SOLUTIONS

WEEKLY TEST-6

RBA

(JEE MAIN PATTERN)

Test Date: 03-09-2017

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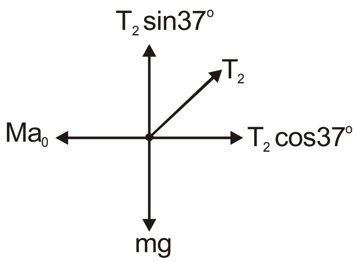
PHYSICS

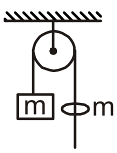
1. **(D)**



$$T_2 \sin 37^\circ = mg$$

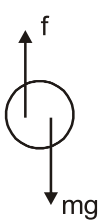
$$\Rightarrow T_2 = \frac{5mg}{3}$$

2. **(A)**

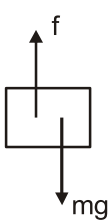


m = 2 kg

f = 10 N



$$a = \frac{mg - f}{m} = \frac{20 - 10}{2} = 5\,m/s^2$$



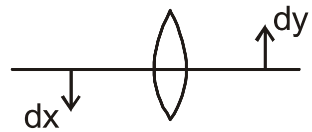
$$a = 5\,m/s^2$$

$$\therefore \text{rel acc} = 5 - 5 = 0\ m/s^2$$

3. (D)

vel of object when it passes through the parincipal axis of the lens

$= \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10\,m/s$



$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$

$\Rightarrow\ v = 30\,cm$

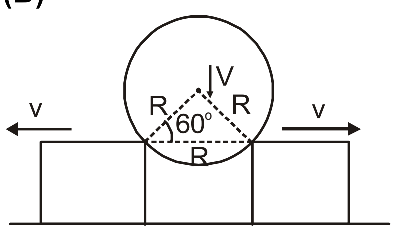
$\frac{dy}{dx} = m = \frac{30}{15} = 2$

$dy = 2.dx$

$\frac{dy}{dt} = 2 \times \frac{dx}{dt} = 2 \times 10 = 20\,m/s$

velocity of image = 20 m/s

4. (B)

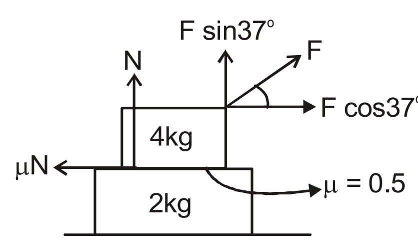


$V\cos 30^o = v\cos 60^o$

$\Rightarrow V = \frac{v}{\sqrt{3}}$

5. (D)

$a = \frac{F\cos 37^o}{4+2}$

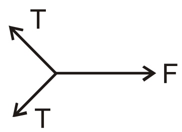


$F\cos 37^o - \mu N = 4 \times a$

$\Rightarrow\ F\cos 37^o - 0.5\left(40 - F\sin 37^o\right) = 4 \times \frac{F\cos 37^o}{6}$

$\Rightarrow F = \frac{600}{17}N$

6. (B)



$$2T\cos 30^\circ = F$$

$$T = \frac{F}{\sqrt{3}}$$

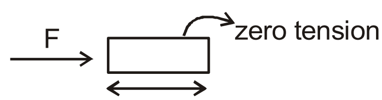
Tension in the rod

$$= T\cos 60^\circ$$

$$= \frac{F}{\sqrt{3}} \times \frac{1}{2} = \frac{F}{2\sqrt{3}}$$

7. (C)

$$a = \frac{4F}{M}$$



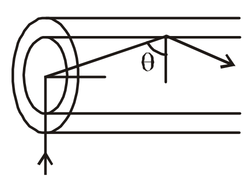
$$F = \frac{M}{\ell}.x.a$$

$$F = \frac{M}{\ell}.x.\frac{4F}{M}$$

$$\Rightarrow x = \frac{\ell}{4}$$

8. (A)

$$1\sin 90^\circ = 2\sin r \Rightarrow r = 30^\circ$$



$$\theta = 60^\circ$$

$$\theta \ge \theta_C \Rightarrow 60^\circ \ge \sin^{-1}\left(\frac{\mu}{2}\right)$$

$$\Rightarrow \frac{\sqrt{3}}{2} \ge \frac{\mu}{2}$$

$$\Rightarrow \mu \le \sqrt{3}$$

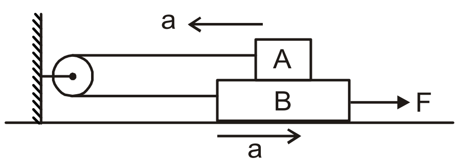
$\mu_{max} = \sqrt{3} = 1.73$

9. (B)

Motion will start when

$F = F_0 = 50N$

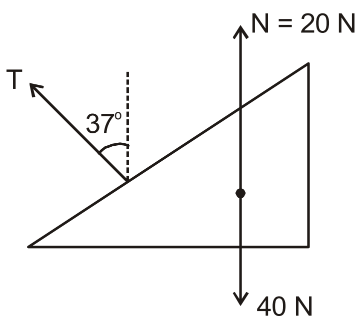
for $F = 100N$



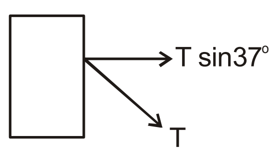
$a = \frac{100}{10} = 10 m/s^2$

10. (D)

