SOLUTIONS

PHASE TEST-1

RB-1804 TO 1806,

RBK-1802 & 1803

RBS-1801 & 1802

(JEE ADVANCED PATTERN)

Test Date: 03-09-2017

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CHEMISTRY

1. (B)

En = x ; Ea = y

$EN = \frac{IP + EA}{2} = x = \frac{IP + y}{2} = \therefore I.P. = 2x - y$

2. (A)

	$2NH_3(g) \longrightarrow$	$N_2(g) +$	$3H_2(g)$
Before sparking	76	0	0
After sparking at equilibrium	$76 - 2x$	x	3x

Increase in pressure 2x = 18 ; x = 9 cm Hg

Partial pressure of $H_2 = 3 \times 3 = 9$ cm Hg

3. (A)

Moles of $BaSO_4 = \frac{1.22}{233.3}$

moles of $M_2(SO_4)_3 = \frac{1.22}{23.3} \times \frac{1}{3} = 1.743 \times 10^{-3}$

wt. of $M_2(SO_4)_3 = 0.596$

$\therefore \quad 1.743 \times 10^{-3}\ (2M + 96 \times 3) = 0.596$

$M = 26.9$

4. (B)

When the compound, on heating dissociate into its at least one compound then these solubility of that compound is decided by % ionic chr. higher the ionic chr. great will be thermal stability.

(A) $LiCO_3 < Na_2CO_3 < K_2CO_3$

(B) $Li_2O > Na_2O > K_2O$ | order of there stability

(C) HF > HCl > H Br

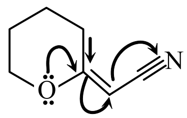
(D) $BeSO_4 < MgSo_4 < CaSO_4$

5. (B)

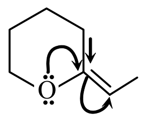
6. (A)

Indicated bond is a double bond maximum delocalisation (by Resonance & Hyperconjugation) maximum single bond character hence minimum rotational energy barrier.

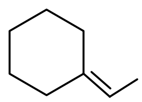
Rotational energy barrier $\propto$ bond strength.



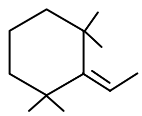
* High delocalisation
* Higher single bond character



* Moderate delocalisation



* 7 Hyper conjugating structure



* 3 Hyper conjugating structure
* Less single bond character

7. (B)

When phenolphthalein is used as an indicator : 0.05x = 20 × 0.1 × 1 ; x = 40 mL.

when methyl orange is used as an indicator.

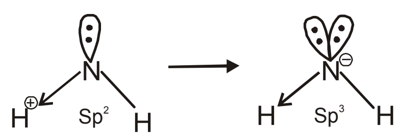
0.05 y = 20 × 0.1 × 3

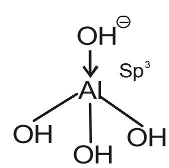
y = 120 mL

$\therefore$ y – x = 80 mL

8. (A,B,C)

9. (A), (B), (D)

(A) 

(B) 

Tetrahedral

(C) Sp^2 'S' Chr = 33 %

'P' Chr = 66%

(D) Hybridized orbitals always from 6 (sigma) bond due to π bond form by pure orbital only.

10. (A, B, C)

$$17g\,/\,L\ H_2O_2 = \frac{17}{34}\,mol\,L^{-1}\,H_2O_2$$

Molarity of $H_2O_2 = \frac{1}{2}$

$M = \frac{V}{11.2} \Rightarrow V = \frac{1}{2} \times 11.2 = 5.6$

5.6 volume H_2O_2 means 1 mL will give 5.6 mL at 273 K and 1 atm

$P_1 V_1 = P_2 V_2$

$1 \times 5.6 = 2 \times V_2$

$V_2 = 2.8\,\text{mL}$

11. **(A, B, C, D)**

For Boyle's law

T = constant

PV = K = constant

12. **(C)**

Ortho effect causes less resonance.

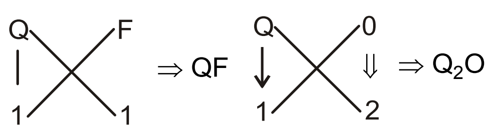
13. **(C)**

Two group showing resonance in similar direction so resonance energy is higher.

14. **(A)**

15. **(B)**

Q = Alkali metal F = Fluorine



16. **(A)**

1st ΔHion of Q and R is very low.

