

where t_2 is the time of flight for the BC section of trajectory (C is the peak of the parabola). The corresponding horizontal displacement BF is

$v_2 t_2 \cos\beta = R \sin \beta$.

Multiplying these equations together we obtain

$$v_2^2 = \frac{gR}{\cos\beta}.$$

Again on solving we get,

$v_1^2 = v_2^2 + 2gR(1 + \cos\beta)$

$= \frac{gR}{\cos\beta} + 2gR(1 + \cos\beta)$

$= 2gR\left(1 + \cos\beta + \frac{1}{2\cos\beta}\right)$.

We can calculate the minimum value of v_1 using differential calculus. However, there is a less complicated method available which uses the inequality between arithmetic and geometric means :

$$\frac{1}{2}\left(\cos\beta + \frac{1}{2\cos\beta}\right) > \sqrt{\cos\beta \frac{1}{2\cos\beta}} = \frac{\sqrt{2}}{2}.$$

So the minimum value of cosβ + 1/(2cosβ) is equal to $\sqrt{2}$ and, therefore, β = 45º. The case β = 0 requires a larger initial velocity since 1.5 > $\sqrt{2}$; it follows that the trajectory with the minimum initial speed does not in fact touch the trunk at its topmost point. The gravitational potential energy of the grasshopper is greater at the peak of the parabola than at the uppermost point of the trunk, but its kinetic energy and total energy are smaller than they would be for a top-touching trajectory.

The numerical value of the minimal initial speed is

$\mathbf{v_1^{min}} =$ $\sqrt{2gR(1+\sqrt{2})}$

15. (6)

$\vec{V}_{B\ell}$ = 4 m/s ↑

$\vec{V}_{B\ell} = \vec{V}_B - \vec{V}_\ell$

4 m/s = $\vec{V}_B$ – 2m/s

$\vec{V}_B$ = 4 + 2 = 6 m/s.

16. (2)

$\frac{dx}{dt} = v_0 - \frac{v_0 t}{5}$

$\Rightarrow x = v_0 t - \frac{v_0 t^2}{10}$

where x can be either +10 or –10.

$10t - t^2 = 10 \Rightarrow t = \frac{10 \pm \sqrt{60}}{2}$

$10t - t^2 = -10 \Rightarrow t = \frac{10 \pm \sqrt{140}}{2}$

for 2nd event, $t = \frac{10 + \sqrt{60}}{2}$

and for 3rd event, $t = \frac{10 + \sqrt{140}}{2}$

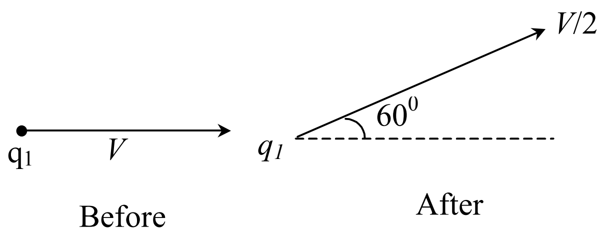
$\Delta t \approx 2$ sec

17. (2)

Since potential is decreasing, so q_1 is +ve and q_2 is –ve

$\frac{Kq_1}{\left(a + \frac{a}{3}\right)} = \frac{Kq_2}{\left(a - \frac{a}{3}\right)}$, $\frac{q_1}{q_2} = \frac{2}{1}$, $q_1 : q_2 =$ **2 : 1**

18. (3)



$\therefore$ Change in linear momentum of the particle of mass m_1 and charge q_1