$T\cos 37^\circ + 20 = 40$



$\Rightarrow T = 25N$



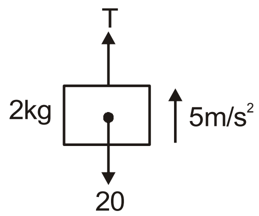
$a = \frac{T\sin 37^\circ}{3} = 5 m/s^2$

$F = (3+4) \times 5 = 35N$

11. (B)

acc of 2 kg & 4 kg $= \frac{10+40-20}{4+2}$

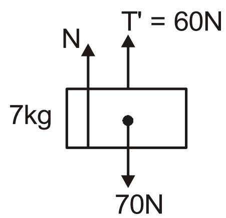
$= 5\ m/s^2$



$T - 20 = 2 \times 5$

$\Rightarrow \quad T = 30N$

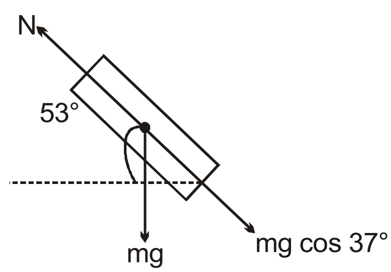
$\therefore \quad T' = 2 \times 30 = 60N$



$\therefore \quad N + 60 = 70$

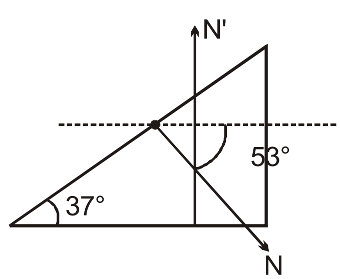
$\Rightarrow \quad N = 10N$

12. (A)



$N = mg\cos 37° = 5 \times 10 \times \frac{4}{5}$

= 40 N



N' = 100 + N × 4/5

= 100 + 40 × 4/5

= 132 N

$\mu N' = N\cos 53°$

$\Rightarrow \mu(132) = 40 \times \frac{3}{5}$

$\Rightarrow \mu = \frac{2}{11}$

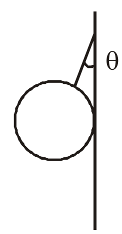
13. B)

$\sin\theta = \frac{\ell}{2} = \frac{1}{2}$

$\theta = 30°$

$T\sqrt{3}/2 = 20$

$T\frac{1}{2} = N = \frac{1}{2} \times \frac{40}{\sqrt{3}} = \frac{20}{\sqrt{3}}$



$2 \times a = \frac{20}{\sqrt{3}}$

$a = \frac{10}{\sqrt{3}}$

$F = 6 \times \frac{10}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 34.6\text{N}$

14. (C)

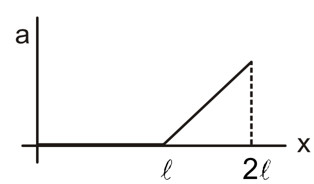
acc $\propto$ extension

acc $\propto$ $(x - \ell)$ for $x > \ell$

x = length of chord

for $x < \ell$, acc becomes zero.

$\therefore$ Graph is



15. (D)

$$\frac{4}{3}\sin\theta = \frac{3}{2}\sin r$$

$$\sin r = \frac{Q}{9}\sin\theta$$

$$r > \theta_c$$

$$\sin r > \frac{2}{3}$$

$$\sin\theta > \frac{9}{8}\times\frac{2}{3} = \frac{3}{4}$$

16. (C)

17. (A)

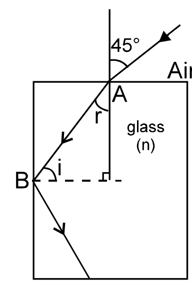
For TIR at B, the angle of incidence $i > c$

& $r + i = 90 \Rightarrow i = 90 - r$

by snell's law at pt A,

$\sin 45° = n \sin r = n \cos i$

Now $\because i > c \Rightarrow \sin i > \sin c$



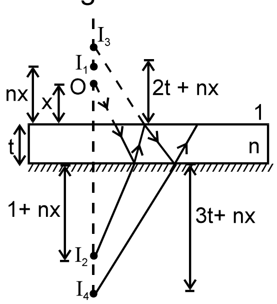
$$\Rightarrow \cos r > \frac{1}{n} \Rightarrow n > \frac{1}{\cos r}$$

$$\Rightarrow n > \frac{1}{\sqrt{1-\sin^2 r}} \Rightarrow n > \frac{1}{\sqrt{1-\frac{1}{2n^2}}}$$

$$\Rightarrow n > \frac{\sqrt{2}\,n}{\sqrt{2n^2-1}} \Rightarrow 2n^2 - 1 > 2 \Rightarrow n > \sqrt{\frac{3}{2}}$$

18. (B)

The figure shows the image formation



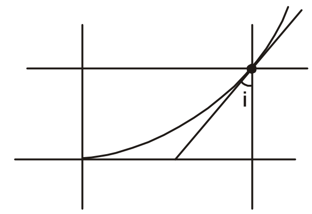
in different steps. Given 2 t = 4 $\Rightarrow$ t = 2 cm

19. (A)

By snell's law.