17. **(9)**

$P_f = 1 + \frac{36}{76} = \frac{112}{76}$ atm. Final height = 19 cm

P_i = 1 atm, initial length = h_i cm

$\therefore$ Boyle's law $P_i V_i = P_f V_f$

$1 \times h_i A = \frac{112}{76} \times 19A$

$h_i = 28\,\text{cm}$

$\therefore$ The length by which the Hg column shifts down = $h_i - h_f$ = 28 – 19 = 9

18. (3)

19. (9)

V, n constant.

$$\frac{P_i}{T_i} = \frac{P_f}{T_f} \Rightarrow P_f = \frac{T_f}{T_f} P_i = \left(\frac{109}{100}\right) P_i \qquad \Rightarrow \text{Pincrease } \Delta P = P_f - P_i = \frac{9}{100} P_i$$

$\therefore$ % Pressure increases

$$= \frac{\Delta P_f}{P_i} \times 100 = \frac{9P_i}{100P_i} \times 100\%$$

X% = 9%

X = 9

20. (6)

Moles of $H_2O = 1$

$$3Fe + 4H_2O \longrightarrow Fe_3O_4 + 4H_2$$

Consumed moles of $Fe = \frac{3}{4}$

Mass of $Fe = \frac{3}{4} \times 56 = 42$, $\Rightarrow \frac{w}{7} = \frac{42}{7} = 6$

21. (4)

$$\underset{0.220\,g}{M_2O_x} \text{ (Metallic oxide)} + H_2 \longrightarrow M(s) + \underset{0.045\,g}{H_2O(\ell)}$$

$\Rightarrow$ POAC on O

$$\left(\frac{0.220}{2M + 16x}\right) x = \left(\frac{0.045}{18} \times 1\right)$$

$$\Rightarrow 3.96x = 0.09M + 0.72x \qquad \Rightarrow E = \frac{M}{x} = 36$$

$$\frac{E}{9} = 4$$

22. (4)

At constant temperature

$$\frac{P_A}{P_B} = \frac{d_A}{d_B} \cdot \frac{M_B}{M_A} = \frac{3}{1.5} \times 2 = 4$$

23. (3)

PHYSICS

24. **(A)**

Let tension be *T* then. $T = ma$. For block *M*, $F - T = MA \Rightarrow A = \frac{F - ma}{M}$

25. **(B)**

Common $acc^n = 2ms^{-2}$.

Frictional force on lower block,

$f = 10 \times 2 = 20N \le \mu_s N = 50N$

$\Rightarrow f = 20N$

26. **(B)**

$u = -x, v \to \infty$

$\mu_1 = 1, R = 10, \mu_2 = 2$

$\frac{2}{\infty} - \frac{1}{-x} = \frac{1}{10}$

$\Rightarrow x = 10cm$

27. **(D)**

$f_k = mg\sin\theta$

During up the incline motion,

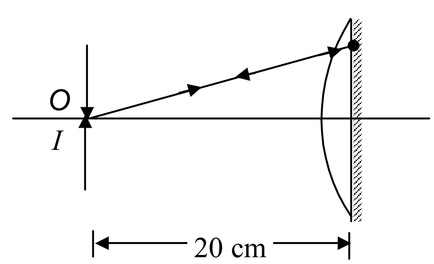
$u = v_0, a = -2g\sin\theta, v = 0$

$\Rightarrow O^2 - v_0^2 = 2(-2g\sin\theta)S$

$S = \frac{v_0^2}{4g\sin\theta}$

28. **(D)**

When the object is placed at the focus of the lens, the refracted rays will be incident normally on the silvered surface. So, they will retrace their path.

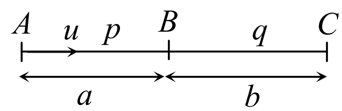


Hence, the image will be formed at the location of the object. In this way, the combination behaves as a concave mirror of radius of curvature (R) = 20 cm

$\therefore \quad f = \frac{R}{2} = 10$ cm

29. (A)

Let retardation be f and initial velocity be u.



For AB,

$a = up - \frac{1}{2}fp^2$ … (i)

For AC, $a + b = u(p+q) - \frac{1}{2}f(p+q)^2$ … (ii)

From

(i) and (ii) $\Rightarrow \quad a + b = \frac{(a + \frac{1}{2}fp^2)}{p}(p+q) - \frac{1}{2}f(p+q)^2$

$\frac{a+b}{p+q} = \frac{a}{p} + \frac{1}{2}fp - \frac{1}{2}fp - \frac{1}{2}fq, \quad \frac{1}{2}fq = \frac{a}{p} - \frac{a+b}{p+q} \quad = \frac{ap + aq - ap - bp}{p(p+q)}$

$\frac{1}{2}fq = \frac{aq - bp}{p(p+q)}, \quad f = \frac{2(aq - bp)}{pq(p+q)}$

$\therefore$ **(A)**

30. (A)