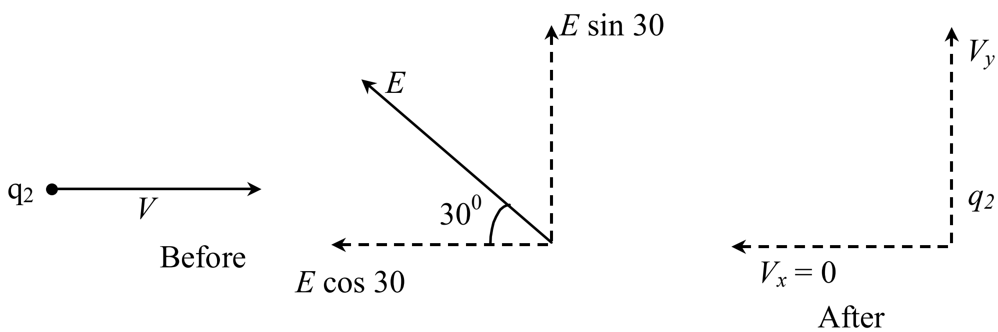
$$\Delta p_1 = \left(\frac{mV}{4} - mV\right)\hat{i} + \frac{\sqrt{3}mV}{4}\hat{j}$$

$$\Delta p_1 = -\frac{3}{4}mV\hat{i} + \frac{\sqrt{3}mV}{4}\hat{j}$$

$\therefore$ tan θ = $-1/\sqrt{3}$

i.e. electric field $\vec{E} = -E_x cos\, 30\hat{i} + E_y sin\, 30\hat{j}$

For second charge



As q_2 is finally moving $\perp$ to initial direction

$\therefore$ time taken by charge q_2 to come to rest along the initial direction of motion

$$t = \frac{V}{Eq_2\sqrt{3}/2m_2}$$

in this time velocity will have value

$$V_y = 0 + \frac{Eq_2}{2m_2}\frac{V}{Eq_2}.\frac{2m_2}{\sqrt{3}} = \frac{V}{\sqrt{3}}$$

CHEMISTRY

19. (C)

$H_2S \longrightarrow \alpha \simeq 90^o$

$NH_3 \longrightarrow \beta < 109.5^o$

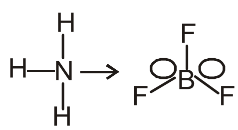
(due to l.p. – B.P. > B.P – B.P)

$SIH_4 \longrightarrow \gamma = 109.5^o$

$BF_3 \longrightarrow \delta = 120^o$

Bond angle order = $\delta > \gamma > \beta > \alpha$

20. (A)



$\left.\begin{array}{l} N \rightarrow Sp^3 \\ B \rightarrow Sp^3 \end{array}\right\}$

21. (A)

Due to +R effect of $-NH_2$ group at $=\ddot{N}-H$ in option A

22. (D)

23. (A)

At A and D the temperatures of the gas will be equal, so

$\Delta E = 0$, $\Delta H = 0$

Now $w = w_{AB} + w_{BC} + w_{CD} = -P_0V_0 - 2P_0V_0 \ln 2 + P_0V_0 = -2P_0V_0 \ln 2$

and $q = -w = 2P_0V_0 \ln 2$

24. (A), (C), (D)

25. (A), (C)

The adiabatic equations are

1. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$
2. $P_1V_1^{\gamma} = P_2V_2^{\gamma}$
3. $\dfrac{T_1^{\gamma}}{P_1^{\gamma-1}} = \dfrac{T_2^{\gamma}}{P_2^{\gamma-1}}$

26. (C), (D) **27. (A), (C)**

28. (A), (B), (C)

Solubility of fluorides of IIA

$BeF_2 > BaF_2 > SrF_2 > CaF_2 > MgF_2$

(BeF_2 is most soluble)

29. (A), (B), (C) **30. (B), (C), (D)** **31. (B), (C), (D)**

32. (4)

$$\gamma_{mix} = \frac{n_A C_{P_A} + n_B C_{P_B}}{n_A C_{V_A} + n_B C_{V_B}} = \frac{2(4R) + 4(5R/2)}{2(3R) + 4(3R/2)} = \frac{18R}{12R} = \frac{3}{2}$$

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ $(\gamma = \gamma_{mix} = 1.5)$

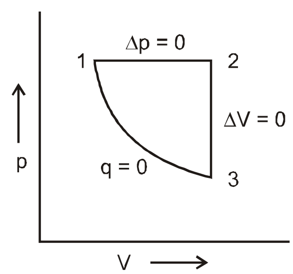
$$\therefore \quad T_2 = 320 \left(\frac{2}{8}\right)^{1.5-1} = 320 \times \frac{1}{2} = 160 \text{ K}$$

$\therefore \quad W = \frac{nR}{\gamma - 1}(T_2 - T_1) = \frac{6R}{1.5 - 1}(160 - 320) = -1920\text{ R} = 1920 \times 2 = -3840$ calories. $= -3.84$ Kcal.