1 × sin 30º = ... = ... = n sin i

where n is R.I. at y and i is angle of incidence at y.



$$\tan(90-i)=\frac{dy}{dx}=8x=4\sqrt{y}$$

$$\cot i=4\sqrt{y}=4\sqrt{\frac{1}{2}}=2\sqrt{2} \;\; \Rightarrow \sin i=\frac{1}{3}$$

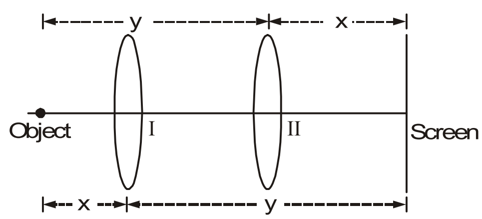
$$\therefore \;\; n = \frac{\sin 30^o}{\sin i}=\frac{1/2}{1/3}=1.5\,\text{Ans.}$$

20. (A)

$$\left(\frac{n_A-1}{1}\right)\frac{2}{R_A}=\left(\frac{n_B-1}{1}\right)\frac{2}{R_B} \text{ or } \frac{0.63}{R_A}=\frac{n_B-1}{R_B} \quad \text{or } n_B=1.7$$

21. (B)

At first position of lens, let the distance of lens from object and screen be x and y respectively.



$\therefore$ x + y = 100 ...(1)

At second position of lens the distance of lens from object and screen shall be y and x respectively.

$\therefore$ y – x = 40 ...(2)

Solving equation (1) and (2) we get

$$y = 70 \text{ cm} = \frac{70}{100}\text{m} \text{ and } x = 30 \text{ cm} = \frac{30}{100}\text{m}$$

$\therefore$ The power of lens is,

$$\frac{1}{f}=\frac{1}{y}+\frac{1}{x}=\frac{100}{70}+\left(\frac{100}{30}\right)=\frac{100}{21}\approx 5\,\text{diopters}$$

22. (B)

Cutting a lens in transverse direction doubles their focal length i.e. 2f.

Using the formula of equivalent focal length $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\frac{1}{f_4}$

We get equivalent focal length as $\frac{f}{2}$

23. (C)

24. (C)

Here direction of light is given by normal vector $\vec{n}$ = $\hat{i}+2\hat{j}+3\hat{k}$

= angle made by the $\vec{n}$ with y-axis is given by $\cos\theta$ = $\frac{2}{\sqrt{1^2+2^2+3^2}}=\frac{2}{\sqrt{14}}$

25. (C)

For 100th max.

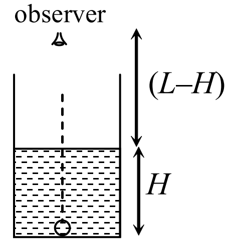
d sin ϕ = 100 λ

sin ϕ = $\frac{100\times5000\times10^{-9}}{1\times10^{-3}}$ = $\frac{5\times10^{-4}}{10^{-3}}$ = 0.5 = $\frac{1}{2}$

$\therefore$ y = D tan ϕ = 1 × tan 30 = $\frac{1}{\sqrt{3}}$

26. (A)

$$X_{app}=L-H+\frac{H}{\mu}$$



$$\frac{dX_{app}}{dt}=\frac{dH}{dt}\left(\frac{1-\mu}{\mu}\right)$$

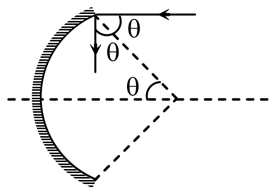
$$\pi r^2 H=V$$

$$\therefore\quad \frac{dH}{dt}=-\frac{2H}{r}\frac{dr}{dt}=\frac{2KH}{r}$$

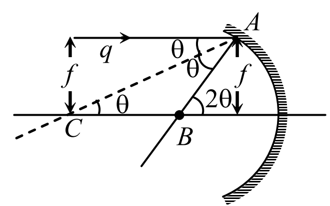
$$\therefore\quad \frac{dX_{app}}{dt}=\frac{2KV}{\pi r^3}\left(\frac{(1-\mu)}{\mu}\right)$$

27. (B)

For multi reflection $2\theta \geq 90°$, $\theta \geq 45°$, $\theta_{min} = 45°$



28. (A)



$$\frac{f}{R} = \sin\theta$$

$$\frac{R/2}{R} = \sin\theta \Rightarrow \theta = 30°$$

$$\sin 2\theta = \frac{f}{AB} = \frac{f}{BC}$$

$$\frac{BC}{f} = \frac{1}{\sin 2\theta} = \frac{2}{\sqrt{3}}$$

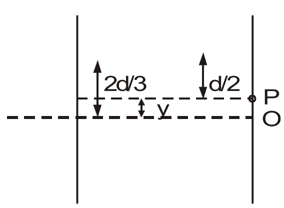
29. (A)

Optical path difference between the waves = $(n_3 - n_2)t$

$\therefore$ phase difference = $2\pi \frac{(n_3 - n_2)t}{\lambda_{(vacuum)}} = 2\pi \frac{(n_3 - n_2)t}{n_1 \lambda_1}$

30. (D)

The nearest white spot will be at P, the central mxima.