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2}$$

For maximum repulsion force $\frac{dF}{dq} = Q - 2q = 0$

$$\frac{Q}{q} = \frac{2}{1} = 2$$

31. (B, C)

v_x= 3 m/s

a_x = – 1.0 m/s^2

$\therefore \quad v_x^2 = u_x^2 + 2a_x.x$ or $0 = (3)^2 + 2(-1)(x)$ or $x = 4.5$ m

Also $v_x = u_x + a_x t$

$0 = 3 - (1.0)t$ or t = 3s

$y = u_y t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-0.5)(3)^2 =$ -2.25 m

and $v_y = a_y t = (-0.5)(3) = -1.5$ m/s

$\therefore \quad \vec{v} = v_x\hat{i} + v_y\hat{j} = 0 - 1.5\hat{j} = (-1.5\hat{j})$m/s

and $\vec{r} = x\hat{i} + y\hat{j} = (4.5\hat{i} - 2.25\hat{j})$m

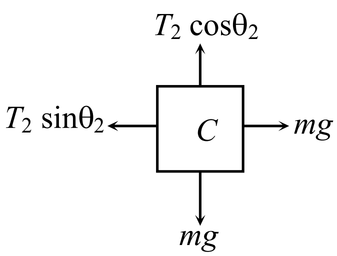
32. (A, C, D)

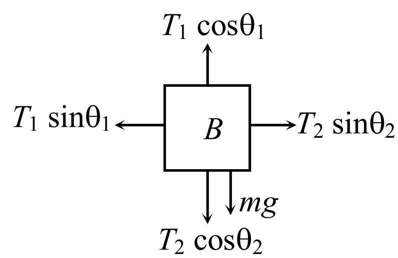
$T_2 \sin\theta_2 = mg$ … (i)

$T_2 \cos\theta_2 = mg$ …(ii)

$T_1 \cos\theta_1 = mg + T_2\cos\theta_2$

$T_1 \sin\theta_1 = T_2 \sin\theta_2$





33. (A) (B) (C)

$a_1 > 0$ when $\frac{F}{4} > 50$, $F > 200$

$a_2 > 0$ when $\frac{F}{4} > 100$, $F > 400$

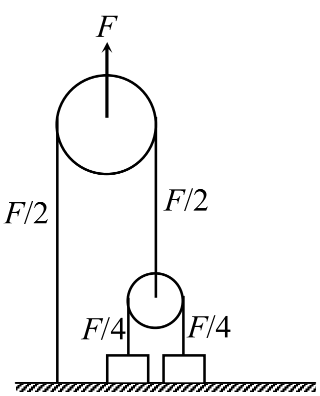
F = 300 N

$a_1 = \frac{F/4 - 50}{5} = \frac{300/4 - 50}{5} =$ 5 m/s^2

$a_2 = 0$

If $F = 500$ N

$a_1 = 15$ m/s^2, $a_2 = 2.5$ m/s^2



34. **(A, B)**

For microwaves,

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10^6} = 300 \text{ m.}$$

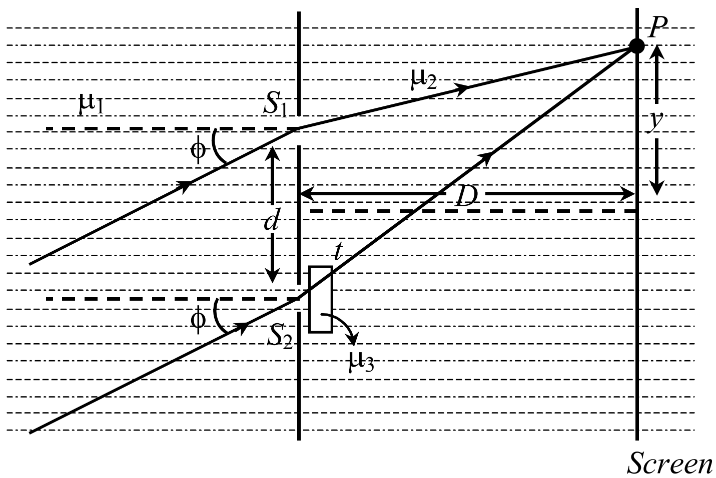
$\Delta x = d \sin\theta$

$$\phi = \frac{2\pi}{\lambda} d\sin\theta = \frac{2\pi}{300}(150\sin\theta) = \pi \sin\theta$$

$$I_\theta = I_0 \cos^2\left(\frac{\pi\sin\theta}{2}\right)$$

Solution for [35 To 37]