33. (5) **34. (8)**

35. (6)

The process can be described on a p-V diagram as



At 1 : p = 10 atm T = 400 K $V = V_1$

At 2 : p = 10 atm T = 800 K $V = V_2 = 2V_1$

At 3 : p = ? $T = T_3$

$V = V_3 = V_2 = 2V_1$

Therefore,

$W_{12} = -p\Delta V = -nRT = -400\text{ R}$

$W_{23} = 0$ $[\because \Delta V = 0]$

Between 3 and 1 ; $TV^{\gamma-1}$ = constant

$T_3 (2V_1)^{\gamma-1} = 400(V_1)^{\gamma-1}$

$\Rightarrow$ $T_3 = 400\left(\frac{1}{2}\right)^{2/3} = 252$ K

$\Rightarrow$ $W_{31} = \Delta E_{31} = nC_V(T_1 - T_3) = \frac{3}{2}R(400 - 252) = 222$ R

$\Rightarrow$ $W_{12-31} = W_{12} + W_{23} + W_{31} = -178\text{ R} = -178 \times 2 = -356$ cal

36. (9)

Heat evolved by 101.972 watt bulb in 15 minutes = w × t (sec)

= 101.972 × 60 × 15 = 91,774.85

Volume of room = volume of air = 5×4×3 = $60m^3$

= 60×10^6 mL

$\therefore$ Mass of air in room

= $60 \times 10^6 \times 1.22 \times 10^{-6}$ kg = 73.2 kg

Since, heat given by bulb = Heat taken by (roof + wall) + heat taken by air

or, $91{,}774.8 = [50 \times 10^3 \times \Delta T] + [73.2 \times 10^3 \times 0.71 \times \Delta T]$

$\Delta T = 0.9K = 9 \times 10^{-1}K$

MATHEMATICS

37. (B)

Let f (x) + g (x) = F (x)

f (x) – g (x) = G (x)

since $\lim\limits_{x\to a} F(x)$ and $\lim\limits_{x\to a} G(x)$ exists

hence $\lim\limits_{x\to a} \frac{F(x)+G(x)}{2}$ and $\lim\limits_{x\to a} \frac{F(x)-G(x)}{2}$ must exist i.e.

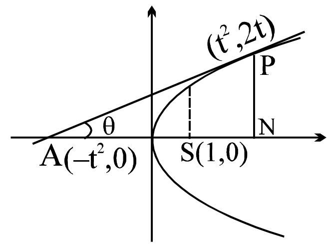
$\lim\limits_{x\to a} f(x)\cdot g(x)$ also exists.]

38. (D)

$$y = 3\sin^{-1}\sin\left(x+\frac{\pi}{2}\right) = \begin{cases} 3\left(x+\frac{\pi}{2}\right), & \frac{-\pi}{2} \le x+\frac{\pi}{2} \le \frac{\pi}{2} \\ 3\left[\pi - \left(x+\frac{\pi}{2}\right)\right], & \frac{\pi}{2} < x+\frac{\pi}{2} \le \frac{3\pi}{2} \end{cases} = \begin{cases} 3x+\frac{3\pi}{2}, & -\pi \le x \le 0 \\ \frac{3\pi}{2} - 3x, & 0 < x \le \pi \end{cases}$$

Clearly function is not differentiable at x = 0 and hence by property of periodic function it is not differentiable at $x = n\pi$; $n \in I$

39. (B)



Tangent : $ty = x + t^2$, $\tan\theta = \dfrac{1}{t}$

Area $A = \dfrac{1}{2}(AN)(PN) = \dfrac{1}{2}(2t^2)(2t)$

$A = 2t^3 = 2(t^2)^{3/2}$

$\because\ t^2 \in [1,4]$ & A_{max} occurs when $t^2 = 4 \Rightarrow A_{max} = 16$