$\therefore$ y = $\frac{2d}{3} - \frac{d}{2} = \frac{d}{6}$

CHEMISTRY

31. (D)

$$\overset{+2\ -1}{2FeS_2} + \frac{11}{2}O_2 \longrightarrow \overset{+3}{Fe_2}O_3 + 4\overset{+4}{S}O_2$$

[n = 1 + (2 × 5) = 11]   (n = 2)   (n = 5)

Equivalents of O_2 = Equivalents of FeS_2 = y

Moles of $FeS_2 = \frac{y}{11}$, Moles of $SO_2 = \frac{2y}{11}$

32. (A)

In 100 ml of given coal gas contain H_2 = 50 ml, CH_4 = 30 ml, CO = 14 ml, C_2H_4 = 6 ml

$H_2(g) + \frac{1}{2}O_2(g) = H_2O(\ell)$

50 ml 25 ml 50 ml

$CH_4(g) + 2O_2(g) = CO_2(g) + 2H_2O(\ell)$

30 ml 60 ml 30 ml 60 ml

$CO(g) + \frac{1}{2}O_2(g) = CO_2(g)$

33. (B)

$N_2 + 3H_2 \rightarrow 2NH_3$

0.05 mole 0.25mole

0 0.1 mole left 0.1 mole

$3PCl_3 + S_2Cl_2 \rightarrow 2PSCl_3 + PCl_5$

1mole 1/3 mole

$nPCl_5 + nNH_4Cl \xrightarrow[120°C]{\Delta} (PNCl_2)_n + n\ HCl$

1/3 mole 0.1 mole 0.1/ n mole formed

ma $nPCl_5 + nNH_4Cl \xrightarrow[120°C]{\Delta} (PNCl_2)_n + n\ HCl$

1/3 mole 0.1 mole 0.1/ n mole formed

mass of $(PNCl_2) = \frac{0.1}{n} \times 116n = 11.6g$

34. (B)

35. (B)

Pressure of dry air 200-93=107mm when volume is doubled the presure drop to 53.5 mm.

Total pressure 53.5+93=146.5mm

36. (A)

The mean free path of gas molecules,

$$\ell = \frac{1}{\sqrt{2}\pi n\sigma^2}; \quad \ell \propto \frac{1}{n\sigma^2}; \ \ell \propto \frac{1}{P\sigma^2}$$

$$7000 \text{ cm} \propto \frac{1}{10^{-6} \times 4^2}; \quad x \text{ cm} \propto \frac{1}{10^{-3} \times 2^2}$$

$$7000/x = \frac{10^6}{16} \times 10^{-3} \times 4 = 250 \qquad \therefore x = 7000/250 = 28 \text{ cm.}$$

37. (B)

We know that

$P_A V_A = nRT, P_B V_B = n_B RT$

and $P_f(V_A + V_B) = (n_A + n_B) RT$

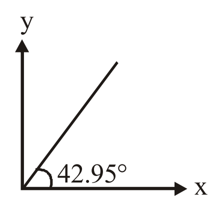
$P_f(V_A + V_B) = P_A V_A + P_B V_B$

$$\therefore \quad P_f = \left(\frac{P_A V_A + P_B V_B}{V_A + V_B}\right) = \frac{1.4 \times 0.1 + 0.7 \times 0.15}{0.1 + 0.15} \text{MPa} = 0.98 \text{ MPa}$$

38. (B)

$$3A \longrightarrow 2B$$

$$1 - \alpha \qquad \frac{2\alpha}{3}$$



Since slope = $\frac{nR}{V}$

$$\tan 42.95 = \frac{(1-\alpha/3)R}{0.0821} = 0.8$$

$$\frac{\alpha}{3} = 0.2 \Rightarrow \alpha = 0.6$$

39. (A)

Total K.E. for 2 mole = $\frac{1}{2} m_{He} N v_{He}^2 + \frac{1}{2} m_{Ne} N v_{Ne}^2$

Since $N = N_A$ & $v_{He} = v_{Ne}$

$$\frac{1}{2}v^2\,[M_{He} + M_{Ne}] = \frac{1}{2}\times 16\times 10^4\,(4+20)\times 10^{-3}$$

K.E. per mole = $\frac{1}{2}\times\frac{1}{2}\times 16\times 10^4\times 24\times 10^{-3}$ = 960 J/mol

40. (C)

Mass of water filled in the glass vessel,

m_1 = (148.0 – 50.0) g = 98.0 g

Volume of glass vessel, $V = \frac{m_1}{\rho} = \frac{98.0\text{ g}}{0.98\text{ g ml}^{-1}}$ = 100 mL = 0.1 dm^3

Mass of gas filled in the vessel, m = (50.5 – 50.0)g = 0.5 g

If M is the molar mass of the gas, we will have

$$pV = nRT = \frac{m}{M}RT \quad \text{or} \quad M = \frac{mRT}{pV} = \frac{(0.5\,g)(8.314\,J\,K^{-1}\,mol^{-1})(300\,K)}{(101.325\,kPa)(0.1\,dm^3)} = 123\text{ g mol}^{-1}$$

41. (A)

+M group most stabilises carbocation.

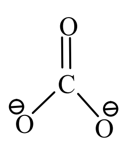
42. (D)

Less stable carbocation can rearrange to form more stable carbocation.

43. (D)

$\overset{-}{C}\equiv\overset{+}{O}$ has lowest bond length due to highest bond order.

O = C = O has second lowest bond length due to doubled bond.


has highest bond length due to lowest bond order which is due to resonance.

44. (D)

Resonance stabilisation

45. (C)

10 electron is not possible in the valence shell of the second period element.