At a general point P on the screen $\Delta x = \mu_2(S_2P - t) + \mu_3 t - (\mu_1 d\sin\phi + \mu_2 S_1 P)$



$$\Delta x = \mu_2(S_2P - S_1P) + t(\mu_3 - \mu_2) - \mu_1 d\sin\phi \Rightarrow \Delta x = \frac{\mu_2 yd}{D} + t(\mu_3 - \mu_2) - \mu_1 d\sin\phi$$

35. For central maxima $\Delta x = 0$, $y = \dfrac{D}{\mu_2 d}(\mu_1 d\sin\phi - (\mu_3 - \mu_2)t) \Rightarrow y = 0$.

$\therefore$ **(B)**

36. If slab is removed, $\mu_1 d\sin\phi = \mu_2 d\sin\theta$

$$\sin\theta = \frac{3}{7}$$

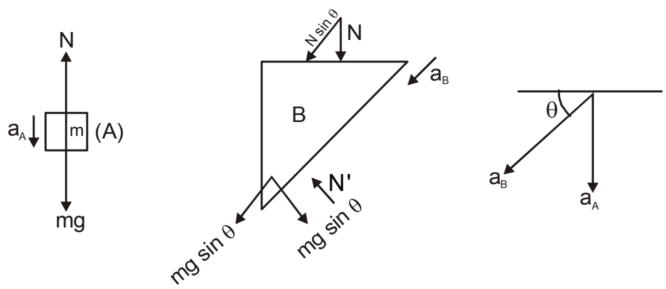
$\therefore$ **(A)**

37. $\beta = \dfrac{\lambda D}{\mu_2 d} = \dfrac{5}{14}$ mm.

$\therefore$ **(B)**

38. **(A)**

39. **(B)**



$mg - N = ma_A$ …(i) $mg\sin\theta + N\sin\theta = ma_B$ ….(ii)

$a_A = a_B\sin\theta$ ... (iii)

from (I), (II) & (III), $mg\sin\theta + (mg - ma_A)\sin\theta = m\dfrac{a_A}{\sin\theta}$

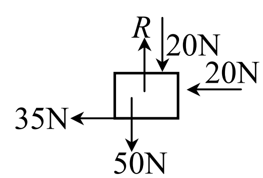
$\Rightarrow a_A\left(\dfrac{m}{\sin\theta} + m\sin\theta\right) = mg\sin\theta + mg\sin\theta$

$\Rightarrow a_A = \dfrac{(m+m)\,g\sin^2\theta}{m + m\sin^2\theta}$ and $a_B = \dfrac{(m+m)\,g\sin\theta}{m+m\sin^2\theta}$

$\Rightarrow N = mg - \dfrac{(m+m)\,mg\sin^2\theta}{m+m\sin^2\theta} = \dfrac{mmg + m^2g\sin^2\theta - 2\,m^2g\sin^2\theta}{m+m\sin^2\theta}$

$\Rightarrow N = \dfrac{mmg - mmg\sin^2\theta}{m+m\sin^2\theta} = \dfrac{m\,mg\cos^2\theta}{m+m\sin^2\theta} = \dfrac{mg\cos^2\theta}{1+\sin^2\theta}$

40. **(0)**



a = retardation of block = 11 m/s^2

u = initial velocity = 33 m/s

By $v = u - at$

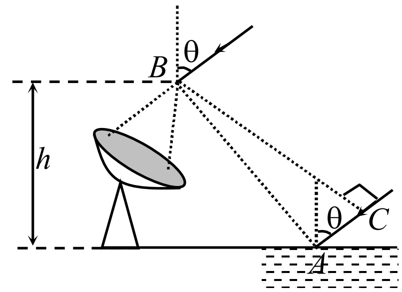
$t = \dfrac{u}{a}$ (time after which its velocity becomes zero)

= 3 s

So block will come to rest after **3 s**. After that the applied force is 20 N towards left which is less than the maximum limiting friction. So it will remain at rest after that.