40. (B)

$f\left(f(x)\right) = x$

$$\frac{a\left(\dfrac{ax+b}{cx+d}\right)+b}{c\left(\dfrac{ax+b}{cx+d}\right)+d} = x$$

$c(a+d)x^2 + \left(d^2 - a^2\right)x - b(a+d) = 0$

$\Rightarrow a + d = 0 \Rightarrow a = -d$

Now, $f(1) = 1 \Rightarrow c = 2a + b.$

& $f(5) = 5 \Rightarrow 25c = 10a + b$

& hence $a = 3c \Rightarrow b = -5c$

$\therefore\ f(x) = \dfrac{3x-5}{x-3}$

41. (C)

$r_1 + r_2 + r_3 + r = 4R + 2r = 2.(2R) + 2r = 2\times 10 + 2\times 2 = 24$ unit

$\left(\because r = (s-a)\tan A/2 = (12-10)\tan 45^\circ = 2 \text{ and } 2R = 10\right)$

42. (A,B,C,D)

(A) Let $f(x) = e^{[x]}$ and $g(x) = e^{\{x\}}$

$\lim\limits_{x\to 0} e^{[x]} = \begin{cases} \text{R.H.L.} = 1 \\ \text{L.H.L.} = \dfrac{1}{e} \end{cases}$ where [x] denotes G.I.F.

$$\lim_{x\to 0} e^{\{x\}} = \left\langle \begin{array}{l} R.H.L. = 1 \\ L.H.L. = e \end{array}\right.$$ where {x} denotes fractional part of x.

but $\lim_{x\to 0} e^{[x]+\{x\}} = \lim_{x\to 0} e^{x} = 1$ (exists)

(B) Let f(x) = [x] and g(x) = {x}

$$\lim_{x\to 0} [x] = \left\langle \begin{array}{l} R.H.L. = 0 \\ L.H.L. = -1 \end{array}\right.$$ where [x] denotes G.I.F.

$$\lim_{x\to 0} \{x\} = \left\langle \begin{array}{l} R.H.L. = 0 \\ L.H.L. = 1 \end{array}\right.$$ where {x} denotes fraction part of x.

$\lim_{x\to 0} \left[\{x\}\right] = 0$ (exists)

(C) Let f (x) = $\dfrac{\sin x}{x}$ and g (x) = [x], where [·] denotes G..I.F.

$$\lim_{x\to 0} g(f(x)) = \lim_{x\to 0} \left[\frac{\sin x}{x}\right] = 0 \text{ (exists)}$$

(D) f (x) = $\dfrac{1}{x-1}$ and g (x) = x + 1

Both $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ exists, but

$$\lim_{x\to 0} f(g(x)) = \lim_{x\to 0} \frac{1}{(x+1)-1} = \lim_{x\to 0} \frac{1}{x} = \text{does not exist.]}$$

43. (B, D)

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \cos \pi x = -1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{-\sin(x-1)}{x-1} = -1$$

$$f(1) = -1$$

$$f'(1^-) = \lim_{x\to 1^-} \frac{\cos \pi x + 1}{x-1} = 0$$

$$f'(1^+) = \lim_{x\to 1^+} \frac{-\dfrac{\sin(x-1)}{x-1} + 1}{x-1}$$

$$= \lim_{x\to 1^+} \frac{-\sin(x-1)+x-1}{(x-1)^2}$$

$$= \lim_{x\to 1^+} \frac{1-\cos(x-1)}{2(x-1)} = 0$$

44. (C, D)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{e^x f(h) + e^h f(x) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^x f(h)}{h} + f(x) \lim_{h\to 0} \frac{e^h - 1}{h}$$

$$= e^x \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} + f(x) \qquad \{\because f(0) = 0\}$$

or $\quad f'(x) = e^x f'(0) + f(x) = 3e^x + f(x)$

$\Rightarrow\ f'(x) - f(x) = 3e^x \qquad$ ----------(1)

also, $\displaystyle \lim_{n\to\infty} \sum_{k=1}^{n} \frac{3}{f'(k)-f(k)} = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{e^k} = \frac{1}{e-1}$

from (1) taking f(x) = y we have

$$\frac{dy}{dx} - y = 3e^x \Rightarrow ye^{-x} = \int 3dx$$

$\Rightarrow\ y = 3xe^x + ce^x \Rightarrow f(x) = 3xe^x + ce^x$

putting f(0) = 0 we have f(x) = $3xe^x$

now, $\displaystyle \lim_{n\to\infty} \sum_{k=1}^{n} \frac{3k}{f(k)} = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{e^k} = \frac{1}{e-1}$