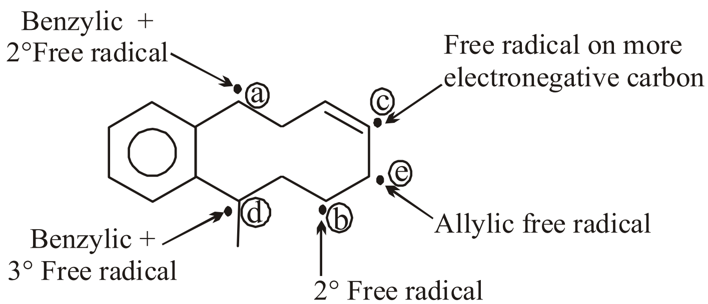
46. (B)

Electron shifting in that direction where both ring should be aromatic. Hence more polarity.

47. (B)

d-orbital resonance.

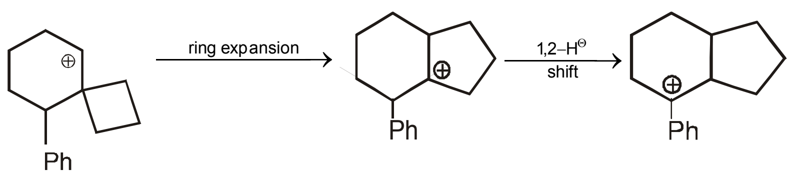
48. (C)



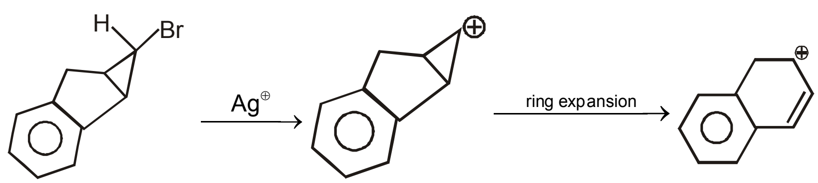
$$\text{Bond Energy } \alpha \frac{1}{\text{Stability of Free radical}}$$

c > b > e > a > d

49. (C)



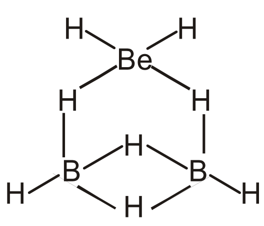
50. (B)



51. (D)

$CH_4 < CCl_4 < CF_4$: Electronegativity of central 'C'-atom

52. (C)

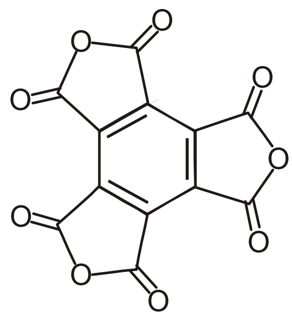


53. (A)

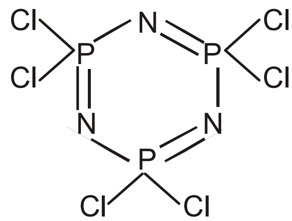
(IV) $LiClO_4 > NaClO_4 > KClO_4 > RbClO_4 > CsClO_4$

54. (A)

(iii) $C_{12}O_9$ – only sp^2 ;



(iv) $N_3P_3Cl_6$ – sp^2 & sp^3 ;



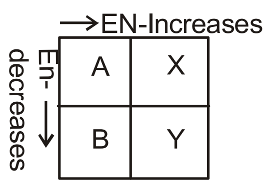
55. (A)

(i) LiF > NaF > KF > RbF : Lattice energy

(iii) $Li^+ < Mg^{2+} < Al^{3+}$:Hydration energy

56. (A)

57. (B)



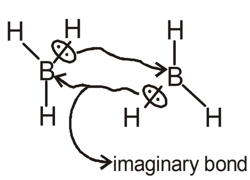
$\Delta E_n < 1.7 =$ Covalent bond (Least polar)

58. (C)

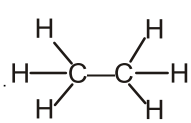
Due to size of Nitrogen is smaller than another.

59. (D) 1 & 3 have x – x bond absent.

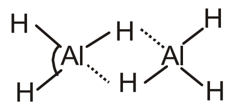
(1) $B_2 H_6$

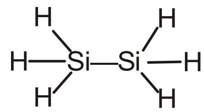


(2) C_2H_6 ....



(3) Al_2H_6



(4)  x-x bond are present

60. (C)

MATHEMATICS

61. (C)

Let $\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)=\alpha$. Then $\cos\alpha=\frac{\sqrt{5}}{3}$, where $0<\alpha<\frac{\pi}{2}$

We have to find $\tan\frac{\alpha}{2}=\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}=\sqrt{\frac{1-5/3}{1+5/3}}$ $=\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}=\sqrt{\frac{(3-\sqrt{5})^2}{9-5}}=\frac{1}{2}\left(3-\sqrt{5}\right)$

62. (B)

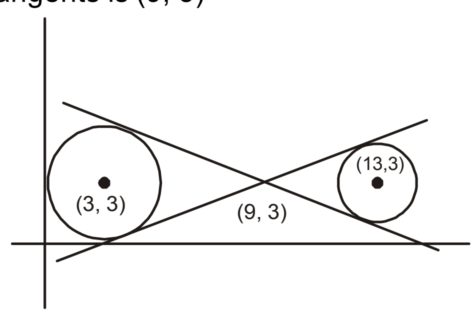
$$\frac{\cos x}{1+\sin x}=\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

63. (D)

Maximum and minimum possible slopes of PQ is along transverse common tangents of the circles.