41. (4)

Path difference of the two rays (one coming directly to antenna and other after reflection from water surface



$$\Delta x = AB + AC + \frac{\lambda}{2}$$

$\Rightarrow$ $\Delta x = \dfrac{h}{\cos\theta} + \dfrac{h}{\cos\theta}\cos 2\theta + \dfrac{\lambda}{2}$

= $\dfrac{h}{\cos\theta}(1+\cos 2\theta) + \dfrac{\lambda}{2}$

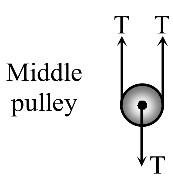
or $\Delta x = 2h\cos\theta + \dfrac{\lambda}{2}$

For constructive interference $\Delta x = n\lambda$

$\Rightarrow$ $2h\cos\theta + \dfrac{\lambda}{2} = n\lambda$ or $h = \dfrac{\left(n-\frac{1}{2}\right)\lambda}{2\cos\theta}$ = $\dfrac{\lambda}{4\cos\theta}, \dfrac{3\lambda}{4\cos\theta}$,........ etc

42. (0)

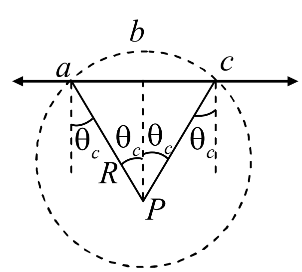
Tension in string is zero.



2T = T $\Rightarrow$ T = 0

43. (1)

The light escape is confined within a cone of apex angle '$2\theta_c$' where θ_c is the critical angle. Imagine a sphere with source of light as its centre and the surface area *abc* is *A*.



here $A = \int_0^{\theta_c} 2\pi R^2 \sin\theta\, d\theta = 2\pi R^2(1-\cos\theta_c)$

$= \pi R^2 \qquad \left[\therefore \theta_c = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^{\circ}\right]$

$\therefore$ Power transfer $= P \times \frac{A}{4\pi R^2}$

$= 4 \times \frac{1}{4} = 1\,\text{W}$

44. (7)

$P_1 = \pi\left(10^{-3}\right)^2 \times \frac{10}{\pi} \times \frac{10}{100} = 10^{-6}\,\text{W}$

$P_2 = \pi\left(2 \times 10^{-3}\right)^2 \times \frac{10}{\pi} \times \frac{10}{100} = 4 \times 10^{-6}\,\text{W}$

Optical Path Difference,

$\Delta x = (\mu - 1)t = 10^{-7}\,\text{m}$

$\Rightarrow \Delta\phi = \frac{2\pi\Delta x}{6000 \times 10^{-10}} = \frac{\pi}{3}$

$\Rightarrow P_{net} = P_1 + P_2 + 2\sqrt{P_1 P_2}\cos\Delta\phi = 7 \times 10^{-6}$ Watt

45. (4)

46. (8)

Common acceleration,

$a = \frac{18}{6} = 3\,\text{ms}^{-2}$

$f - 2 = 2 \times 3 \Rightarrow f = 8\,\text{N}$

MATHEMATICS

47. (C)

$$\sum_{A \in X} \min(A) = 1.(n-1) + 2(n-2) + ... + (n-1)1.$$

$$\sum_{r=1}^{n-1} r(n-r) = {}^{n+1}C_3$$

48. (A)

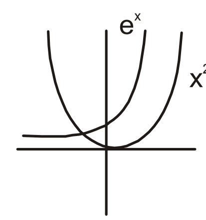
$f(x) = [x] + 1 = x + 1, x \in I$

$\Rightarrow f(f^{-1}(x)) = x \Rightarrow f^{-1}(x) + 1 = x$

$\Rightarrow f^{-1}(x) = x - 1$

49. (C)

Consider $\frac{e^x}{x^2} = 1, \ x > 0$



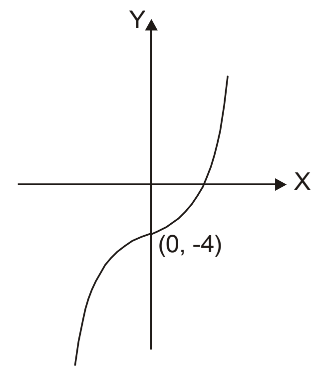
Let $f(x) = \frac{e^x}{x^2} \Rightarrow f'(x) = \frac{e^x x(x-2)}{x^4}$

$\therefore$ f(x) is $\downarrow$ in (0,2); and $\uparrow$ in $(2, \infty)$.

$\Rightarrow$ Min (f(x)) = $\frac{e^2}{4} > 1 \Rightarrow$ No solution for x > 0

$\Rightarrow e^x = x^2$ has 1 real solution from graph

Again $g(x) = x^3 + 2x - 4$ and $g'(x) = 3x^2 + 2$ with g(0) < 0



$\Rightarrow$ g(x) = 0 has 1 real positive root.