Also, $\displaystyle \prod_{k=1}^{n} f(k) = \prod_{k=1}^{n} 3ke^k = 3^n \underline{|n}\, e^{\frac{n(n+1)}{2}}$

45. (B, C)

$$\lim_{x\to\infty} \frac{e^{2x}(3x^2+1)^{1/2}}{x^{k-1}(k+2x)e^{2x}} = \frac{\sqrt{3}}{2} \Rightarrow k = 1$$

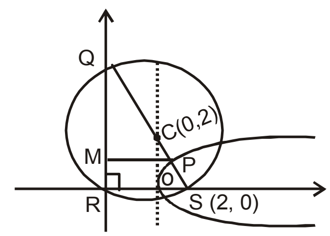
$$\lim_{x\to\infty} \frac{2x^2+3x+5}{(2x+1)(x+1000)} = 1 \ \ \& \ \ \lim_{x\to 0}\left[-\frac{\sin x}{x}\right] = -1$$

46. (A,B,C)

According to the given coordinates of C and co-ordinates of focus we can see that if we plot the diameter through C and S the other end (say Q) lies on directrix of parabola hence the circle must pass through R. As equation of (CS) is x + y = 2 and parabola is $y^2 = 8x$

$\Rightarrow$ x - co-ordinate of P is $(6-4\sqrt{2})$

$\Rightarrow$ $PM = 2+6-4\sqrt{2} = 8-4\sqrt{2}$



CP = (Radius of circle – (SP))

$= 2\sqrt{2} - (8-4\sqrt{2}) = 6\sqrt{2} - 8$

Slope of $CQ \times$ slope of $CR = -1$

$\Rightarrow$ (C) is also correct

47. (A,B,C,D)

Point of contact will lie on y = x (say (α, α)) and slope of tangent can be ±1 and

$\alpha = a\alpha^2 + a\alpha + \frac{1}{24}$. Eliminating α, we get

$$\left(\frac{\pm 1 - a}{2a}\right) = a\left(\frac{\pm 1 - a}{2a}\right)^2 + a\left(\frac{\pm 1 - a}{2a}\right) + \frac{1}{24}$$

On solving we get a = $\frac{2}{3}, \frac{3}{2}, \frac{13 \pm \sqrt{601}}{12}$

48. (A,B,C)

For given condition, $f(x+y) = f(x)f(y) \quad \forall x, y, \in R$

we get $f(x) = e^{2x}$

$$\int_0^{\ell n3} [f(x)e^{-x}]dx = \int_0^{\ell n3} [e^x]\,dx = \ell n2 + 2(\ell n3 - \ell n2) = \ell n(4.5)$$

$\underset{x \to 0^+}{Lt} [e^{2x}] = 1, \ \underset{x \to 0^-}{Lt} [e^{2x}] = 0$ $\therefore$ limit not exist

$f^{-1}(x) = \ell n\sqrt{x}, \ x > 0$

49. (B, C)

$x^2 + y^2 - 8x - 16y + 60 = 0$ ....(i)

Equation of chord of contact from (–2, 0) is – 2x – 4 (x – 2) – 8x + 60 = 0

3x + 4y – 34 = 0 ....(ii)

From (i) and (ii)

$$x^2 + \left(\frac{34-3x}{4}\right)^2 - 8x - 16\left(\frac{34-3x}{4}\right) + 60 = 0$$

$16x^2 + 1156 - 204x + 9x^2 - 128x - 2176 + 960 = 0$

$5x^2 - 28x - 12 = 0 \quad \Rightarrow (x-6)(5x+2) = 0$

$x = 6, -\frac{2}{5}$

$\therefore$ points are $(6, 4)$, $\left(-\frac{2}{5}, \frac{44}{5}\right)$.