Points of intersection of transverse common tangents is (9, 3)

Hence equation of PQ is



$y - 3 = m(x - 9)$

$\Rightarrow mx - y - 9m + 3 = 0$

It is tangent to circle C_1

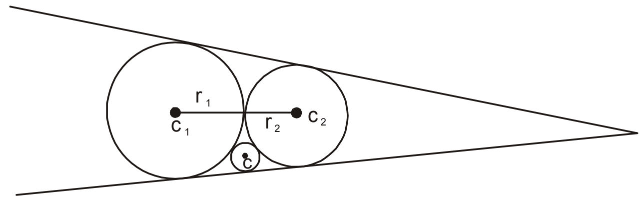
Therefore $\frac{|6m|}{\sqrt{m^2+1}}=3 \Rightarrow 3m^2=1$

$\Rightarrow m=\pm\frac{1}{\sqrt{3}}$

$m\in\left[\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right]$

64. (A)

$\because 2\sqrt{r r_1} + 2\sqrt{r r_2} = 2\sqrt{r_1 r_2}$



or $2\sqrt{r \times 36} + 2\sqrt{r \times 9} = 2\sqrt{9 \times 36}$

or $2 \cdot 6\sqrt{r} + 2.3\sqrt{r} = 2 \times 3 \times 6$

or $12\sqrt{r} + 6\sqrt{r} = 36$

or $18\sqrt{r} = 36$ or $\sqrt{r} = 2 \therefore r = 4$

65. (B)

$y = (\sec^{-1}x)^2 + (\text{cosec}^{-1}x)^2$

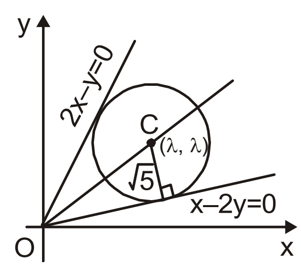
$= (\sec^{-1}x + \text{cosec}^{-1}x)^2 - 2\sec^{-1}x\ \text{cosec}^{-1}x = \frac{\pi^2}{4} - 2(\sec^{-1}x)\left(\frac{\pi}{2} - \sec^{-1}x\right)$

$= \frac{\pi^2}{4} + 2(\sec^{-1}x)^2 - \pi\sec^{-1}x = 2\left(\sec^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{8} \geq \frac{\pi^2}{8}$

66. (A)

Pair of lines are $x - 2y = 0$ and $2x - y = 0$,equation of the bisectors of the pair of lines will be

$\frac{x^2 - y^2}{2-2} = \frac{xy}{-5/2}$ i.e. $y = \pm x$



Centre $C \equiv (\lambda, \lambda)$

$\therefore \left|\frac{\lambda - 2\lambda}{\sqrt{1^2 + 2^2}}\right| = \sqrt{5}$

$\therefore \lambda = \pm 5$

Circle lies in the first quadrant $\because \lambda = 5$

$\therefore$ eqn. of circle $(x-5)^2 + (y-5)^2 = (\sqrt{5})^2 \Rightarrow x^2 + y^2 - 10x - 10y + 45 = 0$

67. (B)

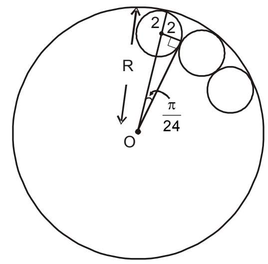
$\cot\{\cot^{-1}3 + \cot^{-1}7 + \cot^{-1}13 + \cot^{-1}21\}$

$= \cot\left\{\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \tan^{-1}\frac{1}{21}\right\}$

$= \cot\{\tan^{-1}2 - \tan^{-1}1 + \tan^{-1}3 - \tan^{-1}2 + ... + \tan^{-1}5 - \tan^{-1}4\}$

$$= \cot\left[\tan^{-1}5 - \tan^{-1}1\right] = \frac{3}{2}$$

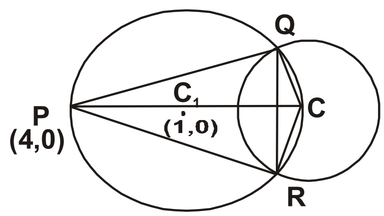
68. (A)



$$\sin\frac{\pi}{24} = \frac{2}{R-2}$$

$$\Rightarrow R = 2\left(1 + \operatorname{cosec}\frac{\pi}{24}\right)$$

69. (C)



Let 'C' be the centre of the circle S = 0, then circumcircle of the ΔPQR will pass through C. Hence, (1,0) centre of the circumcircle of PQR is mid-point of PC. Hence, C is (–2, 0) so equation of S = 0 is $(x+2)^2 + y^2 = (2\sqrt{3})^2$,

Hence, $(-5, \sqrt{3})$ will be on the circle S = 0

70. (C)