$\therefore$ m + n = 2

50. (C)

$$\lim_{n\to\infty} \cos(2\pi\sqrt{n^2+1}) = \lim_{n\to\infty} \cos(2\pi\sqrt{n^2+1} - 2n\pi) = \lim_{n\to\infty} \cos\left(\frac{2\pi}{\sqrt{n^2+1}+n}\right) = 1$$

51. (A)

$$l = \lim_{n\to\infty} n^2 \ln(n \operatorname{cosec}^{-1} n) = \lim_{x\to 0^+} \frac{1}{x^2} \ln\left(\frac{\sin^{-1} x}{x}\right) = \lim_{x\to 0^+} \ln\left(\frac{\sin^{-1} x}{x}\right)^{\frac{1}{x^2}}$$

$$\because \lim_{x\to 0^+}\left(\frac{\sin^{-1}x}{x}\right)^{\frac{1}{x^2}} = e^{\lim_{x\to 0^+}\left(\frac{\sin^{-1}x - x}{x^3}\right)}$$

$$\therefore\ l = \lim_{x\to 0^+}\frac{\sin^{-1}x - x}{x^3} = \lim_{x\to 0^+}\frac{\theta-\sin\theta}{\theta^3} = \lim_{x\to 0^+}\frac{1-\cos\theta}{3\theta^2} = \frac{1}{6}$$

52. (A)

Let $L_1 \equiv 2kx + y - 2 = 0,$

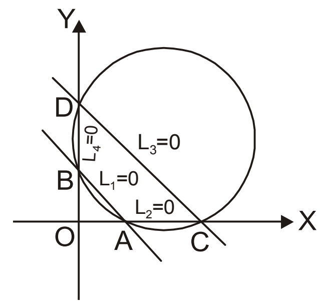
$L_2 \equiv y = 0,$

$L_3 \equiv x + (k+2)y - 3 = 0$

and $L_4 \equiv x = 0$,

then the equation of the circumcircle is

$L_1L_3 + \lambda L_2L_4 = 0,$



Provided coefficient of x^2 = coefficient of y^2 and coefficient of xy = 0.

i.e. $(2kx + y - 2)(x + (k+2)y - 3) + \lambda yx = 0$, where 2k = k + 2 $\Rightarrow$ k = 2

and $2k(k+2) + 1 + \lambda = 0 \Rightarrow \lambda = -17$

i.e. $4x^2 + 4y^2 - 14x - 11y + 6 = 0$

53. (C)

$\sin x \cos 2y \le 1$ and $\cos x \sin 2y \le 1$

$\Rightarrow (a^2-1)^2 + 1 \le 1$ and $a + 1 \le 1$

$\Rightarrow a = \pm 1$ and $a \le 0$

Hence a = –1

54. (A, C, D)

$$0 < \frac{e^x - 1}{e^x - \tan x} \le 1 \Rightarrow \frac{e^x-1}{e^x-\tan x}\left(\frac{e^x-1}{e^x-\tan x} - 1\right) \le 0 \text{ and } x \ne 0$$

$$\Rightarrow \frac{(e^x-1)(\tan x - 1)}{(e^x - \tan x)^2} \le 0 \text{ and } x \ne 0$$

Case I : x > 0

$$\Rightarrow \tan x \le 1 \text{ and } e^x \ne \tan x \Rightarrow x \in \left(0, \frac{\pi}{4}\right] \cup \left(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{4}\right], n \in I^+$$

$$(\because x > 0 \Rightarrow e^x > 1, \text{ but } \tan x \le 1 \ \therefore\ e^x \ne \tan x)$$

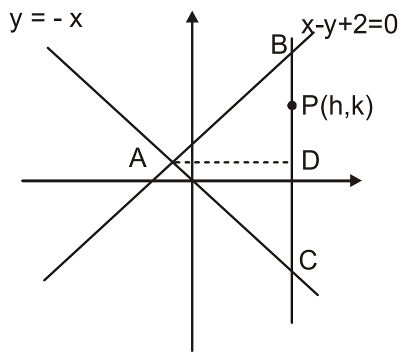
Case II : x < 0

$\Rightarrow \quad \tan x \geq 1$ and $e^x \neq \tan x \Rightarrow x \in \left[n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{2}\right), n \in I^-$

$(\because x < 0 \Rightarrow e^x < 1$, but $\tan x \geq 1 \ \therefore \ e^x \neq \tan x)$

55. (A, B)

BC = | 2h + 2 | ; AD = | h + 1|



So, $(h + 1)^2 = a^2$.

56. (A, B)

For $\lambda = 0, \frac{x}{1} + \frac{y}{3} - 1 = 0$ and $\frac{x}{2} + \frac{y}{4} - 1 = 0$ are one pair of possible lines

$\frac{x}{1} + \frac{y}{4} = 1, \frac{x}{2} + \frac{y}{3} = 1$ are the other pair of possible lines.