50. (8)

put x = 1/t

$$\lim_{t\to 0^+} t^{-p}\left(\left(\frac{1}{t}+1\right)^{1/3}+\left(\frac{1}{t}-1\right)^{1/3}-2\left(\frac{1}{t}\right)^{1/3}\right)$$

$$\lim_{t\to 0^+} t^{-p-\frac{1}{3}}\left((1+t)^{1/3}+(1-t)^{1/3}-2\right)$$

Using expansion, the limit will be finite non-zero real number provided $-p-\frac{1}{3}=-2 \Rightarrow p=\frac{5}{3}$

51. (5)

$g(x) = x^3 - 3x + \frac{1}{2}$

$g'(x) = 3x^2 - 3 = 0, \quad x = 1, -1$

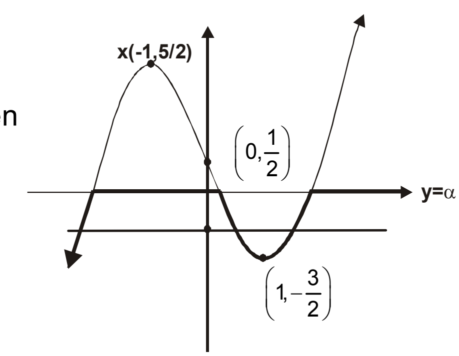
Now f(x) can have 3 points of non-differentibility only when

$g(1) < \alpha < g(-1)$

$\Rightarrow \alpha \in \left(\frac{-3}{2}, \frac{5}{2}\right)$

$\Rightarrow [\alpha] = -2, -1, 0, 1, 2$

Here is the graph is y = f(x)

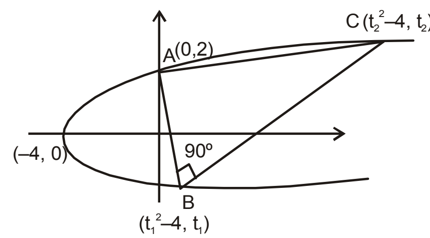


52. (6)

$$\tan^{-1}\frac{1}{\sqrt{2}} - \left(\tan^{-1}\sqrt{3} - \tan^{-1}\sqrt{2}\right)$$

$$\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$$

53. (4)



Now $\left(\frac{t_2-t_1}{t_2^2-t_1^2}\right)\left(\frac{t_1-2}{t_1^2-4}\right)=-1$

$\Rightarrow \frac{1}{(t_1+t_2)(t_1+2)}=-1$

$\Rightarrow t_1^2+t_1t_2+2t_1+2t_2+1=0$

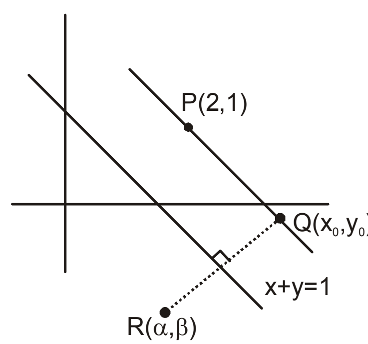
Quadratic in t_1 should have real roots

$\Rightarrow (t_2+2)^2-4(2t_2+1)\geq 0$

$\Rightarrow t_2^2-4t_2\geq 0 \qquad \Rightarrow t_2\in(-\infty,0]\cup[4,\infty)$

Least positive value of t_2 is 4

54. (8)



$PQ=3\sqrt{2}$

$x_0=2+3\sqrt{2}\,\frac{1}{\sqrt{2}}=5$,

$y_0=1-3\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)=-2$

Distance of Q (5, –2) from the line x + y = 1 is $\frac{|5-2-1|}{\sqrt{2}}=\sqrt{2}$

$QR=2\sqrt{2}$

$x_R=5-2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)=3$, $y_R=-2-2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)=-4$

$\Rightarrow \alpha=3,\ \beta=-4$

$\therefore 4\alpha+\beta=8$