Let $g(x) = \sin^{-1}|\sin x| + \cos^{-1}(\cos x)$

$$= \begin{cases} 2x & ; \quad 0 \le x \le \frac{\pi}{2} \\ \pi & ; \quad \frac{\pi}{2} < x \le \frac{3\pi}{2} \\ 4\pi - 2x & ; \quad \frac{3\pi}{2} < x \le 2\pi \end{cases}$$

g (x) is periodic with period 2π and is constant in the interval $\left[2n\pi + \frac{\pi}{2}, 2n\pi + \frac{3\pi}{2}\right]$, $n \in I$

now, $f(x) = g(\lambda x)$

$\therefore$ f(x) is constant in the interval $\left[\frac{2n\pi}{\lambda} + \frac{\pi}{2\lambda}, \frac{2n\pi}{\lambda} + \frac{3\pi}{2\lambda}\right]$

$\therefore 2\pi = \frac{3\pi}{2\lambda} - \frac{\pi}{2\lambda} \Rightarrow \lambda = \frac{1}{2}$

71. (A)

Let equation of circle is

$(x-\alpha)^2 + (y-\beta)^2 = r^2$

$\left|\frac{a\alpha + b\beta + 1}{\sqrt{a^2+b^2}}\right| = r$

$a^2\alpha^2 + b^2\beta^2 + 1 + 2ab\alpha\beta + 2a\alpha + 2b\beta$

$= r^2a^2 + r^2b^2$

Comparision with the given result

$\alpha = 3, \beta = 0, r = \sqrt{5}$

72. (B)

$S_1 - S_2 = 0 \quad \Rightarrow 5ax + (c-d)y + a + 1 = 0$

and $5x + by - a = 0$ represents the same line

$\therefore \frac{a}{1} = \frac{c-d}{b} = \frac{a+1}{-a}$

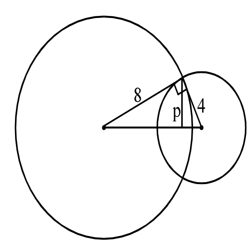
$\Rightarrow ab = c - d$ and $a^2 + a + 1 = 0$

$\Rightarrow$ no real value of a.

73. (A)

Distance 'd' between the centres is = $\sqrt{8^2+4^2} = 4\sqrt{5}$

Also $4\sqrt{5} \cdot p = 8 \cdot 4 \Rightarrow p = \frac{8}{\sqrt{5}}$



$\Rightarrow$ length of common chord is $\frac{16}{\sqrt{5}}$

74. (D)

$\sum_{r=1}^{n} \tan^{-1}\left(\frac{(r+1)! - r!}{1 + (r+1)! \cdot r!}\right)$

$= \sum_{r=1}^{n}\left(\tan^{-1}(r+1)! - \tan^{-1}(r!)\right) = \tan^{-1}(n+1)! - \frac{\pi}{4}$

75. (A)

normals are (y – 2)(y – 2x) = 0 should pass through centre (a, b)

$\Rightarrow$ 2a = b and b = 2 $\Rightarrow$ a = 1 and b = 2

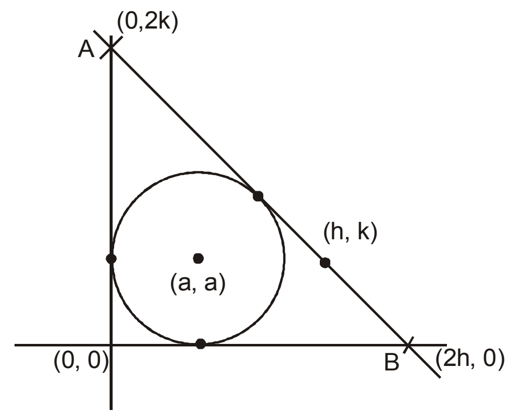
$\therefore$ equation is $x^2 + y^2 - 2x - 4y + c = 0$

circle passes through (2, 1)

$\therefore$ c = 3

76. (C)

(h, k) circumcentre (mid point of AB)



$\Rightarrow$ equation of AB is $\frac{x}{2h} + \frac{y}{2k} = 1$

AB is tangent to circle

$$\Rightarrow \frac{\left|\frac{a}{2h} + \frac{a}{2k} - 1\right|}{\sqrt{\frac{1}{4h^2} + \frac{1}{4k^2}}} = |a|$$

$$\Rightarrow |ak + ah - 2hk| = |a|\sqrt{h^2 + k^2}$$

Locus passes through (38, –37) $\Rightarrow a^2 - 2a = 2812$

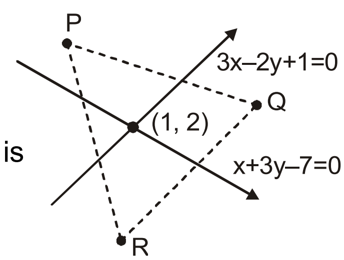
77. (B)

PA+PC is minimum when P is collinear with A and C. PB+PD is minimum when P is collinear with B&D.

$\therefore PA + PB + PC + PD$ is minimum when P is the point of intersection of diagonals AC & BD and its minimum value is $AC + BD$

78. (C)

Circumcentre of ΔPQR is (1, 2).