$\therefore \quad 2\left(1 - \frac{y}{3}\right) + \frac{y}{4} = 1 \Rightarrow y = \frac{12}{5}$; $x = \frac{2}{5}$

Also $\left(\frac{2}{5} + \frac{4}{5} - 1\right)\left(\frac{1}{5} + \frac{3}{5} - 1\right) + \lambda \frac{2}{5} \cdot \frac{12}{5} = 0 \Rightarrow \lambda = \frac{1}{24}$

57. (A, B, C)

cos x + cos y = a

$\cos^2 x + \cos^2 y + 2\cos x \cos y = a^2$ .....(1)

cos2x + cos2y = b

$\therefore \quad \cos^2 x + \cos^2 y = \frac{b+2}{2}$ .....(2)

$\therefore$ From (1) and (2), $\cos x \cos y = \frac{a^2}{2} - \frac{b+2}{4}$

Now, cos 3x + cos 3y = c

$\therefore$ $4\cos^3 x - 3\cos x + 4\cos^3 y - 3\cos y = c$

$4(\cos x + \cos y)(\cos^2 x + \cos^2 y - \cos x \cos y) - 3(\cos x + \cos y) = c$

$\Rightarrow$ $2a^3 + c = 3a(1 + b)$.

Solution for 58 to 60

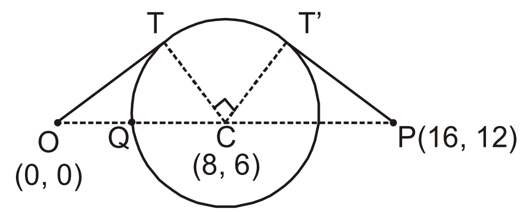
Centre (8, 6) is mid point of O and P

$\therefore$ Length of tangents form O and P to the circle are same and equal to $\sqrt{50}$

$r = CT = \sqrt{50}$

$\therefore \angle COT = \angle T'PC = 45^o$

$\therefore \angle TCT' = 90^o$



shortest path from O to P is along tangent OT, then arc $\overset{\frown}{TT'}$ then $T'P$

Length of arc $\overset{\frown}{TT'} = \frac{2\pi r}{4} = \frac{2\pi\sqrt{50}}{4}$

58. (B)

$\sqrt{50} + \frac{2\pi\sqrt{50}}{4} + \sqrt{50}$

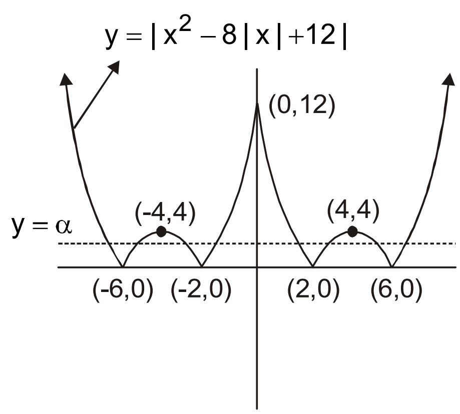
59. (C)

$\text{Area} = \left(\frac{1}{2} \times CT \times OT\right) \times 2 + \frac{\pi r^2}{4}$

60. (D)

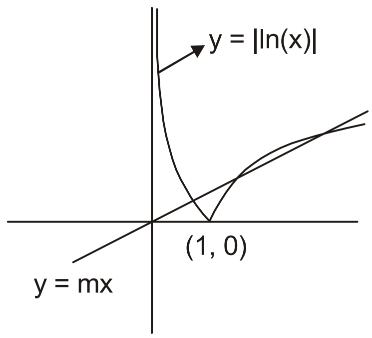
Required length = arc $\overset{\frown}{QTT'} + T'P$

61. (D)



62. (B)

y = mx touches y = ln (x) at $p(\alpha, \ln(\alpha))$, then $y' = \frac{1}{\alpha}$ and tangent is $x - \alpha y = \alpha - \alpha\ \ln(\alpha)$ which passes through origin



$\Rightarrow\ \alpha - \alpha \ln(\alpha) = 0\ \Rightarrow\ \alpha = e$

Hence, slope of tangent through origin is $= \frac{1}{e}$

Hence, all the lines of positive slope less than $\frac{1}{e}$ intersect y = | ln (x) | at 3 distinct points.

63. (3)

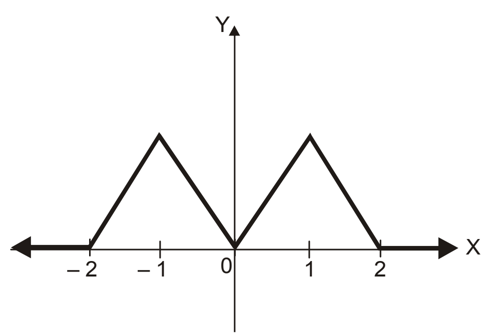
Suppose n(A) = n, then number of reflexive relations on A is $= 2^{n^2 - n}$

and number of symmetric relations on A is $= 2^{\frac{n^2+n}{2}}$

$\Rightarrow\ n^2 - 3n = 0\ \Rightarrow\ n = 3$.

64. (5)

Graph of y = g(x)



There are five points where y = g(x) is not differentiable.

65. (3)

$$2\sin\frac{A+B}{2}\cos\frac{A-B}{2} + 2\sin\frac{C+D}{2}\cos\frac{C-D}{2} = 4$$

$\Rightarrow \quad \sin A + \sin B + \sin C + \sin D = 4$

$\therefore \quad A = B = C = D = 90^\circ$

$\Rightarrow \quad \sum \cos\frac{A}{2}\cos\frac{B}{2} = 6\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = 6\cdot\frac{1}{2} = 3$

66. (3)

Let $x = -i\cot\frac{k\pi}{n},\ k \in \{1, 2, ..., n-1\}$, then

$$\frac{x}{1} = \frac{\cos\frac{k\pi}{n}}{i\sin\frac{k\pi}{n}} \Rightarrow \frac{x+1}{x-1} = \frac{\cos\frac{k\pi}{n} + i\sin\frac{k\pi}{n}}{\cos\frac{k\pi}{n} - i\sin\frac{k\pi}{n}} = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}$$

$$\Rightarrow \left(\frac{x+1}{x-1}\right)^n = \cos 2k\pi + i\sin 2k\pi = 1 \Rightarrow (x+1)^n - (x-1)^n = 0$$

$$\Rightarrow {}^nC_1 x^{n-1} + {}^nC_3 x^{n-3} + ... = 0$$

roots of the above equation are $-i\cot\frac{k\pi}{n}, k \in \{1, 2, ..., n-1\}$

$$\therefore \sum_{k=1}^{n-1}\left(-i\cot\frac{k\pi}{n}\right)^2 = \left(\sum_{k=1}^{n-1}\left(-i\cot\frac{k\pi}{n}\right)\right)^2 - 2\cdot\sum_{1\le p<q\le n-1}\sum\left(-i\cot\frac{p\pi}{n}\right)\left(-i\cot\frac{q\pi}{n}\right) = 0 - 2\cdot\frac{{}^nC_3}{{}^nC_1}$$

$$\Rightarrow -f(n) = \frac{-2n(n-1)(n-2)}{6n} \Rightarrow f(x) = \frac{(n-1)(n-2)}{3}$$

$$\therefore \lim_{n\to\infty}\frac{f(n)}{n^2} = \frac{1}{3}$$

67. (2)

The point B is (2, 1). Image of A(1, 2) in the line x – 2y + 1 = 0 is given by $\frac{x-1}{1} = \frac{y-2}{-2} = \frac{4}{5}$

$\Rightarrow$ Co-ordinates of the image is $\left(\frac{9}{5}, \frac{2}{5}\right)$

(2, 1) and $\left(\frac{9}{5}, \frac{2}{5}\right)$ lie on BC

$\therefore$ Equation of BC is 3x – y – 5 = 0

$\therefore$ a + b = 2.

68. (5)

Both circles cut each other orthogonally. 'C' and 'D' will be centres of two circles also CD will be diameter of circumcircle of quadrilateral ACBD.

Diameter $=\sqrt{3^2+4^2}=\sqrt{25}=5$.

69. (4)

The given equation is $\left(\sqrt{3}\sin x+\cos x\right)^{\sqrt{\sqrt{3}\sin 2x-\cos 2x+2}}=4$

$$\Rightarrow \quad \left[2\sin\left(x+\frac{\pi}{6}\right)\right]^{\sqrt{3\sin^2 x+\cos^2 x+2\sqrt{3}\sin x\cos x}}=4 \quad \Rightarrow \quad \left[2\sin\left(x+\frac{\pi}{6}\right)\right]^{\left|2\sin\left(x+\frac{\pi}{6}\right)\right|}=4$$

Hence $2\sin\left(x+\frac{\pi}{6}\right)=\pm 2 \Rightarrow \sin\left(x+\frac{\pi}{6}\right)=\pm 1$

$$\Rightarrow \; x+\frac{\pi}{6}=2n\pi\pm\frac{\pi}{2}\;;\; x=2n\pi\pm\frac{\pi}{2}-\frac{\pi}{6}$$

$\therefore$ Least positive value of x is $\pi/3$