Straight line through it of slope 'm' is y – 2 = m(x – 1)

intersect axes at $A\left(1 - \frac{2}{m}, 0\right)$ and $B(0, 2 - m)$. Now area of ΔOAB is

$$= \frac{1}{2}\left(1 - \frac{2}{m}\right)(2 - m) = \frac{1}{2}\left(4 - m - \frac{4}{m}\right) \geq 4.$$

79. (D)

$$a - 2\sqrt{bc} = b + c$$

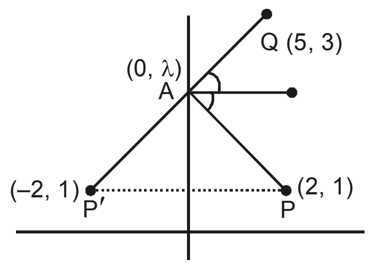
$$\Rightarrow \left(\sqrt{b} + \sqrt{c}\right)^2 - \left(\sqrt{a}\right)^2 = 0 \Rightarrow \sqrt{b} + \sqrt{c} - \sqrt{a} = 0$$

$\Rightarrow \sqrt{a}\ x + \sqrt{b}\ y + \sqrt{c} = 0$ passes through fixed point (– 1, 1)

80. (A)

Equating slopes of P ′A and P ′Q are equal

$$\frac{\lambda - 1}{2} = \frac{3-1}{7}$$



$$\Rightarrow \ \lambda = \frac{11}{7} \ \Rightarrow A = \left(0, \frac{11}{7}\right)$$

81. (D)

$$\sin^{-1}(x^2 - 2x + 2) + \cos^{-1}(4x^2 - 4x + 2) = \frac{\pi}{2}$$

$$\Rightarrow \quad x^2 - 2x + 2 = 4x^2 - 4x + 2$$

$$\Rightarrow \quad 3x^2 - 2x = 0 \Rightarrow x = 0, \frac{2}{3}$$

But for x = 0 and $\frac{2}{3}$

$x^2 - 2x + 2 > 1$ and $4x^2 - 4x + 2 > 1$

Hence No solution

82. (D)

For domain $\ell n\left(\cot^{-1} x\right) > 0$

$\Rightarrow \cot^{-1} x > 1$

$\Rightarrow x < \cot 1$

83. (B)

$f(x) = 2x^3 - 3(p+2)x^2 + 12px - 7$, $-5 \le p \le 5$, $p \in I$

$f'(x)=6(x^2-(p+2)x+2p)\ =6(x-p)(x-2)$

Hence for f(x) to be invertible , p must be 2 only

84. (D)

$\sin^{-1}(\sin 8)=3\pi-8=b$

$\tan^{-1}(\tan 8)=8-3\pi=-b$

f (b) + f(–b) = K

$\therefore$ K = 4

85. (D)

$x^2+4x+\alpha^2-\alpha\geq 0\ \forall\ x\in R$

According to given condition we must have D = 0 $\Rightarrow\quad \alpha=\frac{1\pm\sqrt{17}}{2}$

86. (D)

$$2x=\frac{2a}{1-a^2}+2\frac{(a+a^3)}{1-a^4}\qquad (\therefore\ a\notin\{-1,1\})$$

$$=\frac{2a}{1-a^2}+\frac{2a}{1-a^2}$$

$$\Rightarrow\quad x=\frac{2a}{1-a^2}=\tan(2\tan^{-1}a)$$

Also $x-a^2x=2a\ \Rightarrow\ a^2x+2a=x$

87. (B)

$$\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{12}$$

$$\Rightarrow\frac{\pi}{2}-\tan^{-1}x-\frac{\pi}{2}+\tan^{-1}(x+2)=\frac{\pi}{12}$$

$$\Rightarrow\tan^{-1}(x+2)-\tan^{-1}x=\frac{\pi}{12}$$

$$\Rightarrow\tan^{-1}\frac{(x+2)-x}{1+(x+2)x}=\frac{\pi}{12}\quad\Rightarrow\frac{2}{x^2+2x+1}=2-\sqrt{3}$$

$$\Rightarrow x^2+2x+1=2\left(2+\sqrt{3}\right)\ \Rightarrow x^2+2x-\left(3+2\sqrt{3}\right)=0$$

88. (D)

$\sin^{-1}x, \cos^{-1}x$ exist for $-1 \le x \le 1$. But for $\sin^{-1}(1-x)$ we must have $0 \le x \le 2$. So the equation may hold for $0 \le x \le 1$. Therefore, the options a, b and c are incorrect.

89. (B)

Given equation reduces to $(x-2)^4 + \left(\tan^{-1}(y-1) - \frac{\pi}{4}\right)^2 = 0$

$\Rightarrow$ x = 2, y = 2 $\Rightarrow$ x + y = 4.

90. (D)

$$\left(2^{\frac{\pi}{\sin^{-1}x}} - 4\right)\left(2^{\frac{\pi}{\sin^{-1}x}} - 2a\right) < 0$$

Now, $2^{\frac{\pi}{\sin^{-1}x}} \in \left(0, \frac{1}{4}\right] \cup [4, \infty)$

for $2^{\frac{\pi}{\sin^{-1}x}} \in \left(0, \frac{1}{4}\right]$

We have $2^{\frac{\pi}{\sin^{-1}x}} - 2a > 0$ for some $x \in R$ $\Rightarrow 2a < \frac{1}{4} \Rightarrow a < \frac{1}{8}$

Similarly for $2^{\frac{\pi}{\sin^{-1}x}} \in [4, \infty), a > 2$ we get $a \in \left(-\infty, \frac{1}{8}\right) \cup (2, \infty)$